"Non-Josephson" generation in a resonator with a superconducting Josephson junction

V. N. Gubankov, L. S. Kuz'min, K. K. Likharev, and V. K. Semenov

Institute of Radio Engineering and Electronics, USSR Academy of Sciences
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The possibility is investigated of the appearance, in a resonator with a Josephson junction, of "non-Josephson" generation at the resonator frequency simultaneously with generation of the usual Josephson oscillations. The conditions for the appearance of non-Josephson generation are investigated, and its amplitude, frequency and effect on the current-voltage characteristics of the junction are calculated. The non-Josephson generation is investigated and the results are compared with the theory. Noise is shown to affect considerably the characteristics of the phenomenon. The validity of the Josephson relation at various definitions of the "Josephson frequency" is discussed.

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1. INTRODUCTION

In the nonstationary Josephson effect[1] the average frequency $\bar{f}$ of the Josephson generation is connected with the applied voltage $\bar{V}$ by the known relation

$$\bar{f}=2e\bar{V}/h, \tag{1}$$

which is fundamental. So far, no mechanisms have been observed capable of resulting in a deviation of the coefficient of $\bar{V}$ from the value 2e/h.[2] However, when a Josephson junction is placed in a resonant electrodynamic system there can occur, in addition to the Josephson oscillations, also generation at the natural frequency of the system, so that the frequency of the "non-Josephson" generation is not equal to $\bar{f}$. This generation was observed by Ulrich[3] in a resonator with a superconducting point junction indirectly, from its effect on the current voltage characteristics. In all probability, a similar phenomenon was observed also in tunnel structures[4], but its investigation in this case is made difficult by the complicated character of the interaction of the Josephson current and the fields in the distributed resonator made up by the structure itself.[5]

A simpler picture can be realized in the case of a low capacitance Josephson junction (e.g., a superconducting point junction or a film bridge of small dimensions[6]), for when such a junction is included in an external resonator the oscillation energy is localized mainly outside the junction. In addition, in such a system the effect of non-Josephson generation turns out to be quite strong, i.e., it takes place in a rather wide range of variation of the parameters. As shown by a reduction of the earlier experiments with low-capacitance Josephson junctions, the changes of the current-voltage characteristics of Josephson junctions included in resonators can be attributed only to the onset of such a generation.

The present paper is devoted to a theoretical and experimental investigation of non-Josephson generation in a resonant system with a low-capacitance Josephson junction. It is shown in the Appendix that the Josephson-generation frequency may not be equal to $\bar{f}$ even in the absence of this phenomenon.

2. THEORY

2.1. The Essence of the Phenomenon

The onset of non-Josephson generation is due to strong nonlinear interaction of oscillations in a resonator with Josephson generation of the junction. This can be accompanied by energy pumping into oscillations at the natural frequency ω_r of the resonator. This interaction can be represented as partial synchronization of Josephson generation with the oscillations produced at the resonator frequency[3,6,7] in the course of beats between them.

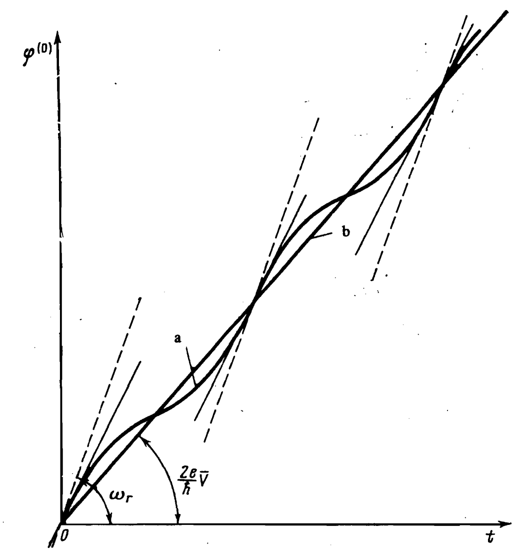

FIG. 1. Non-Josephson generation in the time representation: a) time dependence of the phase difference averaged over the fast ($\sim\bar{f}$ oscillations $\varphi^{(0)}$ in the case of non-Josephson generation; b) the same in the case of Josephson generation, $d\varphi^{(0)}/dt = 2e\bar{V}/\hbar$.

Figure 1 shows schematically the time dependence of the difference of the phases φ of the superconducting electrodes making up the junction, averaged over the fast Josephson oscillations ($\varphi^{(0)}$), in the case of the onset of non-Josephson generation. The instantaneous frequency of the voltage oscillations across the junction is $d\varphi^{(0)}/dt$. In the case of Josephson generation, $\varphi^{(0)}$ is a linear function of the time in accordance with fundamental Josephson relation (1).

As will be shown below (in Sec. 2.4), if non-Josephson generation appears in the resonator, then it exerts a synchronizing action on the Josephson generation, and the function $\varphi^{(0)}(t)$ is altered (Fig. 1a). The instantaneous frequency of the oscillations on the junction $d\varphi^{(0)}/dt$ then comes closer during part of the period of the beats to the resonator frequency ω_r; it is precisely during that time that the energy input to the oscillations in the resonator takes place. During part of the beat period, $d\varphi^{(0)}/dt$ becomes lower than the Josephson frequency, so that the time variation of the phase is determined on the average by the Josephson relation.

It turns out that the existence of non-Josephson generation is possible only in a certain interval of the average rates of change of the phase below the natural resonator frequency ω_r, i.e., at average junction voltages $\overline{V} < V_r = \hbar\omega_r/2e$. The onset of such a process alters strongly not only the spectrum of the oscillations in the junction, but also the shape of its current-voltage characteristics.

2.2. Equation for a Resonator with a Josephson Junction

To calculate the processes in the system we assume that a low-capacitance Josephson junction can be described by the resistive model.[8] In this case the following equations are valid for a junction placed in a resonant system:

$$J_0 \sin\varphi + \frac{V(t)}{R} = \overline{J} + J_r + J_f, \qquad (2)$$

$$V(t) = \frac{\hbar}{2e}\frac{d\varphi}{dt}. \qquad (3)$$

Here J_0 is the critical current of the junction, R is the resistance of the junction in the resistive state, V(t) is the voltage across the junction, $\overline{J}$ is the d.c. bias current, J_r is the current through the resonator, and J_f is the effective fluctuation current. For a resonator we have

$$J_r = -\hat{G}[V(t)], \qquad (4)$$

where $\hat{G}$ is the linear operator of the resonator conductivity.

We confine ourselves to the case of sufficiently high resonator frequencies $\omega_r \gtrsim \omega_0$, where $\omega_0 = 2eV_0/\hbar$, V_0 is the characteristic voltage of the junction ($V_0 = J_0R$); this approximation allows us to find analytically the main characteristics of the system. Averaging Eqs. (2)–(4) over times $\sim\overline{f}^{-1}$, i.e., over the fast oscillations, we obtain a system of truncated equations[9]1)

$$\left[(1+r_r)+\frac{r_r}{h}\frac{d}{dt}\right]x = ie^{i\chi}, \qquad (5)$$

$$\dot{\chi} = \delta\overline{j} + j_f + \operatorname{Im}(xe^{-i\chi})/2\Omega_r. \qquad (6)$$

x is the complex amplitude ($x = ae^{i\theta}$) of the oscillations in a resonator of frequency Ω_r:

$$j_r = \operatorname{Re}\{x\exp(i\Omega_r t)\}, \qquad (7)$$

χ is the deviation of the oscillation phase, $\varphi^{(0)}$ averaged over the times $\overline{f}^{-1}$, from $\Omega_r t$:

$$\chi = \varphi^{(0)} - \Omega_r t + \pi; \qquad (8)$$

r_r is the resonator loss resistance, $2h = Q^{-1}\Omega_r$ is the bandwidth of the resonator without allowance for the losses in the junction, Q is the resonator intrinsic Q; $\delta\overline{j} = \overline{j} - \overline{j}_a(\Omega_r)$, where $\overline{j}_a(\Omega_r)$ is the value of the bias current of the autonomous junction at a voltage $\overline{v} = \Omega_r$ across it.

Expression (6) is the self-consistency equation for the phase χ. In the absence of oscillations in the resonator (x = 0) and fluctuations ($j_f = 0$) Eq. (6) yields the Josephson relation for the time dependence of the phase $\varphi^{(0)}$ on the junction (Fig. 1b). The last term in (6), proportional to $|x| = a$, describes the action on the junction by the oscillations in the resonator.

We shall consider the case of greatest interest, when j_f is white noise:

$$\langle j_f(t_1) j_f(t_2)\rangle = 2\Gamma\delta(t_1 - t_2),$$

where Γ is the line width of the Josephson generation of the autonomous junction, and the resonator band width h is much smaller than $\max[\Gamma, |\Delta|]$, where $\Delta = \dot{\chi} \equiv \overline{v} - \Omega_r$ is the difference between the Josephson-generation frequency and the natural frequency of the resonator.

2.3. Condition for the Onset of Non-Josephson Generation

For small oscillations in the resonator $|x/2\Omega_r| \ll 1$, averaging in (5) over the times $\max\{\Gamma, |\Delta|\}^{-1} \ll t \ll h^{-1}$ and using (6), we obtain[9]

$$[r_r + z(0) + r_r h^{-1} d/dt]x = ie^{i\chi_0}. \qquad (9)$$

Here z(0) is the effective impedance of the transition from small oscillations at the frequency of the resonator ω_r:

$$z(0) = 1 + (\Delta - i\Gamma)/4\Omega_r(\Delta^2 + \Gamma^2); \qquad (10)$$

χ_0 denote the value of the phase χ as $\chi \to 0$, so that the term in the right-hand side of (9) describes the action on the resonator by the unperturbed Josephson generation.

As seen from (9), Re z(0) becomes negative at $\Delta \lesssim 0$, i.e., at $\overline{v} \lesssim \Omega_r$. This anomalous behavior of the impedance of the Josephson junction was predicted theoretically in[7,8] and then observed experimentally.[11,12] Excitation of non-Josephson oscillations at the natural frequency of the resonator, as follows from (10)), occurs at

$$\operatorname{Re} z(0) \leqslant -r_r. \qquad (11)$$

This condition corresponds to complete cancellation of the active resonance losses by the energy input from the Josephson oscillations (Sec. 2.1).

2.4. Non-Josephson Generation in the Absence of Fluctuations

We now determine the spectral composition of the oscillations in the case of small fluctuations: $\Gamma \ll |\Delta|$. By virtue of the condition $h \ll |\Delta|$, the amplitude and the phase of the oscillations in the resonator can be regarded as constant during a time on the order of the period of the beats ($|\Delta|^{-1}$) between the Josephson generation and the natural oscillations in the resonator. Equation (6) coincides in this case

$$\dot{\chi}_1 + a\sin\chi_1/2\Omega_r = \delta\overline{j}, \qquad \chi_1 = \chi - \theta, \qquad a = \text{const} \qquad (12)$$

in form with the equation of Aslamazov and Larkin for the total phase in an autonomous junction.[8] Its solution can be readily obtained:

$$\chi_1 = 2\operatorname{arctg}\left\{\left[\frac{\delta\overline{j} + a/2\Omega_r}{\Delta}\right]^{-1}\operatorname{tg}\left(\frac{\Delta}{2}t\right)\right\} + \frac{\pi}{2}, \qquad (13)$$

$$|\Delta| = |\overline{\dot{\chi}_1}| = [\delta\overline{j}^2 - (a/2\Omega_r)^2]^{1/2}. \qquad (14)$$

It follows from (13) that the plot of $\varphi^{(0)}(t)$ [Eq. (8)] takes the form shown in Fig. 1a.

Let us determine the amplitude and frequency of the

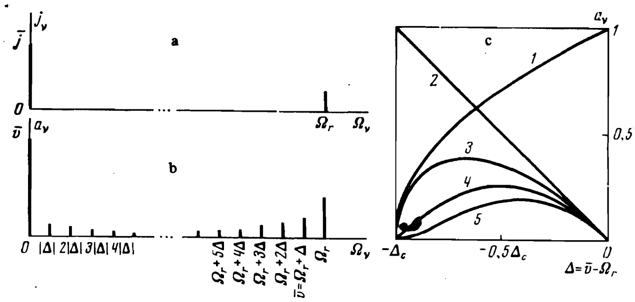

FIG. 2. Non-Josephson generation in the spectral representation: a) spectrum of the current; b) spectrum of the voltage; c) dependence of the amplitudes of the spectral components on the beat frequency Δ. The designations of the voltage components a_ν are as follows: curve 1 for $a_\nu = [(1 + r_r)/r_r]a$, where a is the component at the resonator frequency (synchronizing force); curve 2 is for $a_\nu = a_{\Omega_r+\Delta}$ – the components at the Josephson frequency $\bar{\nu} = \Omega_r + \Delta$; curve 3 for $a_\nu = a_{\Omega_r+2\Delta}$ – the components at the frequency $\Omega_r + 2\Delta, \ldots$. The amplitudes of the components at the frequencies $\Omega_r - \Delta$, $\Omega_r - 2\Delta$, $\Omega_r - 3\Delta, \ldots$ vanish identically at all $\Delta < 0$.

non-Josephson generation and the form of the current-voltage characteristic. Multiplying (5) by $e^{-i\theta}$ and averaging its real part, we obtain

$$(1+r_r)a = -\overline{\sin\chi_1}. \qquad (15)$$

Substituting in it $\overline{\sin\chi_1}$ from (12), we get

$$(1+r_r)a = (a/2\Omega_r)^{-1}[\Delta - \delta\bar{j}], \qquad (16)$$

which makes it possible, in conjunction with (14), to express a^2 and $\delta\bar{j}$ as functions of the average junction voltage $\bar{V} = \Omega_r + \Delta$:

$$\delta\bar{j} = -\frac{1}{2\Omega_r(1+r_r)} - \Delta, \qquad (17)$$

$$a^2 = \frac{4\Omega_r}{1+r_r}\left[\Delta + \frac{1}{4\Omega_r(1+r_r)}\right]. \qquad (18)$$

The quantity a^2 yields the non-Josephson-generation power dissipated in the resonator: $P = J_0V_0a^2r_r$.

We note now that the quantities $\delta\bar{j}$ and a^2 coincide at $\Delta = 0$ with their values for induced Josephson oscillations in the resonator.[6] In addition, at the critical detuning determined from (10) and (11)

$$\Delta = -\Delta_c = -1/4\Omega_r(1+r_r) \qquad (19)$$

a vanishes, and $\delta\bar{j}$ becomes equal to the value for the autonomous junction Δ_c. It is seen therefore, as well as from the linear dependence of $\delta\bar{j}$ and a^2 on Δ, that the change of the current-voltage characteristic of a junction after it is included in a resonator, and also the dependence of the power in the resonator on the voltage, have an asymmetrical triangular shape (Fig. 3 below).

The differential resistance R_d on the left slope of the triangular dip on the current voltage characteristic is equal to (−R). It can be shown that R_d is equal in absolute value to the differential resistance of the autonomous junction at arbitrary Ω_r, provided only that $R_r \gg R_d$. Such triangular dips on the current voltage characteristic, with differential resistance equal to $(-R_d)$, were distinctly observed in Longacre's experiments.[13] In addition, similar dips, smeared out by fluctuations (see below), were observed in results by others (see the bibliography in[6]). In all probability, the change of the current voltage observed in these experiments at $\bar{V} < \hbar\omega_r/2e$ ($\Delta < 0$) was due to excitation of non-Josephson generation in the resonator.

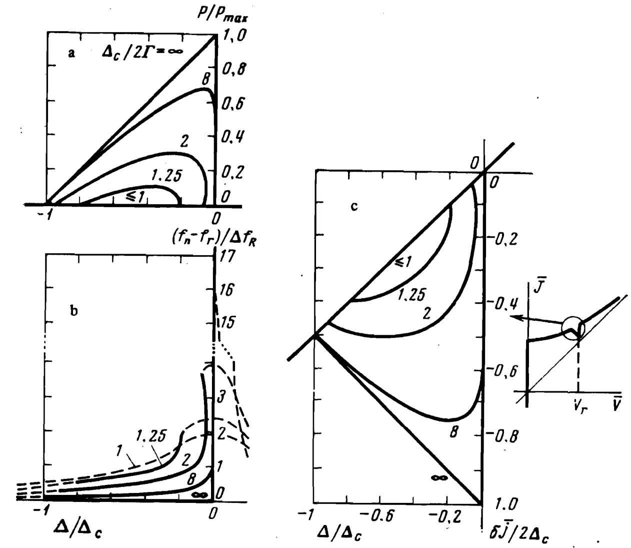

FIG. 3. Theoretical curves for $\Omega_r \gtrsim 1$: a) power of non-Josephson generation, normalized to $P_{max} = J_0V_0r_r/(1 + r_r)^2$; b) shifts of its frequency relative to the exchange frequency f_r of the resonator, normalized to the resonator bandwidth with the junction in the normal state, $(f_n - f_r)/\Delta f_R = [r_r/h(r_r + 1)]\bar{\theta}$, the dashed curve shows the resonator frequency with the junction in the absence of non-Josephson oscillations (10); c) change of the current-voltage characteristic of the junction following the onset of the non-Josephson generation, with the general location of the dip on the current-voltage characteristic shown on the right. The parameter is the relative level of the fluctuations $(\Delta_c/2\Gamma)$. At $(\Delta_c/2\Gamma) < 1$ the non-Josephson generation is completely suppressed by the fluctuations.

2.5. Spectrum of Oscillations in the System

The oscillation spectra in non-Josephson generation are significantly different for different quantities. As already noted, the complex amplitude x of the oscillations in the resonator remains constant during the time of the beats and therefore (see (4)) the spectrum of the current in the resonator has only one component, at the frequency Ω_r.

To determine the spectral composition of the junction voltage it is necessary to resolve V(t) into a low-frequency component (~| Δ |) and a high-frequency component ($\sim\Omega_r$), as was done earlier[9], and to find their spectra separately. This operation reduces in fact to a substitution of the function $\varphi^{(0)}(t)$ obtained from (8) and (13) into Eqs. (2) and (3), which yield respectively the high- and low-frequency parts of the V(t) spectrum. A typical spectrum is shown in Fig. 2. The high-frequency part of the spectrum turns out to be one-sided (there are no components at the frequencies $\Omega_r - \Delta$, $\Omega_r - 2\Delta, \ldots$), a behavior typical of partial synchronization.[14] The dependences of the amplitudes of the spectral voltage components on the voltage detuning (the beat frequencies) are shown in Fig. 2c. We note that as $\Delta \to 0$ ($\bar{V} \to (\hbar/2e)\omega_r$) the amplitude of the component with the resonator frequency Ω_r, which plays the role of the synchronizing force, increases, and the amplitude of the component with the Josephson frequency tends to zero.

2.6. Influence of Fluctuations

In the presence of small fluctuations, the phenomenon of non-Josephson generation is on the whole preserved. Significant differences begin only at[2)] $\Gamma \sim |\Delta_c|$. At

$\Gamma \geq \Delta_c/2$, according to (10) and (11), the non-Josephson generation is completely suppressed.

To calculate the main characteristics of the non-Josephson generation in the presence of fluctuations at $\Gamma < 2\Delta_c$ it is necessary to take into account the fluctuation current j_f in Eq. (6), which leads to the appearance of an analogous term in (12). It is known[15,16] that in this case it is necessary to use not Eq. (14), which describes the current-voltage characteristic near the current step, but the formula

$$\Delta = \left(\frac{\Gamma_a}{\pi}\right)\left(\frac{\pi}{2}\right)\mathrm{sh}\left(\frac{\pi\delta\bar{j}}{\Gamma_a}\right) \Big/ \int_0^{\pi/2} \mathrm{ch}\left(\frac{2\delta\bar{j}}{\Gamma_a}\right) J_0\left(\frac{2}{\Gamma_a \cos y}\right) dy, \quad (20)$$

where $\Gamma_a = 2\Omega_r\Gamma/a$. This expression together with (13) yields the non-Josephson generation power P and the current-voltage characteristics. The dependences of P and $\delta\bar{j}$ on Δ are shown in Figs. 3a and 3b for different fluctuation levels ($1 < \Delta_c/2\Gamma < \infty$).

The fluctuations cause also a shift of the non-Josephson generation f_n on the natural frequency f_r of the resonator. This is seen already from formula (10) of the small-signal impedance of the junction, in which the imaginary part of z is negative at all values of $\bar{V}$ if $\Gamma \neq 0$; this leads to an upward shift of the resonant frequency of the system. The expression for the frequency shift of the non-Josephson generation is obtained by averaging the imaginary part, multiplied by $e^{-i\theta}$, of Eq. (5):

$$r_r \bar{\dot{\theta}} a/h = \overline{\cos\chi_1}, \quad (21)$$

if account is taken of the fact that $\dot{\theta} = (f_n - f_r)/f_0$. The results of the numerical calculation of the frequency shift $(f_n - f_r)$ by formulas (20) and (21) are given in Fig. 3b.

3. EXPERIMENTAL INVESTIGATION OF NON-JOSEPHSON GENERATION

3.1. Experimental Procedure

For an experimental investigation of the non-Josephson generation we used as a low-capacitance Josephson superconducting Nb-Nb point contacts formed in liquid helium. For the experiments we usually chose contacts with low resistance in the normal state, R = 0.08 to 0.2 Ω, since the natural noise of the contact increased with increasing R. Thus, a contact with R = 0.2 Ω and $V_0 = 60\ \mu V$ should have at T = 4.2°K and at a frequency 10^{10} Hz a relative Josephson-generation width not smaller than 1.5×10^{-2}.[1] In addition, the voltage of the noise induced from the outside increases with increasing R.

Taking into account this choice of R, to ensure relation (11) we attempted to minimize the loss resistance (R_r) in the external resonant electrodynamic system, referred to the location of the junction. To this end we used the usual[3,7,11,13] method of placing the junction in a coaxial half-wave resonator (f_r = 9280 MHz, resonator bandwidth without junction loss Δf = 14 MHz). The resistance R_r is the sum of the losses R_0 in the resonator and the resistance R_c due to coupling of the resonator with the waveguide R_0 is equal to $\pi Z_0/4Q$, where Z_0 is the wave impedance of the resonator, determined by the relation $Z_0 = 60\ \ln(D/d)[\Omega]$. In our resonator we had D = 10 mm, d = 4 mm. Z_0 = 72 Ω, Q ≈ 700, and R_0 = 0.10 ± 0.02 Ω.

One of the main requirements for the satisfaction of the excitation conditions (11) is a high impedance of the resistance and of the shunt, compared with R, at the fundamental combination frequencies $f_r - \bar{f}$, $\bar{f}$, and $2\bar{f} - f_r$ (see Fig. 2). If we use a resonator with the indicated value of Q, connected in series with the junction, even in a regime with a set voltage (with a shunt having the parameters $R_S = 10^{-3}\ \Omega$ and $L_S = 10^{-8}$ H), the impedance of the resonator and of the shunt is of the order of $Z_0 \gg R$ so long as $|\bar{V} - V_r|/V_r \gtrsim 5 \times 10^{-3}$, i.e., practically in the entire interval of voltages in which the non-Josephson generation might be excited.

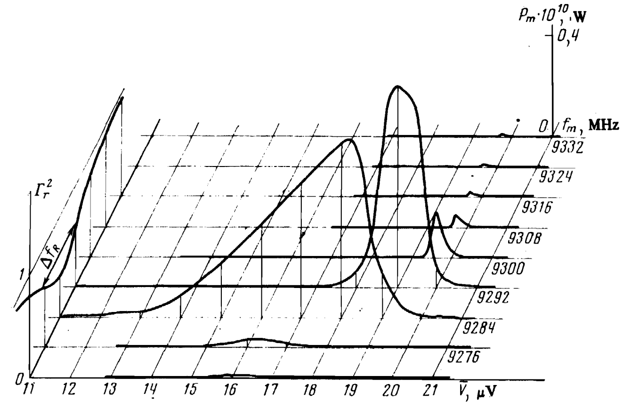

FIG. 4. Experimental dependences of the power radiated from the resonator in the receiver band on the junction voltage $\bar{V}$ at different settings of the receiver frequency f_m near the natural resonator frequency f_r = 9284 MHz. On the left is shown the amplitude-frequency characteristic of a resonator operating in reflection, Γ_r^2, with the junction in the resistive state. The coupling between the resonator and the waveguide is less than critical. The junction parameters are R = 0.13 ± 0.03 Ω, J_0 = 450 μA, V_0 = 60 ± 20 μV, Ω_r = 0.3 ± 0.1, and T = 4.2°K.

The energy was drawn away from the resonator by a coupling post; the resistance R_c could be varied in the course of the experiment from 0.05 to 0.8 Ω (≈R_0) with the aid of the waveguide plunger. The power radiated from the resonator was registered with a 3-cm superheterodyne P5-10 receiver having a bandwidth 8 MHz and a sensitivity $2 \times 10^{-15}\ W/Hz^{1/2}$.

3.2. Experimental Results and Discussion

We have carried out a comparison with the theory only for junctions having a current-voltage characteristic of near-hyperbolic form (this form follows from the resistive model[8]), having on the current-voltage characteristic steps produced by the microwave irradiation, and having not too large values of V_0 (not too small Ω_r). We present below experimental results typical of junction of this kind.

Figure 4 shows the dependence of the power P_m radiated from the resonator in the receiver band, on $\bar{V}$ when the receiver frequency f_m is tuned to a sequence of frequencies spaced 8 MHz apart at a set voltage. The junction parameters are: R = (0.13 ± 0.03) Ω, J_0 = 450 μA, V_0 = (60 ± 20) μV, and Ω_r = 0.3 ± 0.1. The large error in the determination of Ω_r is due to the fact that Ω_r is assumed to be the average of the results of dc experiments ($V_0 = J_0R$) and of microwave experiments reduced by the method described in[17]. The coupling of the resonator with the waveguide is minimal (R_c = 0.05 Ω).

Figure 5a shows the results of the measurement of the shift of the frequency of the non-Josephson genera-

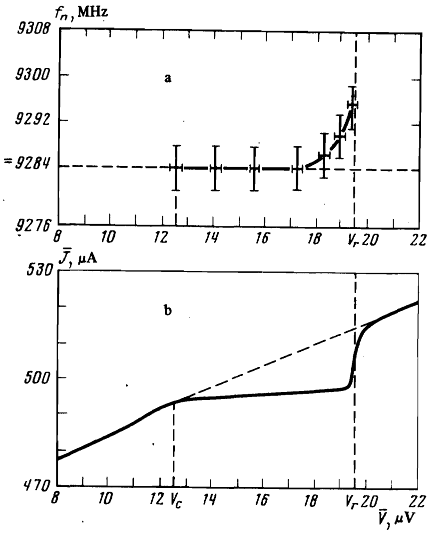

FIG. 5. Experimental curves: a) shift of non-Josephson generation frequency f_n; b) change of current-voltage characteristic of the junction upon excitation of non-Josephson generation. The curve in Fig. a was plotted under conditions when the voltage $\bar{V}$ was set, and the curve in Fig. b was plotted at a set current $\bar{J}$. The junction is the same as in Fig. 4.

tion as a function of $\bar{V}$. The frequency f_n was defined as the frequency at which the power P_m is maximal at fixed $\bar{V}$. The resonator bandwidth with junction in the normal state is $\Delta f_R = 25$ MHz. The current-voltage characteristic of the same junction (plotted at a set current $\bar{J}$) is shown in Fig. 5b.

In the analysis of the results it must be borne in mind that, as seen from Fig. 4, the width of the non-Josephson generation line is small ($\lesssim 8$ MHz), i.e., it lies within the receiver bandwidth. To the contrary, the Josephson generation has a bandwidth ~ 300 MHz (see below), and the receiver bandwidth can span only about 3% of the total power of the Josephson generation, so that its influence on the results is small.[3)]

The presented experimental results (Figs. 4 and 5) agree qualitatively with the conclusions of the theory developed above: one can distinctly observe generation at the resonator natural frequency when the junction voltage $\bar{V}$ is varied (together with the Josephson generation frequencies (1)) in a rather wide range. In addition, the theoretically predicted shift of the frequency of the non-Josephson generation upward is indeed observed as $\bar{V}$ approaches V_r. The curves of Figs. 4 and 5 are close in form to the theoretical ones (Fig. 3).

The small deviations of the form of $P_m(\bar{V})$ when the receiver is tuned to the natural frequency of the resonator from the theoretical form (Fig. 3) are due to the shift of the generation frequency at V close to V_r. In the experiment we used a receiver with finite bandwidth (8 MHz) smaller than the really obtained frequency shifts $f_n - f_r$, whereas the theoretical $P(\bar{V})$ dependence corresponds to the total power, i.e., to observation with a receiver having a bandwidth larger than $f_n - f_r$.

We present some quantitative comparisons for the presented data. The excitation voltage V_c must be determined from the condition (11) using in our case ($\Omega_r < 1$) not formulas (10) but the formulas for the impedance of the junction at arbitrary Ω_r[17]:

$$z(0)=1+\frac{(1+\bar{v}^2)^{1/2}-\bar{v}}{2}\left(\frac{1}{\bar{v}-\Omega_r}+\frac{1}{\bar{v}+\Omega_r}\right), \qquad (22)$$

in which, owing to the low noise, we can disregard the influence of the fluctuations. From (11) and (22) we obtain $V_c = 11 \pm 2.5$ μV, which agrees well with the experimental value $V_c = 12 \pm 1$ μV.

To estimate the non-Josephson generation power it is necessary to use the formula for arbitrary Ω_r. Recognizing, however, that the maximum non-Josephson generation power at low fluctuation is equal to the maximum power of the induced Josephson oscillations in the resonator (see Sec. 2.4), we can use the results of[19]. Using the experimental values of Ω_r and $r_r = 1.15$ we obtain from Fig. 1 of[19] $(P_m)_{max} = (1.1 \pm 0.4) \times 10^{-9}$ W. The experimental value $(P_m)_{max}$ with allowance for a loss of 1.5 ± 0.3 dB in the circuit and a transfer coefficient $R_c/(R_c + R_0) = 0.30 \pm 0.06$ from the resonator into the receiver circuit amounted to $(3.5 \pm 1.5) \times 10^{-10}$ W. This small deviation from the theory, towards lower values of the power, is typical of our experiments; its cause is still not clear.

From a comparison of the form of the curves (Fig. 4, Fig. 5a) with the theoretical curves (Figs. 3a, 3b) we can obtain for the regeneration parameter the value $\Delta_c/2\Gamma = 10 \pm 3$. In the theory, the minimum value of this parameter can be estimated by assuming that the noise is purely thermal. In this case we have[1]

$$\Gamma=2ekT/\hbar J_0 \qquad (23)$$

and for our experiment we get $\Gamma = 3.9 \times 10^{-4}$. At $\Omega_r \lesssim 1$, however, it is more correct to use for the Josehson-generation line width not Γ but the quantity Γ_1[19]:

$$\Gamma_1=\Gamma r_d^2(1+\bar{j}^2/2), \qquad (24)$$

which yields $\Gamma_1 \approx 3.4 \times 10^{-3}$ ($r_d \approx 2.3$). From this we get an upper bound for the regeneration parameter $\Delta_c/2\Gamma_1 \lesssim 20$.

The strong discrepancy between the experimental value and the upper estimate is due to the influence of the excess noise and to the externally induced noise, the action of which is close to that of thermal noise. The experimental value of Γ_1 can be determined from the voltage width $\Delta\bar{V}$ of the resonator emission line at large deviations of the receiver frequency from ω_r. It follows from the theory (see formula (26)) that at this value of the detuning we are actually recording the bandwidth of the unperturbed Josephson generation, which turns out to be $\Delta\bar{V} \approx 0.6$ μV. Taking this definition of Γ_1 into account, we obtain for the regeneration parameter the estimate $\Delta_c/2\Gamma = 6 \pm 2$, which is close to the value of $\Delta_c/2\Gamma$ determined from the shape of the non-Josephson generation curves.

For an additional confirmation of the agreement between the observed effects and the developed theory, we performed the following experiments: At fixed parameters of the junction we varied R_c, or else we varied R of the junction at fixed R_c. The form of $P_m(\bar{V})$ was then significantly altered. With increasing R_c or with decreasing R, the start of the generation shifted towards V_r, whereas the voltage on the right-hand slope remained equal to V_r, a fact well explained from the point of view of the theory of non-Josephson generation (see formulas (10) and (11)).

3.3. Deductions from the Experimental Results

The experimental results offer evidence that the theory developed above for non-Josephson generation describes qualitatively the processes that occur in a system comprising a Josephson junction and a resonator, although it does not take into account such aspects of the problem as the influence of smal Ω_r, the finite bandwidth of the resonator, and the deviation of the externally-induced noise from white noise.

The main factor that enabled us to compare correctly the results of the theory and experiment is the high impedance (compared with R) of the resonator and the shunt at the combination frequencies $f_r - \bar{f}$, $2\bar{f} - f_r$, and $\bar{f}$. In all probability, in Kanter's experiments[12] the shunt did not make it possible to satisfy this requirement for the frequency $f_r - \bar{f}$, and this indeed led to an appreciable decrease of the effect.

It should be noted that many junctions produced during the course of the experiments did not conform with the resistive model. Experiments with such junctions yielded a great variety of results; which frequently deviated greatly from the theory. However, the very fact of excitation of non-Josephson generation in the described resonant system was observed by us in most junctions having not too low resistances ($R \gtrsim 0.05\ \Omega$). The effect was so strong, that the generation was excited not only at the fundamental mode of the resonator, but also at its second and third modes.

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APPENDIX

JOSEPHSON AND NON-JOSEPHSON GENERATION

We consider now the question of the validity of the Josephson relation (1) if we replace in this relation the average frequency $\bar{f}$ by the generation frequency f in the conventional sense of this term.

We consider the satisfaction of relation (1) for various definitions of the frequency f, and distinguish between three cases: a) autonomous junction without action of fluctuations and without a resonant system; b) autonomous junction acted upon by fluctuations; c) junction in a resonator but without the action of fluctuations.

1. Let f be that observation frequency f_V at which the spectral density of the function voltage $s_V(\Omega_m)$ is maximal. If there are several maxima of $s_V(\Omega_m)$, then f_V is taken to mean the position of the principal maximum.

a) Relation (1) is satisfied. This follows from the usual resistive model[8]: in this case the voltage spectrum consists of individual harmonic components, and the frequency of the first harmonic is rigorously determined by relation (1).

b) Relation (1) is not satisfied. The smearing of the Josephson generation line by the fluctuations causes f_V to shift relative to $\bar{f}$.[20]

c) Relation (1) is not always satisfied. As seen from Fig. 2, when non-Josephson generation is excited in the resonator, a voltage component appears at the resonator frequency and can exceed the Josephson component.

2. Let f be that observation frequency f_P at which the spectral density of the radiation power from the resonator $s_P(\Omega_m)$ is maximal. In analogy with case 1, if there are several maxima, then f_P should be taken to mean the principal maximum. The value of $s_P(\Omega_m)$ in terms of $s_V(\Omega_m)$ is given by

$$s_P(\Omega_m)=\frac{s_v(\Omega_m)}{|z+z_e|^2}|z_e|, \tag{25}$$

where z_e is the impedance of the external electrodynamic system connected to the junction.

a) Relation (1) is satisfied. The first harmonic component of $s_V(\Omega_m)$ satisfies relation (1) (see case 1), and the impedance factor does not introduce into it any changes.

b) Relation (1) is not satisfied. A shift of the maximum of $s_V(\Omega_m)$ (see Sec. 1b) leads to a shift of the maximum of $s_P(\Omega_m)$.

c) Relation (1) is not satisfied in the region of $\bar{V}$ where non-Josephson generation is excited. This is evidenced by the theory developed in this article for oscillations in a resonator without fluctuations (see 2.4). In the remaining region, where no non-Josephson generation is excited, relation (1) is satisfied (this is seen from formula (12) for the rate $\dot{\chi}_1(t)$ of the phase deviation at a = 0).

It is of interest to consider, for the quantity f_P, the case when the junction is placed in the resonator and fluctuations are present. We consider first the case when the fluctuations are so strong that relation (11) is not satisfied, which leads to the absence of quasi-monochromatic non-Josephson generation. Using the ordinary expression for the Lorentz line of the spectral density $s_V(\Omega_m)$ of the Josephson-oscillation voltage[20] and substituting in (25) the expressions for the impedance z(0) of the junction [Eq. 10)] and for the impedance $z_e = z_r = r_r + i(1 + r_r)\xi$ of the resonant system, where the detuning is $\xi = (\Omega_m - \Omega_r)/h$, we obtain

$$s_P(\Omega_m)=\frac{[r_r^2+(1+r_r)^2\xi^2]^{1/2}}{4\pi(1+r_r)^2(1+\xi^2)(1-2(\Delta_c/2\Gamma)\xi+\xi^2)}\frac{\Gamma_e}{(\Delta_e^2+\Gamma_e^2)} \tag{26}$$

$$\Delta_e=\Delta+\Delta_c/(1+\xi^2),\quad \Gamma_e=\Gamma-2\Delta_c\xi/(1+\xi^2).$$

It is seen from this formula that even in the absence of non-Josephson generation the frequency f_P is shifted upward relative to $\bar{f}$. The shift is most appreciable at $\bar{V} \approx \Omega_r$. Thus for $\bar{V} = \Omega_r - \Delta_c$ the frequency f_P turns out to be equal not to $\Omega_r - \Delta_c$, but to Ω_r, i.e., it is shifted by the rather large value Δ_c.

In the case of excitation of non-Josephson oscillations in the presence of fluctuations, f_P is all the more different from $\bar{f}$. It follows from Fig. 4 that the difference between f_P and the Josephson frequency $\bar{f}$ reached 40% in our experiments. In the region where the non-Josephson generation is not excited, formula (26) becomes valid, and yields as before a deviation of f_P from $\bar{f}$.

3. Let f be the instantaneous frequency $(2\pi)^{-1}d\varphi/dt$.

a) Relation (1) is not satisfied. This follows from the expression for $d\varphi/dt$ in the usual resistive model.[9]

b, c) Not satisfied all the more,

4. Let f be the frequency $(2\pi)^{-1}d\varphi^{(0)}/dt$. The rate of phase change $d\varphi^{(0)}/dt$ is equal to $d\varphi/dt$ averaged over the fast Josephson oscillations.

a) Relation (1) is satisfied. This follows from the averaging of $d\varphi/dt$ in the resistive model[9] over times $\sim\bar{f}^{-1}$.

b) Relation (1) is not satisfied. In the presence of fluctuations, $\dot{\varphi}^{(0)}$ varies relative to its mean value with a frequency $\sim\Gamma$.

c) Relation (1) is not satisfied for the region of $\overline{V}$ where non-Josephson generation is excited. This follows from relation (12) for the rate $\dot{\chi}_1(t)$ of phase deviation. In the region of $\overline{V}$ where non-Josephson generation is not excited we have $\dot{\chi}_1(t) = \Delta$ and $d\varphi^{(0)}/dt = 2\pi\bar{f}$, i.e., relation (1) is satisfied.

5. Let f be the frequency $\bar{f}$ determined by averaging over the period of the lowest frequencies in the system.

Relation (1) is satisfied in any case. This follows from the averaging of relation (3). It is only this definition of the frequency that must be kept in mind when speaking of the fundamental character of relation (1).

Thus, relation (1) is not satisfied for most usual definitions of the generation frequency in the Josephson effect even in the absence of a resonant system. It can therefore be assumed that failure to satisfy this relation in the case of excitation of non-Josephson generation in a resonant system, which is considered in this paper, is in principle a phenomenon that is in no way exceptional.

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1)Here and below the lower-case letters denote quantities normalized to the corresponding junction parameters R, J_0, V_0, ω_0, and $f_0 = \omega_0/2\pi$. The only exception is the time normalized to the characteristic value $\tau_0 = \omega_0^{-1}$, for which we retain the same symbol t as before, and the normalized frequency $\Omega_r = \omega_r/\omega_0 = f_r/f_0$.

2)It is interesting to note that the appearance of non-Josephson generation can also be regarded as a unique single-frequency parametric excitation. [7] It is much more stable to the fluctuations than the usual parametric effects that are suppressed at $\Gamma \sim h \ll |\Delta_c|/2$.

3)The onset of low-frequency relaxation oscillations in the given-voltage (with a shunt) [18] was impossible because the differential resistance of the junction R_d did not assume negative values in the dip (Fig. 5b).

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