Phase transitions in compressible lattices

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(Submitted April 22, 1975)
Zh. Eksp. Teor. Fiz. **69**, 1100–1107 (September 1975)

The effect of anisotropic compressibility on a second-order phase transition in a crystal is considered. It is shown that allowance for the anisotropy of the elastic properties leads to a change in the pattern of the critical fluctuations. At finite values of $T-T_c$ the effective interaction between fluctuation oscillations changes sign. A first-order transition occurs.

PACS numbers: 64.70.K

1. INTRODUCTION

In discussions of critical phenomena in a compressible lattice it is usual to refer to the results of Larkin and Pikin[1]. These authors proved that if the phase transition in a lattice with fixed sites is accompanied by an unlimited increase in the specific heat, and the medium is isotropic in its elastic properties, a first-order transition occurs.

Elastic isotropy is extremely important, since only in this case do the Hamiltonians of the interaction of the critical fluctuations have an analogous form in compressible and incompressible lattices. For an anisotropic medium (and any crystal is anisotropic in the sense of elasticity theory), the interaction of critical fluctuations that arises from the exchange of acoustic phonons depends on the angles between the momentum-transfer vector $\mathbf{k}$ and the crystal axes.

The question of the effect of the angular dependences of the interaction on the character of critical phenomena has not been considered previously. Methodologically, this problem reduces to solving parquet-type equations in which the amplitudes depend not only on a logarithmic (slow) variable, but also on the rapid angular variables, and the right-hand sides of the equations contain integrals over the angles. The problem of a rapid parquet has already been discussed in different physical situations, in the literature[2]. In these cases the authors were interested in the stability of the pole solutions. In the present paper equations are derived for the scattering amplitudes of critical oscillations and the stability of the zero-charge solutions is investigated. It is shown that the above-mentioned angular dependences change the character of the solutions, the amplitudes change sign at finite values of $T-T_c$, and the stability condition is violated. The first-order transition which then occurs is explained, generally speaking, by the resultant effect of the Larkin-Pikin mechanism and a mechanism associated with the change of sign of the amplitudes. Possible cases when the Larkin-Pikin effect can be neglected are discussed.

The cases of a phase transition in a four-dimensional model and in a space of dimensions $d = 4-\epsilon$ are considered.

2. HAMILTONIAN WITH INCLUSION OF THE ACOUSTIC PHONONS

When the interaction with the long-wavelength acoustic phonons described by the deformation field $u^\alpha(\mathbf{r})$ is taken into account, the Hamiltonian of a solid near the phase-transition point can be written in the form

$$\mathscr{H}=\mathscr{H}_0+\mathscr{H}_{int};$$

$$\mathscr{H}_0=\frac{1}{2}\sum_{\mathbf{k}}\left[(\omega_0^2+sk^2)\eta_i(\mathbf{k})\eta_i(-\mathbf{k})+\lambda_{\alpha\beta\gamma\delta}u^{\alpha\beta}(\mathbf{k})u^{\gamma\delta}(-\mathbf{k})\right], \quad (1)$$

$$\mathscr{H}_{int}=-\sum_{\mathbf{k},\mathbf{p}}qu^{\alpha\alpha}(\mathbf{k})\eta_i(\mathbf{p})\eta_i(-\mathbf{k}-\mathbf{p})+\sum b\eta_i(\mathbf{k}_1)\eta_i(\mathbf{k}_2)\eta_j(\mathbf{k}_3)\eta_j(\mathbf{k}_4), \quad (2)$$

where the summation over $\mathbf{k}_1, \mathbf{k}_2, \ldots$ in (2) is restricted by the condition $\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4=0$. Here $u^{\alpha\beta}(\mathbf{k})$ denotes the expression

$$u^{\alpha\beta}(\mathbf{k})={}^1/{}_2 i[k^\alpha u^\beta(\mathbf{k})+k^\beta u^\alpha(\mathbf{k})], \quad i, j=1, 2, \ldots, n.$$

In formulas (1) and (2) η denotes the fluctuations of the order parameter, and $u^{\alpha\beta}$ the elastic deformations. The relationship between the critical fluctuations and the elastic deformations is chosen in its simplest form[1].

3. EQUATIONS FOR THE AMPLITUDE IN THE FOUR-DIMENSIONAL MODEL

In this Section we discuss the question of the phase transition in the four-dimensional model. This case is equivalent to the quantum field theory problem with interaction $\lambda\varphi^4$[3]. As is well-known, this problem reduces to the determination of the effective four-point amplitude. The four-point amplitude can be found by summing the sequence of parquet diagrams. The difference between the field-theory problem and the theory of a phase transition in a compressible lattice lies in the fact that, in the latter case, in addition to the contact interaction γ there is also an interaction between the critical fluctuations, which is due to the exchange of an acoustic phonon. In second order of perturbation theory, this interaction is described by the expression

$$\lambda=\lambda_s+\lambda_t+\lambda_u, \qquad \lambda_s=-q^2/a_l(\mathbf{n}), \quad \mathbf{n}=\mathbf{k}/|\mathbf{k}|. \quad (3)$$

Here $\mathbf{k}$ is the momentum transfer in the s-channel, and λ_t and λ_u correspond to the exchange of an acoustic phonon in the t- and u-channels and are constructed analogously to λ_s; $a_l(\mathbf{n})$ is the square of the longitudinal-sound velocity and depends, generally speaking, on the direction of the vector $\mathbf{n}=\mathbf{k}/|\mathbf{k}|$.

The total bare interaction is made up of the contact interaction γ and the interaction (3). Thus, the bare interaction depends on the angles between the vector $\mathbf{k}$ and the crystal axes. This dependence, naturally, will also be preserved in higher orders of perturbation theory. Therefore, the exact amplitude will depend on the angles. The parquet amplitude can be represented in the form

$$\Gamma(\xi,\theta_s,\theta_t,\theta_u)=\gamma-\lambda_s-\lambda_t-\lambda_u+\sum_{i=1}^{3}J_i(\xi,\theta_i), \quad (4)$$

where $J_i(\xi, \theta_i)$ denotes the sum of graphs that are reducible in the i-th channel only (i = s, t, u). It is obvious that J_i depends only on the angles θ_i, the momentum transferred in the i-th channel, and also on the logarithmic variable $\xi = 1/2 \ln [\omega_D^2/(\omega_0^2 + sk^2)]$. It can be seen from expression (4) that the total amplitude can be represented in the form of the sum

$$\Gamma(\xi, \theta_s, \theta_u, \theta_t) = \sum_{i=1}^{3} \Gamma(\xi, \theta_i). \tag{5}$$

The equation for the amplitude $\Gamma(\xi, \theta)$ in differential form is

$$-\frac{\partial}{\partial \xi}\Gamma(\xi, \theta) = n\Gamma^2(\xi, \theta) + 4\Gamma(\xi, \theta)\overline{\Gamma}(\xi) + 4\overline{\Gamma^2}(\xi), \tag{6}$$

$$\Gamma(0, \theta) = \gamma(\theta) = {}^1/_3\gamma - \lambda(\theta). \tag{7}$$

The bar in the right hand side of expression (6) denotes integration over the angular variables. The general properties of the solution of Eq. (6) are found to be connected neither with the concrete form of the angular dependence of the bare vertex $\gamma(\theta)$ nor with the form of the weight functions appearing in the angular integrals.[1]

Equation (6) is a nonlinear integrodifferential equation. Cases when the sum of the parquet series of diagrams is determined by an equation of such a type (a rapid parquet) have also been encountered earlier[2]. The equations considered in these papers contain integral terms of the convolution type. The integral terms of Eq. (6) do not depend on the angles at all. This makes it possible to make a qualitative study of the

4. QUALITATIVE INVESTIGATION OF THE SOLUTIONS

A. First of all, we shall elucidate in which cases an asymptotic (as $\xi \to \infty$) solution independent of the angles is stable. For this we assume that

$$\Gamma(\xi, \theta) = \overline{\Gamma}(\xi) + \delta\Gamma(\xi, \theta), \quad \overline{\delta\Gamma} = 0, \quad |\delta\Gamma| \ll \overline{\Gamma}.$$

In the asymptotic region,

$$\overline{\Gamma}(\xi) = [(n+8)\xi]^{-1}, \quad -\frac{\partial}{\partial \xi}\delta\Gamma = 2(n+2)\delta\Gamma\overline{\Gamma},$$
$$\delta\Gamma \sim [(n+8)\xi]^{-2(n+2)/(n+8)}. \tag{8}$$

For n > 4 the exponent in formula (8) is greater than unity and therefore a solution that does not depend on the angles is stable. For n < 4 such a solution is unstable. In the case of the pole solution $\Gamma \sim (\xi - \xi_0)^{-1}$ the stability is destroyed when n > 4.

B. We shall demonstrate the impossibility of solutions having the asymptotic form $\Gamma(\xi, \theta) = a(\theta)/\xi$. In fact, in this case Eq. (6) is rewritten in the form

$$a(\theta) = na^2(\theta) + 4a(\theta)\bar{a} + 4\overline{a^2}, \tag{9}$$

where $\bar{a}$ and $\overline{a^2}$ do not depend on the angle θ. Therefore, after solving the quadratic equation (9) for $a(\theta)$ we find that $a(\theta)$ = const = $(n + 8)^{-1}$. The problem of the stability of such a regime was discussed in Subsection A.

C. The derivative $\partial\Gamma(\xi, \theta)/\partial\xi < 0$ for $\xi < \infty$. In fact,

$$-\frac{\partial}{\partial \xi}\Gamma(\xi, \theta) = n\Gamma^2(\xi, \theta) + 4\Gamma(\xi, \theta)\overline{\Gamma}(\xi) + 4\overline{\Gamma^2}(\xi) \geqslant \Gamma^2(\xi, \theta)$$
$$+ 4\Gamma(\xi, \theta)\overline{\Gamma}(\xi) + 4\overline{\Gamma}^2(\xi) = [\Gamma(\xi, \theta) + 2\overline{\Gamma}(\xi)]^2 \geqslant 0. \tag{10}$$

The equality in expression (10) is possible if n = 1 and $\Gamma(\xi_0, \theta) \equiv 0$. But then, by virtue of Eq. (6), we have $\Gamma(\xi, \theta) \equiv 0$ for all ξ.

D. We shall assume that there exists a continuous solution of Eq. (6). It is easy to see that such a solution is majorized by the solution of the equation $y' = -y^2$, if $y(0, \theta) = \gamma(\theta)$. Thus, $\Gamma(\xi, \theta) < \xi^{-1}$. We shall consider the quantity $\Delta(\xi)$ corresponding to the width of the bunch of integral curves of Eq. (6): $\Delta(\xi) = \Gamma_{max} - \Gamma_{min}$. The equation for $\Delta(\xi)$ has the form

$$-\Delta'(\xi) = \Delta(\xi)[n\Gamma_{max}(\xi) + n\Gamma_{min}(\xi) + 4\overline{\Gamma}(\xi)]. \tag{11}$$

From this it follows that

$$\Delta(\xi) = \Delta(0)\exp\left\{-\int_0^{\xi} d\zeta\,[n\Gamma_{max}(\zeta) + n\Gamma_{min}(\zeta) + 4\overline{\Gamma}(\zeta)]\right\}. \tag{12}$$

If we assume that $\Gamma(\xi)$ falls off more rapidly than ξ^{-1}, the integral in the right-hand side in (12) increases more slowly than $\ln \xi$ and, consequently $\Delta(\xi)$ falls off more slowly than any power of ξ which contradicts the majorizability.

Thus, the continuous solution of Eq. (6) has the asymptotic form

$$\Gamma(\xi, \theta) \cong \varphi(\xi, \theta)/\xi, \tag{13}$$

where φ is a bounded function. It can be shown that the only function $\varphi(\xi, \theta)$ that does not violate the condition that $\Gamma(\xi, \theta)$ be monotonic is a constant: $\varphi(\xi, \theta)$ = C. Therefore, if the solution $\Gamma = C/\xi$ is unstable (n < 4) this means that no solution of Eq. (6) that is continuous for all ξ exists at all.

Consequently, for n < 4 there is a singularity at finite $\xi = \xi_0$, and the solution cannot be continued into the region $\xi > \xi_0$. If all the values of $\Gamma(\xi_0, \theta)$ were finite, they could be used as initial conditions for Eq. (6) and the solution could thus be continued into the region $\xi > \xi_0$. Therefore, the only possibility of satisfying the properties that the solution be monotonic and not continuable is that $\Gamma_{min}(\xi) \to -\infty$ as $\xi \to \xi_0$.

E. The possible situations are a standing or a running pole[2]. A standing pole corresponds to a solution of the form $\Gamma(\xi, \theta) = a(\theta)/(\xi - \xi_0)$. By a method analogous to that used in Subsection B, it can be shown that $a(\theta)$ = const. As we have seen, for n < 4 such a solution is stable. In the case of a running pole, as shown in Appendix 1, $\overline{\Gamma}$ falls off more slowly than Γ_{min}, although, in this case too, $\overline{\Gamma} \to -\infty$ as $\xi \to \xi_0$. In Appendix 1 it is also shown that as $\xi \to \xi_0$ the quantity $\Gamma_{max} \to -\infty$. Thus, in the neighborhood of the singularity, $\Gamma(\xi, \theta) < 0$ for all θ.

In Appendix 2, as an illustration, Eq. (6) is solved exactly for n = 0. In this case a standing pole is realized, with residue independent of the angular variable.

5. THE PHASE TRANSITION IN THE ϵ-APPROXIMATION

For most transitions at a finite temperature, a power rather than a logarithmic divergence of the terms in the perturbation-theory series occurs. In this case there is no zero-charge and the interaction at large distances tends to a finite limit Γ_0. We shall study the stability of such a solution against perturbations that depend on the angular variables. For this we write out the equation for the amplitude in the ϵ-approximation[4]:

$$\partial\Gamma(\xi, \theta)/\partial\xi = \varepsilon\Gamma(\xi, \theta) - n\Gamma^2(\xi, 0) - 4\Gamma(\xi, \theta)\overline{\Gamma}(\xi) - 4\overline{\Gamma^2}(\xi). \tag{14}$$

The right-hand side of Eq. (14) vanishes when $\Gamma(\xi, \theta)$

$= \epsilon/(n + 8)$. The linearized equation for $g(\xi, \theta) = \Gamma(\xi, \theta) - \epsilon/(n + 8)$ has the form

$$\frac{\partial}{\partial \xi} g(\xi, \theta) = \frac{\varepsilon}{n+8} [(4-n) g(\xi, \theta) - 12\bar{g}(\xi)]. \quad (15)$$

Decomposing $g(\xi, \theta) = \bar{g}(\xi) + \delta g(\xi, \theta)$, $\overline{\delta g(\xi)} = 0$, we obtain

$$\frac{\partial}{\partial \xi}\bar{g} = -\varepsilon \bar{g}, \quad \bar{g} = \bar{g}(0) e^{-\varepsilon \xi},$$
$$\frac{\partial}{\partial \xi}\delta g = \frac{4-n}{n+8}\varepsilon \delta g, \quad \delta g = \delta g(0) \exp\left\{\frac{4-n}{n+8}\varepsilon \xi\right\}. \quad (16)$$

It can be seen from formulas (16) that the solution $\Gamma = \epsilon/(n + 8)$ is unstable against perturbations depending on the angles, for $n < 4$. In the initial stage the perturbation grows rapidly, until it becomes of order ϵ. After this, the solution of Eq. (14), most probably, behaves analogously to the solution of the parquet equation (6). In Appendix 2 the solution of Eq. (14) for $n = 0$ is investigated. From this study it can be seen that $\Gamma(\xi, \theta)$ changes sign at finite ξ.

Thus, it is evident that for all transitions in a compressible lattice the amplitude changes sign at finite values of $T - T_c$ and the stability condition is violated. Such a situation has often been discussed recently[5]. As has been shown by the authors of these papers, the change of sign of the amplitude is accompanied by a first-order transition.

6. ESTIMATES AND CONCLUSIONS

If the strictional interaction q is sufficiently large, the first-order phase transition occurs in the region of applicability of the Landau theory. But if q is sufficiently small ($\gamma \gg \lambda$), then, in the initial stage, the solution of the parquet equations is close to the zero-charge solution $\Gamma \cong [(n + 8)\xi]^{-1}$, and the width of the bunch of integral curves is $\Delta \cong \lambda[(n + 8)\gamma\xi]^{-2(n+2)/(n+8)}$. At $\xi \sim \xi^*$ the value $\Delta(\xi^*)$ becomes of the order of $\overline{\Gamma}(\xi^*)$ and the amplitudes Γ change sign. Comparing $\Delta(\xi^*)$ and $\overline{\Gamma}(\xi^*)$, we obtain the estimate

$$\xi^* \sim [(n+8)\gamma]^{2(n+2)/(n+8)} \lambda^{-(n+8)/(4-n)}. \quad (17)$$

The Larkin-Pikin effect becomes important if[1]

$$\frac{1}{T_c}\frac{4\mu K_0}{3K_0+4\mu}\left(\frac{\partial T_c}{\partial p}\right)^2 C \gg 1, \quad (18)$$

where K_0 and μ are the bulk and shear moduli and C is the specific heat. Using the estimates (17), (18) and the formula for the specific heat from [3], we can see that the region in which the amplitudes change sign coincides with the region of the Larkin-Pikin effect. The point here is that the exponent of the growth of the relative width $\Delta/\overline{\Gamma}$ of the bunch of integral curves in the leading logarithmic approximation coincides with the specific-heat index. In fact, the right-hand side of the equation for the amplitude J_i that is reducible in the i-th channel only (formula (4)) contains terms of three types: first, the term J_i^2, secondly, the product $J_j J_k$ $(j,k \neq i)$, and thirdly, the cross terms $J_i J_j$ $(j \neq i)$.

Let each such term appear with coefficients A, D and B, respectively. Then the integro-differential equation is written in the form

$$-\frac{\partial}{\partial \xi}\Gamma(\xi, \theta) = A\Gamma^2(\xi, \theta) + 4B\Gamma(\xi, \theta)\overline{\Gamma}(\xi) + 4D\overline{\Gamma}^2(\xi). \quad (19)$$

Equation (6) is a particular case of Eq. (19) ($A = n$, $B = D = 1$). In the region $\Delta \ll \overline{\Gamma}$,

$$\overline{\Gamma} \cong [(A+4B+4D)\xi]^{-1},$$
$$\Delta \sim \xi^{-2(A+2B)/(A+4B+4D)}, \quad (20)$$
$$\Delta/\overline{\Gamma} \sim \xi^{1-2(A+2B)/(A+4B+4D)}.$$

To estimate the specific heat it is necessary to calculate the three-point amplitude $\mathcal{T} = \partial G^{-1}/\partial \omega_0^2$. The equation for $\mathcal{T}$ has the form

$$-\frac{\partial}{\partial \xi}\mathcal{T} = (A+2B)\mathcal{T}\Gamma.$$

Hence, for the vertex $\mathcal{T}$ and specific heat C we obtain the estimates:

$$\mathcal{T} \sim \xi^{-(A+2B)/(A+4B+4D)}, \quad C \sim \xi^{1-2(A+2B)/(A+4B+4D)}. \quad (21)$$

Thus, for all the parquet equations, the relative width $\Delta/\overline{\Gamma}$ of the bunch of integral curves of Eq. (19) grows like the specific heat C. Therefore, in a significant proportion of cases, the first-order phase transition is explained by the resultant action of the Larkin-Pikin mechanism and the mechanism associated with the change of sign of the amplitude. To take these effects into account consistently, estimates are inadequate and it is necessary to calculate $\Gamma(\xi, \theta)$ and all the thermodynamic functions, which can only be done by numerical methods.

However, there exist cases when the Larkin-Pikin effect can be neglected. For a transition at a finite temperature, if the anisotropy of the elastic moduli is not too small, the Larkin-Pikin effect can be neglected in those cases where the specific heat contains a small coefficient. Such a situation arises for transitions of the displacement type, and also for the Ising model with interaction of long range. In all these cases, $\gamma \ll 1$.

The authors are grateful to A. I. Larkin for a valuable comment.

APPENDIX 1

In the neighborhood of a running pole the amplitude $\Gamma(\xi, \theta)$ A can be written in the form

$$\Gamma(\xi, \theta) = \frac{a(\theta)}{\xi_0(\theta) - \xi} + R(\xi, \theta), \quad (A.1.1)$$

where $R(\xi, \theta)$ is a function with a singularity, weaker than a pole, at the point $\xi_0 = \min \xi_0(\theta)$. Using the representation (A.1.1) and Eq. (6), we obtain

$$a(\theta) = -1/n,$$
$$-\frac{\partial R}{\partial \xi} = -\frac{2R}{\xi_0(\theta) - \xi} + R^2 + 4\overline{\Gamma}\left[R - \frac{1}{n[\xi_0(\theta) - \xi]}\right] + 4\overline{\Gamma}^2. \quad (A.1.2)$$

Depending on the weight with which the angular integration is performed, different situations are possible. If $\overline{\Gamma}$ is a bounded function for $\xi \leq \xi_0$, it follows from (A.1.2) that

$$R = -\frac{2}{n}\overline{\Gamma}, \quad -\frac{\partial R}{\partial \xi} = R^2 + 4\overline{\Gamma}R + 4\overline{\Gamma}^2. \quad (A.1.3)$$

It is easy to convince oneself that the relations (A.1.3) are inconsistent with the original equation (6). Consequently, as $\xi \to \xi_0$ the amplitude $\overline{\Gamma} \to -\infty$. In view of the fact that $\overline{\Gamma}$ is monotonic, its sign can be established. It follows from Eq. (6) that

$$\overline{\Gamma}(\xi) = \bar{\gamma} - \int_0^\xi [4\overline{\Gamma}^2(\zeta) + (n+4)\overline{\Gamma^2}(\zeta)] d\zeta. \quad (A.1.4)$$

The integral in the right-hand side of (A.1.4), as we have just proved, diverges as $\xi \to \xi_0$. Therefore, the integral of the second term of the integrand diverges.

Taking this circumstance into account, and also the fact that

$$\Gamma_{max}=\gamma_{max}-\int_0^{\xi}[n\Gamma_{max}^2+4\bar{\Gamma}\Gamma_{max}+4\bar{\Gamma}^2]d\zeta,$$

we can conclude that $\Gamma_{max}\to-\infty$ as $\xi\to\xi_0$. Thus, as $\xi\to\xi_0$ we have $\Gamma(\xi,\theta)\to-\infty$ for all θ.

APPENDIX 2

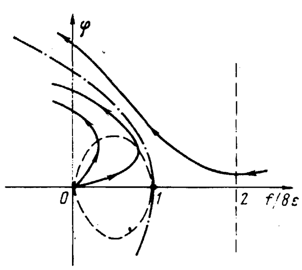
We shall study the properties of Eqs. (6) and (14) for $n = 0$:

$$\frac{\partial}{\partial\xi}y=\varepsilon y-4\overline{y^2}-4y\bar{y}. \qquad (A.2.1)$$

The solution of this equation can be sought in the form $y(\xi,\theta) = f(\xi) + a(\theta)\varphi(\xi)$, $\bar{a} = 0$. The functions f and φ obey the system of equations

$$\varphi'=(\varepsilon-4f)\varphi,\quad f'=\varepsilon f-8f^2-4\overline{a^2}\varphi^2. \qquad (A.2.2)$$

The singular points of the system (A.2.2) are the points $A_1(0.0)$—the center of instability, and $A_2(\epsilon/8.0)$—the saddle point.



As can be seen from the Figure, in which the trajectories of the solutions of the system (A.2.2) are shown, all these trajectories move away into the left half-plane for finite values of ξ.

In the case $\epsilon = 0$, Eq. (A.2.1) can be reduced to quadratures. In fact, in this case the equations of the system (A.2.1) are homogeneous and if we seek the solution in the form $\varphi = f\psi(f)$ the equation for ψ has the form

$$f\frac{d\psi}{df}=\frac{\psi(1+\overline{a^2}\psi^2)}{2+\overline{a^2}\psi}. \qquad (A.2.3)$$

Solving this equation, we obtain

$$f=\pm C_1(1+\overline{a^2}\psi^2)^{1/2}/\psi^2. \qquad (A.2.4)$$

Using Eq. (A.2.2) with $\epsilon = 0$, and also (A.2.3) and (A.2.4), we can obtain the relation

$$\frac{C_1\xi}{4}=\int\frac{\psi d\psi}{(1+\overline{a^2}\psi^2)^{3/2}}. \qquad (A.2.5)$$

The integral in the right-hand side of the relation (A.2.5) converges. Therefore, at finite values $\xi = \xi^*$ the quantity ψ passes through infinity and changes sign. After this ψ increases and vanishes at $\xi = \xi_0$. From the expression (A.2.4) we can conclude that $f(\xi)$ changes sign at $\xi = \xi^*$, and, as $\xi\to\xi_0$, $f(\xi)\sim\psi^{-2}\sim(\xi-\xi_0)^{-1}$. For $\xi\to\xi_0$ the quantity $\varphi(\xi)$ is proportional to $\varphi(\xi) = f\psi\sim\psi^{-1}\sim(\xi_0-\xi)^{-1/2}$. Thus, for $\epsilon = 0$ the solution of Eq. (A.2.1) changes sign at finite values of ξ and moves out into the pole regime. In the neighborhood of the pole the relative width of the bunch of integral curves $y(\theta,\xi)$ tends to zero: $\varphi/|f|\sim(\xi_0-\xi)^{1/2}$.

1) Strictly speaking, the equations obtained are true if the Green function G(k) is not renormalized in the leading logarithmic approximation. In an incompressible lattice this property is a consequence of the contact character of the interaction [3]. In a compressible lattice, and with allowance for the anisotropy, corrections to G can arise in the leading logarithmic approximation and can depend on the directions **k**. However, as can be shown, for the simplest Hamiltonian (1), (2) and anisotropy of the cubic type the Green function is not renormalized. For simplicity, everywhere below we study precisely this case. Such a theory can be applied quite concretely to uniaxial ferroelectrics.

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Translated by P. J. Shepherd
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