Cascade ionization in transparent dielectrics at optical radiation intensities close to the breakdown threshold

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Cascade ionization induced in transparent dielectrics by optical radiation of intensity close to the breakdown threshold is investigated. The dependence of the character of the interaction between the carriers and the phonon field on the carrier energy and on the lattice temperature is taken into account. The extreme case of low lattice temperatures, when the carriers efficiently interact with the zero-point oscillations of the phonon field, and the extreme case of high temperatures, when the carriers are scattered by phonons obeying the equipartition law, are considered. The cascade-development constant and the threshold (with respect to breakdown) values of the light pulse are determined in these cases, with account taken of the energy lost by the carriers to spontaneous phonon generation.

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1. INTRODUCTION

According to the presently-held opinion,[1] cascade ionization is the main process that leads to strong absorption of a high-power light pulse in a transparent dielectric, and is also responsible for the electric breakdown. The development of cascade ionization under the influence of a single pulse of light was investigated quite thoroughly both experimentally and theoretically. In the theoretical analysis, several mechanisms were proposed for the development of the cascade ionization, all of them based on a model proposed by Seitz[2] for electric breakdown in a constant field, namely, the "free" electrons of the conduction band, acquiring in the field an energy $\epsilon \gtrsim I$ (I is the ionization energy), ionize by impact the valence band of the dielectric, after which the process repeats with increasing number of "free" carriers, leading to a cumulative increase of their number.

Molchanov[3] has assumed that the conduction electron acquires an energy $\epsilon \sim I$ in a series of photon–electron–phonon collisions.[1)] Mednis and Fain[4] have developed an approach suitable for stronger electromagnetic fields, and proposed to alternative mechanisms of energy acquisition by the conduction electron, wherein k_0 photons ($k_0 > 1$) are directly absorbed in each collision act from the external electromagnetic field, and the excess momentum is discarded either in a collision with the phonon, as in[3], or in a collision by another conduction electron. As shown by estimates, the fields at which the acquisition mechanisms proposed in[4] become competitive are much higher than the experimentally observed threshold breakdown fields $E_{cr} \sim 10^6$ V/cm, and therefore these schemes of energy acquisition can be disregarded in the field region $E \sim E_{cr}$.[2)]

Finally, Vlasov et al.[5] attempted to refine Molchanov's results[3] by taking into account the carrier energy losses, which were disregarded in[3] and occur in the case of intraband scattering by phonons.[3)] In that case, however, a term that is small at fields close to the breakdown value and contains the cascade constant γ was left out from the kinetic equation for the conduction electron in the electromagnetic-radiation field; therefore the breakdown values of the optical-radiation flux density were determined, in practice, from the condition that the current be stationary with respect to the energy axis, $J(\epsilon)$ = const (corresponding to the absence of electron multiplication). This has led Vlasov et al.[5] to a strong overestimate of the threshold flux density of the radiation energy. It will be shown below that although the correction of the electron energy density, necessitated by the neglect in[5], is small, when it comes to the corresponding corrections to the current $J(\epsilon)$ only the second-order correction is small. The first-order correction, on the other hand, is not small and ensures the possibility of carrier multiplication. It was this correction which determines the cascade constant γ.

In Sec. 2 of this paper we consider the cross section for the absorption of electromagnetic radiation by the conduction-band electrons. In Secs. 3 and 4 we consider cascade ionization in a dielectric at low and high lattice temperatures. We determine also the corresponding cascade-development constants and the threshold (with respect to breakdown) characteristics of the optical radiation. Section 5 contains a comparison of the results with the experimental data and a discussion of the validity of the assumed approximations.

2. ABSORPTION CROSS SECTION AND KINETIC EQUATION FOR ELECTRONS

When a conduction electron absorbs a photon, the energy and momentum conservation laws can be satisfied in the final state by emission or absorption of a phonon; this can occur either before or after the electron absorbs the photon. In the first nonvanishing order of perturbation theory, the transition amplitude M corresponding to absorption of a quantum of electromagnetic radiation of frequency ω by electrons of energy ϵ from the conduction band is the sum of the four diagrams shown in Fig. 1. The matrix element of the transition is calculated in accordance with the usual rules of the diagram technique. During the course of the calculation of M_{fi} one can neglect the energy $\hbar\omega_q \lesssim 2\times 10^{-1}$ of the phonon that takes part in the process, and the momentum k_ω of the radiation quantum in comparison, respectively, with the energy of the optical emission quantum $\hbar\omega \sim 1$ to 2 eV and with the momenta q and p of the phonon and of the electron (from the conservation laws we have $k_\omega/q \sim k_\omega/p \lesssim 10^{-3}$). For the conduction electrons we assume here, in addition, a quadratic dispersion law

$$\varepsilon(\mathbf{p}) = \mathbf{p}^2/2m$$

and the degeneracy is assumed to be small. Under these

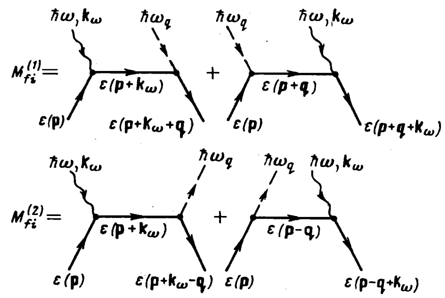

FIG. 1. Graphic representation of the transition amplitude corresponding to absorption of an electromagnetic-radiation quantum by carriers; the wavy, solid, and dashed lines correspond to the photon, electron, and phonon propagators.

approximations, the probability of the process

$$P(\varepsilon,\omega)=\frac{2\pi}{\hbar}\sum_{s,q}|M_{fi}^{(s)}|^2\delta(\mathscr{E}_f^{(s)}-\mathscr{E}_i^{(s)})$$

(where s = 1, 2 corresponds to a transition with absorption and emission of a phonon, respectively and $\mathscr{E}$ is the total energy of the system), after summing over the phonon momentum **q**, turns out to be[4)]

$$P(\varepsilon,\omega)=4\frac{A_\omega(\varepsilon)}{(\hbar\omega)^2}\frac{G_0(p)}{p}f(\varepsilon), \tag{1}$$

where

$$G_0(p)=\frac{Vm}{2\pi\hbar^4}\int_0^{2p}B(q)(2N_q+1)\left(1+\frac{q^2}{4p^2}\right)q\,dq, \tag{2}$$

$B(q)$ is the square of the matrix element of the electron-phonon interaction, $A_\omega(\epsilon)$ is the angle-averaged square of the matrix element of the perturbation produced by a radiation field of frequency ω, $f(\epsilon)$ and N_q are the energy distribution functions of the conduction electrons and of the phonons, V is the volume of the dielectric, and m is the carrier effective mass.

By specifying the optical-radiation field in the form of a plane wave

$$\mathbf{E}=\mathbf{E}_0\cos(\mathbf{k}_\omega\mathbf{r}-\omega t), \tag{3}$$

we get for $A_\omega(\epsilon)$

$$A_\omega(\varepsilon)=\frac{1}{6}\frac{e^2E_0^2}{m\omega^2}\varepsilon, \tag{4}$$

where e is the elementary charge. To obtain the cross section we substitute (4) in (1) and divide the result by the flux density of the incident quanta, which is equal to $\hbar\omega/\overline{W}_n$. Here $\overline{W}_n$ is the average (over the period) value of the Poynting vector in the dielectric. Taking (3) into account, we get

$$\overline{W}_n=\frac{c_n\overline{|\mathbf{E}|^2}}{4\pi}=\frac{c_nE_0^2}{8\pi}$$

where c_n is the velocity of light in the dielectric, and we have for the cross section for the absorption of optical radiation of frequency ω by a conduction electron

$$\sigma_\omega(\varepsilon)=\frac{8}{3}\pi\frac{e^2}{c_nm^2\hbar\omega^3}pG_0(p). \tag{5}$$

The kinetic equation for the energy distribution function $n(\epsilon, t)$ of the conduction electrons[5)] in the field of a light wave takes in the diffusion approximation (with respect energy, $h\omega \ll I$) (see[6,3]) the form

$$\frac{\partial n}{\partial t}=-\frac{\partial J_1}{\partial\varepsilon}+Q, \tag{6}$$

where

$$J_1=nu-D\partial n/\partial\varepsilon,\quad u=D/2\varepsilon,$$
$$D=\overline{W}_n\hbar\omega\sigma_\omega(\varepsilon), \tag{7}$$

and Q describes the electron losses.

Disregarding the region of low energies

$$\varepsilon<mu_l^2\quad(mu_l^2\ll I),$$

where u_l is the velocity of the longitudinal elastic waves, we assume that the predominant process is carrier scattering by acoustic lattice vibrations, an assumption that holds true except for certain compounds with polar bond.[7] Under these conditions the main contribution to Q will be made by the energy losses of the electron to the spontaneous emission of the acoustic phonons, and accordingly[8]

$$Q=\frac{\partial J_2}{\partial\varepsilon},\quad J_2=\frac{1}{p}G_1(p)n,$$
$$G_1(p)=\frac{Vm}{2\pi\hbar^4}\int_0^{2p}B(q)\hbar\omega_q q\,dq, \tag{8}$$

where, according to[7],

$$B(q)=\mathscr{E}_1^2q/2\rho u_lV \tag{9}$$

($\mathscr{E}_1$ is the deformation-potential constant and ρ is the density of the material). Equation (6) takes the form

$$\partial n/\partial t=-\partial J/\partial\varepsilon,\quad J(\varepsilon,t)=J_1(\varepsilon,t)-J_2(\varepsilon,t). \tag{10}$$

The character of the interaction of the conduction electrons with the phonon field depends essentially both on the lattice temperature and on the carrier energy. In the case of predominant acoustic scattering it follows from (2) and (9), with

$$N_q=[\exp(\hbar\omega_q/kT)-1]^{-1} \tag{11}$$

taken into account, that at a carrier energy

$$\varepsilon\gg\varepsilon_0\sim0{,}8(kT)^2/mu_l^2$$

the scattering is due to the zero-point oscillations of the phonon field (effectively we have $N_q \ll 1$ under the integral sign in (2)). Conversely, at $\epsilon \ll \epsilon_0$ the carriers interact with acoustic phonons that obey the equipartition law ($N_q \sim kT/\hbar\omega_q \gg 1$). Owing to the actual form of (11), the transition from one type of scattering to the other occurs in a rather narrow region of ϵ/kT near[7]

$$\varepsilon_0/kT=0{,}8\,kT/mu_l^2,$$

and we can therefore approximately replace the enhanced inequalities by the usual ones. Then, if $0 < \epsilon_0 < I$, it is necessary to take into account in the solution of (6) and (7) the change that takes place in the character of the scattering at $\epsilon \sim \epsilon_0$. The cascade constant γ is determined then not only from the usual boundary conditions considered in the case of cascade ionization[6,3]

$$n(\varepsilon=I)=0,\quad J(\varepsilon=0)=(1+\alpha)J(\varepsilon=I) \tag{12}$$

(where α is the probability that the electrons will pass through the energy region of the inelastic losses and traps), but also from the requirement that n and $\partial n/\partial\epsilon$ (or, more conveniently, n and J) be continuous at $\epsilon = \epsilon_0$. For simplicity we confine ourselves here only to limiting cases, namely, to the case of low lattice temperatures, when $\epsilon_0 \ll I$ and the electrons can be regarded as interacting with the zero-point oscillations of the phonon field in the entire energy region $0 < \epsilon < I$, and the case of high temperatures, when $\epsilon_0 \gtrsim I$ and the carriers in the considered energy region interact with acoustic phonons that obey the equipartition law.

3. LOW TEMPERATURES

Taking into account $N_q \ll 1$ and (9), (2), and (5), we have

$$\sigma_\omega(\varepsilon)=A_0\varepsilon^2, \quad A_0=\frac{8^3}{45}\frac{e^2\mathscr{E}_1{}^2 m}{c_n\hbar^2\rho u_l(\hbar\omega)^3}. \tag{13}$$

We seek a solution of (6) in the form

$$n(\varepsilon, t)=\tilde{n}(\varepsilon)\exp(\gamma_0 t).$$

Then, taking into account (7), (13), and

$$J_2(\varepsilon)=C\varepsilon^{3/2}n, \quad C=4m^3\mathscr{E}_1{}^2/\sqrt{2m}\pi\hbar^4\rho,$$
$$D=2B_0\varepsilon^2, \quad 2B_0=\overline{W}_n\hbar\omega A_0$$

Eq. (6) can be reduced to the form

$$\varepsilon^2\tilde{n}''+(3/2+\eta_0\varepsilon^{1/2})\varepsilon\tilde{n}'-(\varkappa_0+1/2-3/2\eta_0\varepsilon^{1/2})\tilde{n}=0, \tag{14}$$

where $\kappa_0 \equiv \gamma_0/2B_0$ and $\eta_0 \equiv C/2B_0$.

In the region of threshold fields (at $E_0 \gg 10^5$ V/cm) we have $\kappa_0 \ll 1$. We therefore move the term $\kappa_0\tilde{n}$ to the right-hand side of (14) and solve the resultant equation, which is now inhomogeneous with a small right-hand side, by successive approximations[6] (i.e., we seek the solution in the form $n = n_0 + \delta n^{(1)} + \dots$). For $n_0(\epsilon)$ we have

$$n_0(\varepsilon)=\sqrt{\varepsilon}\exp\{-2\eta_0\sqrt{\varepsilon}\}\left(C_1+C_2\int_{-\infty}^{\sqrt{\varepsilon}} z^{-4}\exp\{2\eta_0 z\}\,dz\right). \tag{15}$$

We likewise seek the current $J(\epsilon) = J\{\tilde{n}(\epsilon)\}$ in the form $J = J_0 + \delta J^{(1)} + \dots$. From (10) with allowance for $\partial n/\partial t = \gamma_0 n$, we have

$$\gamma_0 n=-\partial J/\partial\varepsilon. \tag{16}$$

J_0 corresponds to $n_0(\epsilon)$, i.e., it corresponds to $\kappa_0 = 0$ in (14) or to $\gamma_0 = 0$ in (16). Therefore from (16) with $\gamma_0 = 0$ we have $J_0(\epsilon)$ = const, and for $\delta J^{(1)}(\epsilon)$ we obtain

$$\gamma_0 n_0\approx-\partial\delta J^{(1)}/\partial\varepsilon,$$

whence

$$\delta J^{(1)}(\varepsilon)\approx-\gamma_0\int^{\varepsilon} n_0(z)\,dz=-2B_0\varkappa_0\int^{\varepsilon} n_0(z)\,dz. \tag{17}$$

Since $\gamma_0 = 2B_0\kappa_0$ is not small (although $\kappa_0 \ll 1$!), the correction $\delta J^{(1)}$ is not small. All that is small is the second-order correction

$$\delta J^{(2)}\sim\gamma_0\varkappa_0\sim 2B_0\varkappa_0{}^2\ll 1.$$

We now impose the boundary conditions (12). The first of them can be imposed only on $n_0(\epsilon)$, since $\delta n^{(1)}(\epsilon) \sim \kappa_0 \ll 1$. The second, on the other hand, we impose on $J_0 + \delta J^{(1)}(\epsilon)$ since $\delta J^{(1)}$ is not small, whence

$$\delta J^{(1)}(0)=\alpha J_0+(1+\alpha)\delta J^{(1)}(I). \tag{18}$$

Since we have as $\epsilon \to 0$

$$\delta J^{(1)}(\varepsilon)\sim 2B_0\varkappa_0\frac{C_2}{3}\ln\varepsilon,$$

we assume, in order to eliminate the divergence in the left-hand side of (18), $\epsilon = \epsilon_0 \ll I$ instead of $\epsilon = 0$. This cutoff is justified not only because in the derivation of (5) and (8) we have assumed that $\epsilon \gg mu_l^2$, but also by the fact that at $\epsilon < \epsilon_0$ the zero-point-oscillation approximation is violated, and $\delta J^{(1)}_{(0)}$ is finite for the acoustic phonons that obey the equipartition law (see Sec. 4). In addition, at $\epsilon \lesssim mu_l^2$ an important role is assumed by the interaction between the carrier and the optical phonons or the impurities, which also eliminates the divergence of $\delta J^{(1)}(\epsilon)$ as $\epsilon \to 0$. The energy region $0 < \epsilon < \epsilon_0$ is small because $\epsilon_0 \ll I$, and cannot make an appreciable contribution to J (when account is taken of the already mentioned fact that $J(\epsilon)$ is real as $\epsilon \to 0$). We then have from (18), setting the lower limit of the integral in (17) also equal to ϵ_0 for the sake of convenience,

$$2B_0\varkappa_0\int_{\varepsilon_0}^{I} n_0(z)\,dz\approx\frac{\alpha}{1+\alpha}J_0. \tag{19}$$

By direct calculation from (7) and (8), using $n_0(\epsilon)$ from (15), we obtain J_0. Recognizing that the first boundary condition yields

$$C_1=-C_2(2\eta_0)^3\int_{-\infty}^{2\eta_0\sqrt{I}} z^{-4}e^z\,dz,$$

we substitute $n_0(\epsilon)$ in (19). Changing over next to dimensionless integration variables and expanding asymptotically the integrals with respect to the upper limit $2\eta_0\sqrt{I}$ ($2\eta_0\sqrt{I} \gg 1$ in the field region close to threshold, at $E_0 \ll 10^7$ V/cm) up to the principal terms, we obtain

$$\frac{\exp(2\eta_0\sqrt{I})}{(2\eta_0\sqrt{I})^4}-\frac{\ln(2\eta_0\sqrt{\varepsilon_0})}{6}\approx\frac{\alpha}{1+\alpha}\frac{1}{8\varkappa_0}$$

Inasmuch as the argument of the logarithmic term due to the cutoff is $2\eta_0\sqrt{\epsilon_0} \sim 1$, this term can be neglected in comparison with the others. We then have

$$\gamma_0=\frac{1}{8}\frac{\alpha}{1+\alpha}2B_0(2\eta_0\sqrt{I})^4\exp(-2\eta_0\sqrt{I})$$
$$=\frac{3^6 5^3}{2^{11}\pi}\frac{\alpha}{1+\alpha}\frac{\mathscr{E}_1{}^2 m^7 u_l{}^3\omega^6 I^2}{\hbar^4\rho(eE_0)^6}\exp\left[-\frac{45}{8}\frac{m^2 u_l\omega^2\sqrt{I}}{\sqrt{2m}(eE_0)^2}\right] \tag{20}$$

The threshold values (with respect to the onset of the breakdown $E_{0\,cr}$ and $\overline{W}_{n\,cr}$ are then determined from the condition[2]

$$\gamma_0\tau\sim 40, \tag{21}$$

where τ is the duration of the light pulse.

4. HIGH TEMPERATURES

Since the analysis in this case follows Sec. 3 to a considerable degree, we note here briefly only the main results. In the case of high temperatures $N_q \sim kT/\hbar\omega_q \gg 1$. Therefore

$$\sigma_\omega(\varepsilon)=A_T\varepsilon^{3/2}, \quad A_T=8\frac{e^2\mathscr{E}_1{}^2(2m)^{1/2}kT}{c_n\hbar^2\rho u_l{}^2(\hbar\omega)^3}$$
$$D=2B_T\varepsilon^{5/2}, \quad 2B_T=\overline{W}_n\hbar\omega A_T, \quad J_2=C\varepsilon^{3/2}n, \quad C=\frac{4m^3\mathscr{E}_1{}^2}{(2m)^{1/2}\pi\hbar^4\rho}$$

At $n(\epsilon, t) = \tilde{n}(\epsilon)\exp(\gamma_T t)$, Eq. (6) takes the form

$$\varepsilon^2\tilde{n}''+\varepsilon(1+\eta_T\varepsilon)\tilde{n}'-(1/4+\varkappa_T\varepsilon^{1/2}-3/2\eta_T\varepsilon)\tilde{n}=0,$$
$$\varkappa_T=\gamma_T/2B_T, \quad \eta_T=C/2B_T. \tag{22}$$

The term $\kappa_T\epsilon^{1/2} \le \kappa_T I^{1/2} \ll 1$ in the region of breakdown fields (at $E_0 \gg 2\times 10^5$ V/cm).

We seek the solution of (22) in the form $\tilde{n}(\epsilon) = n_0(\epsilon) + \delta n^{(1)}(\epsilon) + \dots$. For $n_0(\epsilon)$ we have

$$n_0(\varepsilon)=\sqrt{\varepsilon}\exp(-\eta_T\varepsilon)\left(C_1+C_2\int_{-\infty}^{\varepsilon} z^{-2}\exp(\eta_T z)\,dz\right). \tag{23}$$

We seek $J(\epsilon)$ in the form $J(\epsilon) = J_0(\epsilon) + \delta J^{(1)}(\epsilon) + \dots$. In this case we obtain

$$J_0(\varepsilon)=\text{const}=-2B_T C_2, \tag{24}$$

$$\delta J^{(1)}(\varepsilon)=-2B_T\varkappa_T\int_{0}^{\varepsilon} n_0(z)\,dz. \tag{25}$$

The correction $\delta J^{(1)}(\epsilon)$ is not small (the small corrections are $\delta J^{(s)}$ with $s \geq 2$), and

$$\lim \delta J^{(1)}(\varepsilon), \quad \varepsilon\to 0$$

is finite. The boundary conditions (12) yield

$$C_1=-C_2\eta_T \int_{-\infty}^{\eta_T I} z^{-2}e^z\,dz, \tag{26}$$

$$\delta J^{(1)}(0)=aJ_0+(1+\alpha)\,\delta J^{(1)}(I). \tag{27}$$

For the sake of convenience we choose a = 0 in (25). We then get from (27)

$$2B_T\varkappa_T\int_0^I n_0(z)\,dz \approx \frac{\alpha}{1+\alpha}J_0.$$

Hence, substituting (23), (24) and (26) and expanding the integrals, in which we first make the integrands dimensionless, in asymptotic series in $\eta_T I$ ($\eta_T I \gg 1$ at $E_0 \ll 2\times 10^7$ V/cm) up to the principal-order terms, we get

$$\frac{\sqrt{\pi}}{2}\frac{\exp(\eta_T I)}{(\eta_T I)^{5/2}} \approx \frac{\alpha}{1+\alpha}\frac{1}{\varkappa_T I^{1/2}}.$$

It follows therefore that

$$\gamma_T=\frac{2}{\sqrt{\pi}}\frac{\alpha}{1+\alpha}2B_T\eta_T^{5/2}I^2\exp(-\eta_T I)$$
$$=\frac{2^{7/2}}{\pi^{3/2}}\frac{\alpha}{1+\alpha}\frac{\mathscr{E}_1^{\,2}(2m)^{1/2}u_l^{\,3}\omega^3m^5I^2}{\hbar^4\rho(kT)^{3/2}(eE_0)^3}\exp\left[-2\frac{u_l^{\,2}\omega^2m^2I}{kT(eE_0)^2}\right], \tag{28}$$

and the threshold values are determined from a condition analogous to (21)

$$\gamma_T\tau\sim 40.$$

5. ESTIMATES OF THE THRESHOLD FIELDS AND DISCUSSION

The calculation of the threshold values $\overline{W}_{cr}$ and E_{cr} (here $E = E_0/\sqrt{2}$ is the effective value of the electric field intensity in (3)) leads to a transcendental equation. In the limit of low lattice temperatures this equation takes the form

$$R_0E_{cr}^{\,6}=\exp(-S_0/E_{cr}^{\,2}), \tag{29}$$

where

$$R_0=\frac{2^{17}\pi}{3^65^2}\frac{1+\alpha}{\alpha}\frac{e^6\rho\hbar^4}{u_l^{\,3}\omega^6m^7\mathscr{E}_1^{\,2}I^2\tau},$$
$$S_0=\frac{45}{16\sqrt{2}}\frac{m^{3/2}u_l\omega^2I^{1/2}}{e^2},$$

and in the case of high temperatures we have

$$R_TE_{cr}^{\,3}=\exp(-S_T/E_{cr}^{\,2}), \tag{30}$$

where

$$R_T=5\pi^{3/2}\sqrt{2}\frac{1+\alpha}{\alpha}\frac{e^3\rho\hbar^4(kT)^{3/2}}{u_l^{\,3}\omega^3m^{11/2}\mathscr{E}_1^{\,2}I^2\tau},$$
$$S_T=m^2u_l^{\,2}\omega^2I/e^2kT.$$

Equations (29) and (30) are similar. (Figure 2 shows a graphic investigation of Eq. (30).) Each of these equations has two roots. It is easy to establish that the larger of the roots is a shortcoming of the assumed approximation, since it leads to E_{cr} and $\overline{W}_{cr}$ that increase with increasing α (in addition, it corresponds already to values of E_0 at which the employed conditions $2\eta_0\sqrt{I} \gg 1$ and $\eta_T I \gg 1$ are violated). The smaller root gives a correct and physically necessary dependence of the breakdown thresholds on α, since these thresholds should decrease with increasing α. After choosing the root, it is easy to trace the temperature dependence of the breakdown

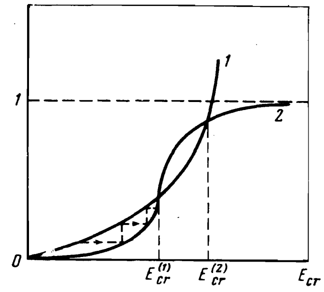

FIG. 2

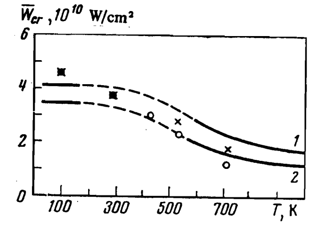

FIG. 3

FIG. 2. Graphic investigation of Eq. (30). Curve 1 corresponds to $R_TE_{cr}^3$ and curve 2 corresponds to $\exp(-S_TE_{cr}^2)$. The root $E_{cr}^{(2)}$ is extraneous. At $\alpha = \alpha_0 \sim 10^{-4}$ the roots come together and go off to the complex plane. The deviation of α_0 from zero is a shortcoming of the assumed approximation. The numerical value of $E_{cr}^{(1)}$ is determined by successive approximation (see the dashed lines with the arrows).

FIG. 3. Dependence of the breakdown threshold on the temperature. Curves 1 and 2 are results of calculations by (29) and (30) in the case of breakdown of leucosapphire and ruby, respectively, by a single pulse from a neodymium laser. The dashed curves denote the extrapolated region of intermediate temperatures; X and O mark the experimental data from [10] for leucosapphire and ruby, respectively.

thresholds. Whereas the threshold values determined by (29) do not depend, naturally, on T, Eq. (30) yields threshold values that decrease with increasing T, namely $E_{cr} \propto T^{-1/2}$ and $\overline{W}_{cr} \propto T^{-1}$. (In addition, in both cases we have $E_{cr} \propto \omega$ and $\overline{W}_{cr} \propto \omega^2$, which agrees with Bloembergen's predictions[1] when account is taken of the fact that $\omega\tau_{eff} \gg 1$.) The temperature region in which the results of the zero-point-oscillation approximation valid is then bounded from below, for reasons given in Sec. 3, by the condition $\epsilon_0 \gg mu_l^2$. and from above by the condition $\epsilon_0 \ll I$. Hence

$$mu_l^{\,2}/k \ll T \ll (mu_l^{\,2}I)^{1/2}/k. \tag{31}$$

For the high-temperature approximation to be valid we need $\epsilon_0 \geq I$, i.e.,

$$T \geq (mu_l^{\,2}I)^{1/2}/k. \tag{32}$$

In the case of breakdown of leucosaphire (ρ = 3.8 g/cm³, $\mathscr{E}_1$ = 11 eV, I = 0 eV, $\alpha \sim 1$, $u_l = 8\times 10^5$ cm/sec, $m \sim m_e$) and ruby (I = 6 eV) by a single pulse from a neodymium laser ($\hbar\omega$ = 1.17 eV, $\tau = 3\times 10^{-8}$ sec), the conditions (31) and (32) represent 4°K ≪ T ≪ 630°K and T ≥ 630°K for leucosaphire and 4K ≪ T ≪ 530°K and T ≥ 530°K for ruby. The results of calculations with (29) and (30), together with the experimental data of Zverev et al.[10] are shown in Fig. 3. The results of the calculation are in good agreement with experiment. The discrepancy between theory and experiment at low temperatures is most probably due to the discarding of $1/6\ln(2\eta_0\sqrt{\epsilon_0})$ when $J(\epsilon)$ is cut off at $\epsilon = \epsilon_0$. On the other hand, the tendency of the discrepancy between theory and experiment to increase with increasing temperature is apparently connected with the lowering of the breakdown threshold, due to the enhanced role of the dynamic heating of the lattice on account of the spontaneous emission of photons by the carriers during the time of action of the pulse.[11] From the results of the calculation it is seen that the strong-field analysis parameter from[4]

$$|eE_0(p-p_0)/\hbar m\omega^2| \ll 1$$

in the region of breakdown fields and therefore the use of ordinary perturbation theory in the derivation of (5) is justified.

Let us now compare (28) with the result of Molchanov.[3] To this end we must put $\eta_T \to 0$ and $C \to 0$. Inasmuch as we used in the derivation of (28) the approximation $\eta_T I \gg 1$, this transition to the limit in the final result is not valid. However, if the limit is taken prior to the use of the asymptotic form, then the result of[3] is reproduced.

We note in conclusion that if the dielectric contains an appreciable amount of impurities, then as the lattice temperature is lowered the carriers become more frequently scattered by the impurities than by the lattice vibrations. The analysis can be carried out in analogy with that given above. The corresponding carrier energy $\epsilon_0(T)$ at which the change mechanisms of carrier scattering change place can be estimated from the condition that the mean free path times of the carriers be the same for scattering by impurities and by lattice vibrations. The character of the dependence of E_{cr} and $\overline{W}_{cr}$ on T then remains the same.

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1)We have in mind impurity-free dielectrics. In the case of a large number of impurities, energy can be acquired via a series of photon-electron-impurity collisions.

2)The mechanisms proposed in [4] become effective at optical-radiation pulse directions $\tau < 10^{-8}$ sec, since the inertia of the ordinary cascade mechanism with $k_0 = 1$, which becomes noticeable with decreasing τ, is cancelled out because $k_0 \geqslant 1$. With further decrease of τ, these mechanisms should go over into the usual multiphoton case.

3)The neglect of the losses in [3] led to an underestimate of the breakdown fields.

4)If the momentum transfer is neglected, the probability $P(\epsilon, \omega)$ can be factored out: (1) breaks up into a product of an electron-photon part by an electron-phonon (G_0) part. G_0 is proportional to the reciprocal free-path time of the carriers in scattering by phonons, τ_{eff}^{-1}. Thus, the factor G_0 effectively separates from the total number of the electron-photon collisions only those in which "simultaneous" scattering by a phonon takes place with loss of momentum, a process that allows the absorption of the electromagnetic quantum.

5)$n = \rho_E f$, where ρ_E is the electron state density per unit energy interval.

6)Equation (14) has an exact solution in the form [9]

$$\tilde{n}(\varepsilon) = \frac{1}{\sqrt{\varepsilon}} \exp(-\eta_0 \sqrt{\varepsilon}) \{C_1 W_{2,m}(2\eta_0 \sqrt{\varepsilon}) + C_2 W_{-2,m}(-2\eta_0 \sqrt{\varepsilon})\},$$

$$2m^2 = 16(\varkappa_0 + 1/2) + 1,$$

where $W_{\lambda,\mu}(z)$ are Whittaker functions. For the sake of uniformity in the analysis of the cases of low and high temperatures (no exact solution of the equation has been obtained in the latter case) and in view of the difficulty of obtaining an explicit expression for the quantity γ in the index of the Whittaker function, we employ here an approximate method.

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