Theta pinch in electron-hole plasma under skin-effect conditions

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The equations of a θ pinch in semiconductors are integrated with a computer for the case when the dimension of the skin layer is smaller than the sample radius. In the case of a strong skin effect, solitary waves of the density and of the magnetic fields are produced in the plasma and move towards the center of the crystal. The position of the front of these waves and the conditions of maximum plasma compression are determined. It is shown that the spatial distribution of the plasma when the θ pinch is concentrated in the skin layer depends strongly on the waveform of the magnetic-field pulse and on the ratio of the characteristic times (of ambipolar diffusion, volume recombination, pulse duration, diffusion of the magnetic field). In the case when the external magnetic field is harmonic, the θ pinch has a number of interesting features due to the trapping of the magnetic field by the plasma and to the formation of a neutral layer (due to cancellation of the fields). During definite stages of the pulse there is produced in this case a magnetic-field wave with a very steep front, and this leads to an abrupt increase of the concentration effect. The results of the developed theory are compared with the experimental data.

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1. The production of a nonequilibrium high-concentration plasma in a semiconductor is of great scientific and technical interest. One of the contactless methods of producing such a plasma is the θ pinch, wherein the intrinsic plasma of the sample is constricted towards the axis under the influence of a longitudinal magnetic field that increases with time.[1] Since the carrier momentum relaxation time in semiconductors is very short (10^{-11}–10^{-12} sec), the compression of the plasma in the θ pinch is the result of ambipolar drift of the electrons and holes in the longitudinal magnetic field H and the azimuthal electric field E_θ induced by it. The character of the compression depends strongly on the ratio of the dimensions δ and R of the skin layer and of the sample in the compression direction. If $\delta \gg R$, then the longitudinal magnetic field can be regarded as constant along the sample cross section during the course of the pulse (unskinned θ pinch). In the opposite case, the magnetic field is highly inhomogeneous (skinned θ pinch).

The theory of the unskinned θ pinch in semiconductors was constructed in[2]. In this case the maximum compression of the plasma is reached near the sample axis and is determined by the amplitude of the magnetic field if the pulse duration τ is much shorter than the volume recombination time τ_r and the ambipolar-diffusion time $\tau_D = R^2/D$:

$$N \approx h \quad (h \gg 1),$$

where $N = n/n_p$; n_p is the equilibrium plasma concentration; $h = (b_e/b_h)^{1/2} H/c$; $b_{i=e,h}$ are the mobilities of the electrons and holes; D is the coefficient of ambipolar diffusion.

If $\tau > \tau_r$, then the compression effect attenuates exponentially ($\sim \exp\{-t/\tau_r\}$). In the region of the maximum plasma compression, the spatial distribution takes the form of a plateau, the dimension of which is $\sim 1/h^{1/2}$ in the case of cylindrical geometry. Experiments[3] performed on germanium ($T = 300°K$, $n_p \approx 2 \times 10^{13}\ cm^{-3}$, conductivity close to intrinsic), confirmed the main conclusions of the theory of the unskinned θ pinch ($\delta/R \approx 100$ in these experiments).

The first investigations of the θ pinch in InSb samples ($T = 300°K$, $n_p \approx 2 \times 10^{16}\ cm^{-3}$) were performed somewhat earlier by Hübner and co-workers.[4] Under the conditions of these experiments, the skin-layer thickness was commensurate with the radius of the sample, so that the measurement results differed greatly from those given in[3]. Thus, maximum plasma compression was reached at a certain distance from the sample axis, a fact due entirely to the skin character of the θ pinch in these experiments. Unfortunately, the theoretical calculations of Bruhns and Hübner[5] for the case when $\Delta n \ll n_p$ (very small compression) cannot explain the main regularities of the skinned θ pinch in intrinsic semiconductors. It is this problem to which this paper is devoted.

2. We consider the case of a degenerate plasma. The initial equations are the equations of motion for the electrons and holes, the continuity equation, and Maxwell's equations:

$$\mathbf{v}_{i=e,h} = -\frac{D_i}{n}\nabla n \mp b_i E \mp \frac{b_i}{c}[\mathbf{v}_i \times \mathbf{H}], \qquad (1)$$

$$\frac{\partial n}{\partial t} + \operatorname{div}(n\mathbf{v}_i) = -\frac{n-n_p}{\tau_p}, \qquad (2)$$

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \qquad (3)$$

$$\operatorname{rot}\mathbf{H} = \frac{4\pi}{c} en(\mathbf{v}_h - \mathbf{v}_e), \qquad (4)$$

where $\mathbf{v}_i$ and D_i are the velocities and the diffusion coefficients of the electrons and holes.

It is assumed that all the quantities depend only on the radius (cylindrical geometry). With the aid of (1) and (4) it is easy to obtain an expression for the velocity of the ambipolar drift:

$$v_r = -\frac{D}{n}\frac{\partial n}{\partial r} - \frac{c^2}{4\pi\sigma} h \frac{\partial h}{\partial r}, \qquad (5)$$

where $\sigma = e(b_e + b_h)n$.

Using (5) and (1)–(4) we can obtain the initial system of equations for the plasma density and the magnetic field:

$$\frac{\partial N}{\partial T} = \frac{\beta^2}{\rho}\frac{\partial}{\partial \rho}\left(\rho h \frac{\partial h}{\partial \rho}\right) + \frac{1}{\tilde{\tau}_D \rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial N}{\partial \rho}\right) - \frac{N-1}{\vartheta}, \qquad (6)$$

$$\frac{\partial h}{\partial T} = \frac{\beta^2}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{1+h^2}{N}\frac{\partial h}{\partial \rho}\right) + \frac{1}{\tilde{\tau}_D \rho}\frac{\partial}{\partial \rho}\left(\rho \frac{h}{N}\frac{\partial N}{\partial \rho}\right), \qquad (7)$$

$$\rho = r/R,\quad T = t/\tau,\quad \tilde{\tau}_D = \tau_D/\tau,\quad \vartheta = \tau_p/\tau,$$
$$\beta^2 = c^2\tau/4\pi\sigma_p R^2 = \delta^2/R^2 = \tau/\tau_M.$$

Here δ and τ_M are parameters with the meaning of the thickness of the skin layer and magnetic-field diffusion coefficient. There is no doubt that the true values of these quantities vary during the course of plasma compression as the local conductivity and magnetization of the plasma change: $\tau_M \sim N/(1 + h^2)$. We note that in the case of a harmonic pulse ($H = H_0 \sin \omega t$) the parameter τ corresponds to the period of the oscillations.

The initial and boundary conditions needed for the solutions take the form

$$N(0,\rho) = \begin{cases} 1, & 0 \leqslant \rho \leqslant 1 \\ 0, & \rho > 1 \end{cases},\quad h(0,\rho) = 0,$$
$$\frac{\partial N}{\partial \rho} = \frac{\partial h}{\partial \rho}\bigg|_{\rho=0} = 0,\quad \int_0^1 N\rho\, d\rho = 0.5 \tag{8}$$

(the condition for the conservation of the number of particles upon compression), and

$$h(T, \rho = 1) = h_0(T), \tag{9}$$

where $h_0(T)$ is the vacuum magnetic field.

In the case of linear volume recombination, the condition for the conservation of the number of particles in the course of the pinch is valid if the rate of surface recombination—generation (s) is small, $s\tau/R \ll 1$.

In the case $\beta^2 \gg 1$ the solutions of Eqs. (6)—(9) coincide with those given earlier[2] (θ pinch without skin effect). In the case of interest to us ($\beta \lesssim 1$), however, it is impossible to obtain analytic solutions, and we shall henceforth present the results of numerical integration of Eqs. (6)—(9) for different waveforms of the vacuum magnetic-field pulse and different values of the parameters in (6) and (7). Certain analytic estimates can nevertheless be obtained even in this case.

3. It is obvious that in the case of strong skinning ($\beta < 1$) there are produced in an electron-hole plasma, during the course of the θ pinch solitary waves of the density and of the magnetic field, with a narrow front, moving towards the center of the crystal (the spatial distribution of the plasma has a tubular character). If we neglect diffusion ($\tilde{\tau}_D \gg 1$), then we can use (5) to estimate the position $a(T)$ of the front of these waves:

$$\int_G N v_\rho \rho\, d\rho = U_G \dot{a} = -\frac{\beta^2}{2}\int_G \frac{\partial h^2}{\partial \rho}\rho\, d\rho, \tag{10}$$

where the integration is over the region of strong compression of the plasma (G), and $U_0 \approx (1 - a^2)/2$ is the number of particles in the density wave.

Recognizing that $N = 1$ and $h \approx 0$ ahead of the front and $N \approx 0$ and $h = h_0$ behind the front we can show with the aid of (10) that the position of the wave front is determined by the transcendental equation

$$-\ln a + \frac{a^2 - 1}{2} = \beta^2 \int_0^T h_0^2(T)\, dT. \tag{11}$$

In the case of planar geometry we have

$$a(T) = 1 - \beta\left[\int_0^T h_0^2(T)\, dT\right]^{1/2}.$$

These estimates are valid if the depth of diffusion of the magnetic field is small enough, $a(T) > \beta T^{1/2}$. In the opposite case the magnetic field ahead of the wave front cannot be regarded as small.

Let us examine the dependence of $a(T)$ on the waveform of the magnetic-field pulse, using the relations obtained above. It can be shown that at a specified magnetic-field amplitude (h_A) and a specified pulse duration the density wave and the magnetic-field wave will come closest to the sample axis at the instant of time T in the case when $h_0 = h_A T^{1/2}$; the position of the front of these waves will be farthest from the center at $h_0 = h_A T^2$, and an intermediate position $a(T)$ corresponds to a linear pulse $h_0 = h_A T$.

On the other hand, if h_A is varied at fixed τ and at a given pulse waveform, then $a(h_0, h_{A1}) < a(h_0, h_{A2})$ at $h_{A1} < h_{A2}$. The reason is that at lower values of the magnetic-field amplitude a certain value of the field h_0 is reached after a longer time. Therefore the density wave and the magnetic-field wave have time to come closer to the sample axis. We note that in the case of a smooth magnetic-field pulse waveform the condition of strong plasma constriction, just as in a θ pinch in the absence of the skin effect, is determined according to (11) by the criterion $h_0^2 \gg 1$.

After the wave front has come sufficiently close to the sample center, the skin-effect stage of the compression terminates and Eq. (11) no longer holds. The new compression stage (which we shall call intermediate) is characterized by a jumplike growth of the magnetic field near the axis, wherein $h(0, T)$ becomes equal to the vacuum value $h_0(T)$ within a very short time interval. This is connected both with the decrease of the plasma dimension and with the increase of the diffusion coefficient of the magnetic field ($\sim h^2$). After the intermediate stage there sets in a regime of a θ pinch without the skin effect.[2] If at the end of the intermediate compression stage ($T = T_{int}$) the density profile takes the form $N_{int}(\rho, T_{int})$, then in the succeeding instants of time ($T > T_{int}$), according to the theory of unskinned θ pinch

$$N(\rho, T > T_{int}) = \left[\frac{1 + h_0^2(T)}{1 + h_0^2(T_{int})}\right]^{1/2} N_{int}\left\{\rho\left[\frac{1 + h_0^2(T)}{1 + h_0^2(T_{int})}\right]^{1/4}, T_{int}\right\}. \tag{12}$$

We note that in the foregoing estimates we did not take into account effects due to ambipolar diffusion: $a^2(1 + h_0^2)/D_T \gg 1$.

Figures 1 and 2 show the results of the numerical integration of Eqs. (6)—(9) for various magnetic-field

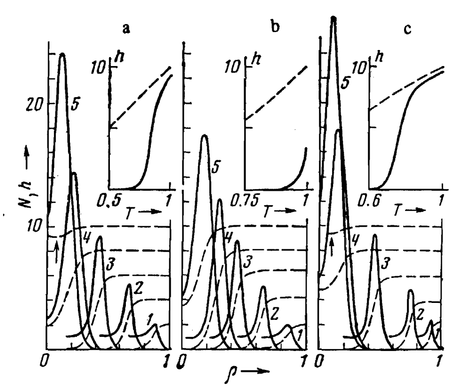


FIG. 1. Spatial distribution of the density (solid lines) and of the magnetic field (dashed) at various instants of time; inserts—time dependences of the magnetic field on the sample axis (solid lines—h, dashed—vacuum field h_0): $\beta = 1/4$, $\tilde{\tau}_D = 100$, $\vartheta = \infty$; the ratio h_0/h_A is equal to T (a), T^2 (b), or $T^{1/2}$ (c); $h_A = 10$; curves 1—5 are for T equal to: (a) 0.2; 0.4; 0.6; 0.8; 1; (b) 0.45; 0.65; 0.8; 0.9; 1; (c) 0.05; 0.45; 0.35; 0.65; 1.

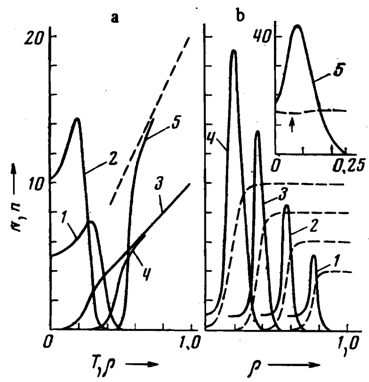


FIG. 2

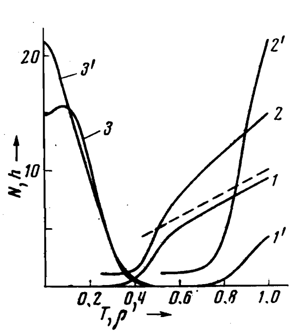


FIG. 3

FIG. 2. Spatial distribution of the density and of the field, and time dependences of the field on the sample axis ($h_0 = h_A T, \tilde{\tau}_D = 100, \vartheta = \infty$): (a) 1, 2–N(ρ, T) at T = 0.5; 1; β = 1, h_A = 10. 3, 4, 5–h(0, T) at β = 1; 0.5; 0.25 and respectively h_A = 10; 20. (b) h_A = 20, β = 0.25; curves 1–5–N(ρ, T), h(ρ, T) and T = 0.2; 0.3; 0.4; 0.5; 0.75.

FIG. 3. Plots of the magnetic field (1, 1′) and of the concentration on the sample axis (2, 2′) against the time; spatial distributions of the density (3, 3′–T = 1): $h_0 = h_A T$, $h_A = 10$, $\tau_D = 10$, $\vartheta = \infty$, β = 0.5 (curves 1, 2, 4) and β = 0.25 (curves 1′, 2′, 3′).

pulse waveforms, for various values of β and various amplitudes of the magnetic field in the case when the volume recombination can be neglected ($\vartheta \to \infty$). The parameter $\tilde{\tau}_D$ was assumed equal to 100. These calculations clearly illustrate the estimates given above. As seen from Fig. 1, at a given magnetic-field amplitude and pulse duration, the wave front velocity (the position of the front corresponds to the maximum of the concentration) is largest in the case $h_0 = h_A T^{1/2}$ and is smallest at $h_0 \sim T^2$. The front position agrees well with the calculated one (11). Naturally, in the case $h_0 \sim T^{1/2}$ the start of the intermediate constriction regime sets in earlier than at $h_0 \sim T$ or $h_0 \sim T^2$ (Fig. 1, see the time dependences of the magnetic field h at the center of the sample). With the aid of Figs. 1a and 2b it is possible to trace also the already mentioned variation of $a(h_0, h_A)$.

It is easy to show that near the sample axis

$$N(0, T) \approx [1 + h^2(0, T)]^{1/2} \tag{13}$$

and $N(0, T) \approx h_0$ at $T > T_{int}$ in accordance with formula (1) of the theory of unskinned θ pinch. These relations agree well with the results given in Figs. 1 and 2.

4. It can be shown that for all the distributions shown in Figs. 1 and 2 the condition $a^2(1 + h_0^2)/D_T \gg 1$ is satisfied. If this condition is violated in the θ pinch, then at sufficiently small β the spatial profiles of the plasma are bell-shaped towards the end of the pulse. The reason is that the ambipolar diffusion contributes to the increase of the plasma density in the axial region prior to the intermediate stage of the pinch, before the field penetrates to the center. If $\tilde{N}(0, T_{int}) > 1$ at the start of this stage, then $N(0, 1) \approx h_0 \tilde{N}(0, T_{int})$ at the end of the pulse, so that in the region next to the axis the plasma concentration may turn out to be larger than at the other points. It is important here that the jump of the magnetic field near the axis start at an instant when the plasma density in this region becomes large enough. Therefore such a singularity of the solutions will arise at small values of β.

Calculations illustrating this singularity were carried out by us for the case $h_0 \sim T$, $\tilde{\tau}_D = 10$, $\vartheta = \infty$ (Fig. 3). As seen from this figure, at $\beta = 1/2$ the intermediate stage of the pinch sets in before the process of ambipolar diffusion leads to a noticeable accumulation of the plasma near the axis, and the tubular character of the distribution is preserved. If $\beta = 1/4$, then the diffusion processes begin to play a noticeable role prior to the onset of the intermediate constriction stage, and this the reason why a bell-shaped profile is produced by the instant T = 0: $N(0, 1) \approx 21$.

Notice must be taken of one more rather interesting singularity of the solution for the magnetic field. This singularity is connected with the allowance for ambipolar diffusion. At small values of β, when $T > T_{int}$, the magnetic field towards the end of the pulse is not a monotonic function of the radius (Fig. 1), and goes through a minimum (marked by the arrow in Fig. 1). This singularity should arise under conditions of pronounced tubular plasma distribution. Indeed, from (7) it follows that at $T > T_{int}$

$$\beta^2 \frac{\partial h}{\partial \rho} = \frac{\rho N}{2(1+h_0^2)} \frac{dh_0}{dT} - \frac{h_0}{\tilde{\tau}_D (1+h_0^2)} \frac{\partial N}{\partial \rho}. \tag{14}$$

At small β the quantity $\partial N/\partial \rho > 0$ can become sufficiently large in the region $\rho \gtrsim 0$ to make the right-hand side of (14) negative. In this case the diffusion component of the azimuthal current prevails in the region near the axis over the drift component. Naturally, this singularity of the solution arises at certain intermediate values of $\tilde{\tau}_D$ when, on the one hand, the tubular profile is sufficiently strongly pronounced ($\tilde{\tau}_D$ is large), but on the other hand the role of ambipolar diffusion is appreciable (small $\tilde{\tau}_D$).

5. It should be noted that in the case of a strong skin effect the volume recombination may not affect the character of the plasma constriction even at $\vartheta \gtrsim 1$, in contrast to the unskinned θ pinch.[2] This is due to the onset of the intermediate stage of plasma constriction ($\beta \lesssim 1$), when the field on the sample axis increases to its vacuum value within a time $\tau' \ll \tau_r$. The maximum plasma constriction near the sample axis ($N \approx h_0$) will be reached in this case at values of the parameter β such that the termination of the intermediate stage corresponds to the instant of time $T \approx 1$ (end of the pulse). The corresponding results of the numerical integration are shown in Fig. 4. Naturally, the constraining role of the volume recombination will be weaker when the pulse is steeper, for in this case the start of the intermediate stage is delayed.

6. Let us note some interesting features of the

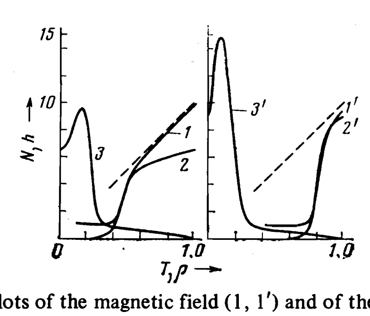


FIG. 4. Plots of the magnetic field (1, 1′) and of the concentration (2, 2′) on the sample axis against the time; spatial density distribution (3, 3′–T = 1): $h_0 = h_A T$, $h_A = 10$, $\tau_D = 100$, $\vartheta = 1$; β = 0.5 (1, 2, 3) and β = 0.25 (1′, 2′, 3′).

skinned θ pinch when the external magnetic field varies harmonically, $h_0 = h_A \sin 2\pi T$. In this case the condition $h_0 \gg 1$ for strong constriction of the plasma, which is necessary both for an unskinned and a skinned θ pinch (in the case of a smooth magnetic-field pulse), is not obligatory. Figures 5a and 5b show the results of the numerical integration ($h_A = 3$, $\beta = 1$, $\tilde{\tau}_D = 100$, $\vartheta \to \infty$), which illustrate this premise clearly. During the first quarter of the period the plasma is pressed towards the sample axis and the density profile at the instant $T = 1/4$ takes the form shown in Fig. 5a. During the second quarter of the period the plasma is pressed towards the surface of the sample. However, at the end of the first half-period, when $h_0 = 0$, the magnetic field in the plasma is $h(\rho < 1) > 0$ (Fig. b) as a result of the skin effect (the plasma captures the field). Therefore at that instant the plasma does not return to the initial distribution ($N = 1$), but is concentrated in the region $\rho < 1$ (Fig. 5a) with an average density $\overline{N} \approx 2$. Beginning with the third quarter of the period, a new constriction stage sets in, but the course of the process is appreciably altered, namely, the field in the central region is opposite in sign to the external field (Fig. 5b, curves 11, 12, 13). As a result the diffusion of the external field into the region near the axis is hindered (mutual cancellation of the fields takes place). This leads to formation of a magnetic wave with a very steep front (Fig. 5b, curves 12 and 13), and at the initial stage of its formation the particles in the plasma start to move in the opposite direction in the region where $h \approx 0$ (neutral layer), and the plasma concentration at these points increases sharply (Fig. 5a). Therefore at the instant of termination of the third quarter of the period the plasma is concentrated in a much narrower region than during the first constriction stage. This is followed again by pressure towards the surface, but at $T = 1$ the dimension of the plasma pinch is much smaller than at $T = 0.5$. During the course of the new constriction stage, even greater densities are attained, etc. Thus, at $T = 1.75$ the plasma concentration at the axis reaches $N = 100$ (Fig. 5a).

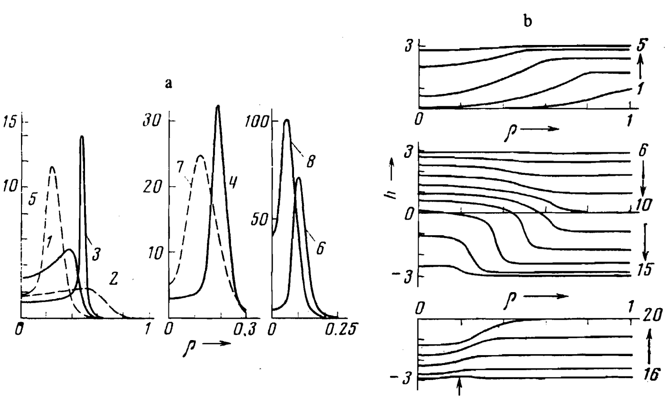
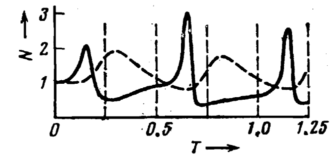


FIG. 5. Spatial distributions of the density (a) and of the magnetic field (b) at various instants of time: $h_0 = h_A \sin 2\pi T$, $h_A = 3$, $\tilde{\tau}_D = 100$, $\beta = 1$, $\vartheta = \infty$; a) curves 1–8, T = 0.25; 0.5; 0.6; 0.75; 1.0; 1.25; 1.5; 1.75; (b) curves 1–20, T = 0.05; 0.1; . . . ; 1; time interval $\Delta T = 0.05$.

FIG. 6. Time dependences of the plasma concentration at the points $\rho = 0.7$ (solid line) and $\rho = 0.16$ (dashed): $h_0 = h_A \sin 2\pi T$, $h_A = 2$, $\vartheta = 1/3$, $\tau_D = 100$, $\beta = 1$.



To a certain degree, the foregoing results are confirmed by the experimental data of Hübner and coworkers.[4] In their experiments, however, recombination played a noticeable role ($\tau_r = 2 \times 10^{-7}$ sec, $\tau = 6 \times 10^{-7}$ sec). We therefore present in Fig. 6 the results of the numerical integration of Eqs. (6)–(9) as applied to these experiments ($h_A = 2$, $\beta = 1$, $\vartheta = 1/3$). The parameter $\tilde{\tau}_D$ was assumed equal to 100 to decrease the calculation time. In the experiments $\tau_D \approx 10^4$, but this difference has practically no effect on the results of the numerical integration. Figure 6 shows plots of the plasma concentration at fixed points of the sample on the time (similar relations were determined in experiment by a laser probe method). The results of these calculations agree well with the experimental data of Hübner et al.[4] It is clearly seen that the volume recombination limits the growth of the plasma concentration in the region of the sections of the magnetic field that increase with time, and leads to a decrease of the attained densities (the limitation effects in the experiments of[4] are due in part to the damped character of the magnetic-field pulse).

In the computer integration of the system (6)–(9) we used a purely implicit difference scheme, the solution of which was obtained by iteration.

We note in conclusion that a similar picture of plasma constriction in a θ pinch should arise also in a weakly ionized gas-discharge plasma. This question is investigated in[6] with an unskinned θ pinch as an example.

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80