Electron cyclotron resonance spectrum in a weakly ionized gas

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A solution of the kinetic equation for electrons in a weakly ionized gas near the cyclotron resonance state is used to show that the experimentally observed splitting of the spectral line is a nonlinear effect and is due to significant irregularities in the energy dependence of the cross section for electron scattering by heavy particles. Under certain conditions these irregularities can be used to produce a negative temperature in an electron gas. Study of the shape of the cyclotron line may make it possible to determine the behavior of the scattering cross section at energies below 1 eV.

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It has recently been discovered that under fairly ordinary experimental conditions cyclotron resonance lines for gaseous plasmas with various chemical compositions often lack the expected simple bell-shaped form. Symmetric splitting relative to the Larmor frequency into two or three lines is characteristic [1]. Despite the simplicity of the physical object (the large mean free path of the electrons, the fact that the interactions between the charged particles may be neglected, etc.), the attempts made in [1] and [2] to explain this effect appear to be unsatisfactory, since they are qualitative, and each refers to a certain specific mechanism that is intimately related to the chemical nature of the plasma.

A consistent interpretation should clearly be based on the fact that an electron gas is only slightly bound to the heavy component of a weakly ionized plasma and, in addition, does not ensure high conductivity of the latter. Therefore, even a low-power variable-field generator creates a sufficient electric-field intensity (on the order of 1 V/cm at pressures of 10^{-1}—10^{-2} Torr) to significantly disturb the thermal equilibrium of the electron gas and to convert it into a nonlinear medium. One of the nonlinear effects is the deformation of the cyclotron absorption line.

In this work a solution of the kinetic equation has been used to show that the spectral line may become a two- or three-peaked curve that is symmetric with respect to the cyclotron frequency when there are appreciable irregularities in the electron scattering cross section σ as a function of the electron energy ϵ and when the electromagnetic field intensity is sufficient for the appearance of nonlinear effects. This phenomenon is due to the variability in the thermal contact between the electron gas and the thermostat. The conditions for the splitting of the spectral line have been obtained and the nonlinear-resonance spectral cuvres calculated for characteristic models of the function $\sigma(\epsilon)$. It has been shown that study of the cyclotron resonance spectrum makes it possible to measure σ at low energies. The possibility of the occurrence of a negative temperature in an electron gas under nonsteady-state conditions is also associated with the deformation of the spectral line.

It is natural in the case of cyclotron resonance to assume that $\tau\omega_0 \gg 1$, where $\tau = (N\sigma v)^{-1}$ is the mean free-flight time of an electron with the velocity v ($\epsilon = mv^2/2$, where m is the electron mass), and ω_0 is the Larmor frequency. The scattering is assumed to occur only from neutral particles of mass M with a concentration N. The alternating field of frequency ω is assumed to be linearly polarized, and its projection on a plane orthogonal to the direction of the magnetic field is denoted by E. In the case of three-dimensional isotropy the kinetic equation in the frequency range $\Omega = |\omega - \omega_0| \ll \omega_0$ for the symmetric part f(v, t) of the distribution function of the electrons is [3]

$$\frac{\partial f}{\partial t}=\frac{1}{2v^2}\frac{\partial}{\partial v}\left[\chi(v)\frac{v^3}{\tau}\left(f+\frac{kT}{mv}\frac{\partial f}{\partial v}\right)+\frac{e^2E^2}{6m^2}\frac{\tau v^2}{1+\tau^2\Omega^2}\frac{\partial f}{\partial v}\right], \quad (1)$$

where e is the electron charge, k Boltzmann's constant, and T the temperature of the neutral particles. The scattering, which is elastic and causes excitation of low-energy states, is replaced by a certain effective elastic scattering [4] characterized by a cross section $\sigma(\epsilon)$ and a mean fraction $\chi(v)$ of the energy lost in one collision. In a purely elastic process we have $\chi = 2m/M \sim 10^{-3}$, but even in the general case $\chi(v)$ is usually small ($\chi \lesssim 10^{-2}$). This weakens the thermal contact of the electron gas with the thermostat and promotes the nonlinear effects, which are hampered in those cases in which a choice must be made between small χ/τ and large E [5]. The changes in the parameters on the right side of Eq. (1) as functions of the time t should be regarded as slow, i.e., as occurring well within a time Δt satisfying the condition $\Delta t \gg \tau$.

The solution of Eq. (1) under the steady-state condition $\Omega\Delta t \gg 1$ is the function

$$f(v)=A\exp\left\{-\int_0^v\frac{mv\,dv}{kT+e^2E^2\tau^2/6m\chi(1+\tau^2\Omega^2)}\right\} \quad (2)$$

with the normalization coefficient A. The power taken from the field by the electrons and transferred to the thermostat (the heavy particles) equals

$$W(\omega,t)=-n\frac{e^2E^2}{12m}\int_0^\infty\frac{\tau v^3}{1+\tau^2\Omega^2}\frac{\partial f}{\partial v}dv\Big/\int_0^\infty v^2f\,dv, \quad (3)$$

where n is the concentration of electrons. In the steady-state problem at a low thermostat temperature, i.e., the value of kT is small in comparison to the following term in Eq. (2), the general equation (3) yields

$$W(\omega)=\frac{mn}{2}\int_0^\infty\frac{\chi(v)}{\tau(v)}v^4f\,dv\Big/\int_0^\infty v^2f\,dv. \quad (4)$$

We can immediately make several statements regarding the shape of the cyclotron line.

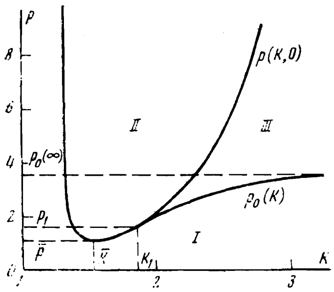

FIG. 1

1. According to (2), $\partial f/\partial v$ is always less than or equal to zero. Near the state of thermal equilibrium, where the term kT in the denominator of the integrand in Eq. (2) is dominant, we obviously have $\partial f/\partial\Omega^2 \approx 0$. Then, according to (3), we have $\partial W/\partial\Omega^2 \leq 0$, i.e., in the absence of nonlinear effects the cyclotron line is described by a plot of $W(\omega)$ that is symmetric with respect to the point $\omega = \omega_0$ and falls off monotonically with increasing $|\omega - \omega_0|$.

2. In the idealized case with τ = const and χ = const, Eq. (3) yields

$$W(\omega)=\frac{ne^2E^2}{4m}\frac{\tau}{1+\tau^2(\omega-\omega_0)^2}. \tag{5}$$

The nonlinearity of the medium has absolutely no effect on the shape of the spectral line, although the temperature of the electrons may increase quite substantially. Here we have

$$f=e^{-mv^2/2U},\quad U=kT+\frac{e^2E^2}{6m\chi}\frac{\tau^2}{1+\tau^2(\omega-\omega_0)^2}. \tag{6}$$

3. In order to illustrate how a decrease in σ with increasing energy affects $W(\omega)$, we take one more idealized case $\tau/\chi = \text{const}\cdot v^2$. Here the peak of the absorption curve becomes compressed, so that W = const up to $\Omega^2 \sim (M/12kT)(eE/m)^2$ under purely elastic scattering.

4. Now we can explain the seemingly fairly strange experimental fact that electron gases absorb less energy from the field at a point closer to the cyclotron frequency than they do at more distant points. Let us assume that the scattering cross section $\sigma(\epsilon)$, or, more precisely, the quantity $\sigma\chi$, has considerable irregularities in the region of nearly elastic processes such as a sharp drop following a section of smooth variation, or else a rapid increase (a descending jog or a pronounced maximum). As Ω increases, the effective acceleration of the electrons decreases, and according to Eq. (2) the fraction of slow particles increases. The contribution to the integral in (4) that determines $W(\omega)$ from the velocity region with large $\sigma\chi$ is enhanced owing to the factor χ/τ. It may be expected that in a certain frequency range the escape of electrons to the low-energy region, where χ/τ is large, will be more dominant than the frequency difference phenomenon, and $W(\omega)$ will have sections that rise along with $|\omega - \omega_0|$.

We shall simulate the irregularities in $\sigma\chi$ with the aid of the two functions

$$\varphi(v)=\begin{cases} p & \text{when } 0\leqslant v<u,\\ 1 & \text{when } u<v<Ku,\\ \infty & \text{when } Ku<v;\end{cases}$$

$$\psi(v)=\begin{cases} 1+pu\delta(u-v) & \text{when } 0\leqslant v<Ku,\\ \infty & \text{when } Ku<v;\ K>1.\end{cases}$$

(In this manner we can obtain many variants by simulating σ and χ separately, as well as by combining φ and ψ with powers of v.) Equations (2)–(4) make it possible to find $W(\omega)$ for each model as a function of the introduced parameters K and p. By next assuming that $\partial W/\partial\omega = 0$, we can calculate the respective critical value $p(K, \Omega) > 0$, above which the function W ceases to decrease monotonically with $|\omega - \omega_0|$ in the vicinity of a given Ω. Since all the frequencies ω are scanned experimentally (but, of course, $\Omega \ll \omega_0$), a minimum value $p_0(K)$ of the function $p(K, \Omega)$ with respect to Ω should be found for each K. Any p that exceeds p(K, 0) fits the splitting of the resonance line exactly at the cyclotron frequency (a two-peaked curve). In addition, there is an absolute minimum $\bar{p}$ (with respect to K and Ω) of the critical value of the parameter p for these models.

The results of such an analysis in the case of the strong field

$$\frac{e^2E^2}{6m}\gg kT\frac{\chi}{\tau^2},\quad \frac{e^2E^2}{6m}\gg\int\limits_0^{Ku}\frac{\chi}{\tau^2}mv\,dv \tag{7}$$

for the model $1/\tau(v) = \psi(v)/\tau_0$, χ = const are presented in Fig. 1. The coordinate axes and the plots of $p_0(K)$ and p(K, 0) delineate three regions for the values of p. In region I the function $W(\omega)$ decreases monotonically with increasing $|\omega - \omega_0|$ and describes a single-peaked absorption line with a maximum at the Larmor frequency ω_0. In region II the point $\omega = \omega_0$ is a minimum in $W(\omega)$, i.e., the spectrum is a two-peaked curve. In region III there is an increase in W as a function of $\Omega = |\omega - \omega_0|$ when $\Omega > 0$; therefore, the plots of $W(\omega)$ are three-peaked curves with one peak at the cyclotron frequency and two symmetrically placed minima.

Plots similar to those in Fig. 1 can be drawn for any sufficiently irregular function of $\sigma\chi$. The main characteristics of these plots and the critical values of p in the strong field (7) for several models are given in the table.

Figure 2 presents several spectral lines constructed with the aid of Eq. (3). All such plots have been observed experimentally[1].

The use of Eq. (1) is equivalent to the assumption that the scattering is elastic or nearly elastic. Inelastic processes introduce new parameters into the problem, and it is wiser to take them into account not by combining models, but by carrying out the calculation for the cross sections that actually exist in a specific situation by the method cited, for example, in[4]. In the general case, however, it is not difficult to prove that the addition of inelastic effects at the point v = Ku in our models (with or without an increase in the elastic cross section $\sigma(\epsilon)$) causes a monotonic decrease in W as a function of

Model Number	Model	Critical values of the parameter p and their coordinates K	Form of the p(K, 0) dependence
1	$\frac{1}{\tau}=\varphi/\tau_0$, $\chi=\text{const}$	$\bar{p}=3.39$, $p_1=4.23$, $p_0(\infty)=7.15$; $\bar{K}=1.29$, $K_1=1.53$	$1+\frac{4K^7}{21K^2-25}$
2	$\sigma=\sigma_0\varphi$, $\chi=\text{const}$	$\bar{p}=5.63$, $p_1=7.84$, $p_0(\infty)=17{,}74$; $\bar{K}=1.29$, $K_1=1.53$	$1+\frac{K^8}{4K^2-5}$
3	$\frac{1}{\tau}=\psi/\tau_0$, $\chi=\text{const}$	$\bar{p}=1.11$, $p_1=1.63$, $p_0(\infty)=3.54$; $\bar{K}=1.53$, $K_1=1.86$	$\frac{4K^7}{35(3K^2-5)}$
4	$\sigma=\sigma_0\psi$, $\chi=\text{const}$	$\bar{p}=1{,}82$, $p_1=2{,}93$, $p_0(\infty)=8{,}15$; $\bar{K}=1.49$, $K_1=1{,}80$	$\frac{K^8}{8(3K^2-5)}$
5	$\tau=\text{const}$, $\chi=\chi_0\psi$	$\bar{p}=1{,}18$	$p(K,0)\to 0{,}15K^{7/2}$, $K\to\infty$
6	$\sigma=\text{const}$, $\chi=\chi_0\psi$	$\bar{p}=1{,}92$	$p(K,0)\to 0{,}16K^4$, $K\to\infty$

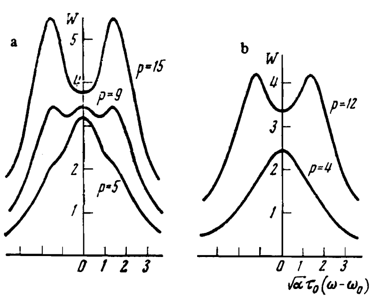

FIG. 2

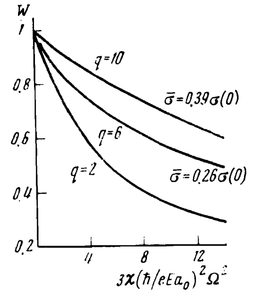

FIG. 3

FIG. 2. Cyclotron absorption lines $W(\omega)$ for the model $1/\tau(v) = \varphi(v)/\tau_0$, τ_0 = const, χ = const; $\alpha = (3\chi/4\tau_0)(mu/eE)^2$; a) $\alpha = 0.1$; $K = \sqrt{5}$; b) $\alpha = 1$; $K = 2$.

FIG. 3. Cyclotron absorption spectrum in nascent hydrogen,

$$q=3\chi\left(\frac{\hbar}{ma_0}\right)^4\left[\frac{mN\sigma(0)}{eE}\right]^2, \quad a_0=0.529\cdot10^{-8}\ \text{cm}$$

$|\omega - \omega_0|$. Now the contact between the electron gas and the thermostat is large not only in the region of low energies, but also in the region of energies exceeding the threshold. The increase in the electron temperature near the cyclotron frequency, however, cannot easily be reconciled with any general weakening of the coupling with the thermostat.

Distinct irregularities in the cross section $\sigma(\epsilon)$ or the value of χ in the region of elastic or nearly elastic processes (at electron energies of about 1 eV) are typical of many atoms and molecules[6]. They may differ in nature, and, in particular, they may be produced by the Ramsauer effect[7]. The cyclotron line in a weakly ionized helium and neon plasma is probably single-peaked, in any case, when the vessel with the gas under investigation in the experimental apparatus is located in an approximately uniform accelerating field. We stress that the latter condition is obligatory in the present work, since kinetic equation (1) does not include differentiation with respect to the coordinates.

Analysis of the shape of the cyclotron line may serve as a fairly effective method for studying the behavior of the electron scattering cross section $\sigma(\epsilon)$ in the low-energy region, which is hardly accessible to other methods.

Let us assume that the temperature of the gas is so low that $12kT \ll \epsilon_1$, where ϵ_1 lies in the region of elastic or nearly elastic processes. We introduce the average values $\bar{\sigma}$ and $\bar{\chi}$ in this region $[0, \epsilon_1]$. Then from Eqs. (2) and (3) we easily obtain

$$W(\omega)=\text{const}\, D_{-3}(\beta)/D_{-3/2}(\beta) \quad \text{for} \quad 12\bar{\chi}\varepsilon_1 kT\ll\left(\frac{eE}{N\bar{\sigma}}\right)^2\ll\bar{\chi}\varepsilon_1^2 \qquad (8)$$

in the case of nonlinear resonance and

$$W(\omega)=\text{const}\,\gamma^2e^{\gamma}\Gamma(-2,\gamma) \quad \text{for} \quad \left(\frac{eE}{N\bar{\sigma}}\right)^2\ll12\bar{\chi}\varepsilon_1 kT. \qquad (9)$$

in the case of linear resonance. Here

$$\beta=\sqrt{3\bar{\chi}}\,m\Omega^2/eEN\bar{\sigma}, \qquad \gamma=m\Omega^2/2kTN^2\bar{\sigma}^2,$$

and the notation for the special functions is conventional. Comparison of the experimental curves with Eqs. (8) and (9) at the appropriate values of the field intensity E readily yields the values of $\bar{\sigma}$ and $\bar{\sigma}/\sqrt{\bar{\chi}}$. In this manner, by probing the region of small ϵ values at fairly low temperatures and small E values, in which $\chi = 2m/M$, we can also determine the scattering lengths. If the frequency difference relative to the resonance frequency is small, curves (8) and (9), when reduced to an amplitude equal to unity at $\Omega = 0$, become the straight lines

$$W(\omega)=\begin{cases}1-0.55\beta & \text{(nonlinear resonance)},\\ 1-\gamma & \text{(linear resonance)}.\end{cases} \qquad (10)$$

Figure 3 presents the nonlinear-resonance spectral curves for nascent hydrogen. The experimental data for the scattering cross section were taken from[7], but at low energies, where these were lacking, the theoretical results from[8] were used. The curves closely approximate straight lines in their initial segments. The slopes of the straight lines were compared with the first formula in (10), and the ratios of $\bar{\sigma}$ to $\sigma(0)$ are given in the figure. We note that in the case of the hydrogen atom $\sigma(\epsilon)$ has a sharp maximum at $\epsilon = 0$, so that $\bar{\sigma}$ is known to be less than $\sigma(0)$.

In conclusion we shall discuss briefly the possibility of creating a negative temperature in an electron gas under nonsteady-state conditions, since a connection with the deformation in the spectral line $W(\omega)$ appears in this case.

Ruling out an interaction with the electromagnetic field, we obtain the following equation from Eq. (1) with $t > 0$ for the distribution function ($T = 0$):

$$\frac{\partial f}{\partial t}=\frac{1}{2v^2}\frac{\partial}{\partial v}\left(\frac{\chi v^3 f}{\tau}\right). \qquad (11)$$

The function $f(v, 0) \equiv f(v)$ at the moment $t = 0$ is assumed assigned. A solution of Eq. (11) is

$$f(v,t)=\frac{\tau(v)}{\chi(v)v^3}\frac{\chi(s)s^3}{\tau(s)}f(s), \qquad (12)$$

where s should be found from the equation

$$t=2\int_v^s\frac{\tau(x)\,dx}{\chi(x)x}.$$

It is not difficult to show that

$$\text{sgn}\frac{\partial f(v,t)}{\partial v}=\text{sgn}\left\{\frac{1}{s^2f(s)}\frac{\partial}{\partial s}\left[\frac{\chi(s)s^3}{\tau(s)}f(s)\right]-\frac{1}{v^2}\frac{\partial}{\partial v}\left[\frac{\chi(v)v^3}{\tau(v)}\right]\right\}.$$

If saturation is approximately achieved in the vicinity of a certain v in the initial state through prior acceleration of the electrons, i.e., $\partial f(v)/\partial v \approx 0$, then for small t, and therefore for $s \gtrsim v$, we have

$$\text{sgn}\frac{\partial f(v,t)}{\partial v}=\text{sgn}\frac{d}{dv}\left[\frac{1}{v^2}\frac{d}{dv}(\chi v^3/\tau)\right].$$

This means that a necessary condition for the appearance of population inversion $\partial f(v, t)/\partial v > 0$ over the course of a certain time interval after a strong accelerating field is switched on is the relationship

$$\frac{d}{dv}\left[\frac{1}{v^2}\frac{d}{dv}\left(\frac{\chi v^3}{\tau}\right)\right]>0,$$

or if the energy variable $\epsilon = mv^2/2$ is used

$$\frac{d}{d\varepsilon}\left[\frac{1}{\sqrt{\varepsilon}}\frac{d(\sigma\chi\varepsilon^2)}{d\varepsilon}\right]>0. \qquad (13)$$

Inequality (13) can be treated as a condition for the concavity of the graph of $\epsilon^2\sigma\chi$ as a function of $\epsilon^{3/2}$. Such sections clearly exist in regions with a sharply irregular cross-section dependence (e.g., the Ramsauer effect).

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