Current flow in long superconducting junctions

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Current states in long superconduction junctions (of the S-C-S type) are investigated. The Josephson model can be used if $a \ll d \ll \xi$ (a is the junction radius, $2d$ is its length, and ξ is the coherence length). The critical current is shown to be determined by the concentration N_s of the superconducting carriers and the current density exceeds the pair-breaking current j_d. At $d \ll \xi$, macroscopic quantization of the system state occurs, thus leading to a nonunique dependence of the current on the phase. Criteria for the realization of branches of the energy $E_n(\varphi)$ and current $I_n(\varphi)$ as functions of the phase φ in the stationary or dynamic regimes are discussed. The amplitude of the alternating Josephson current decreases with junction length like $I_a \sim 1/d$.

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1. INTRODUCTION

The purpose of this work was an investigation of the superconducting currents in inhomogeneous superconductors under conditions when the weak-superconductivity situation is realized. According to the previously adopted classification,[1] weak superconducting junctions include systems of the S-I-S (superconductor-insulator-superconductor), S-N-S (where N is a normal-metal layer), and S-C-S type (where C is a geometrical constriction of radius $\lesssim \xi$) type, and others. The theory of S-I-S and S-N-S junctions has by now been sufficiently well developed (see[1, 2]). As to junctions of the third type (which are possibly of greatest interest for applications[3]), there is no detailed explanation of their properties. Aslamazov and Larkin[4] have shown on the basis of a solution of the Ginzburg-Landau equations that in the dirty limit the constriction between the superconductors can be described by a Josephson model[5] that includes the Josephson current $I_c \sin\varphi$ and the conduction current V/R.

The most interesting are bridges whose dimensions 2d in the current direction are large in comparison with the constriction radius a (we shall call these "long" bridges). With increasing length d, a transition takes place from the behavior typical of a weak junction to the behavior corresponding to a one-dimensional superconducting channel. The main task of the present paper is the analysis of this transition. It will be shown that the current state of the bridge is unique at $d \gg a$, but $d \lesssim \xi(T)$, where ξ is the temperature-dependent coherence length, and a discrete set of allowed states is produced at $d \gg \xi(T)$.

In Sec. 2 we consider the model of a bridge in the form of a narrow superconducting channel in contact with superconducting edges. By analyzing the spreading flow of current injected into a superconductor from an infinitesimally thin filament, we obtain the boundary conditions at the point of contact between the filament and the bulky superconductor. The current states in the S-C-S junction correspond to the value of the current density in the narrowest part of the constriction, which greatly exceeds the pair-breaking current density.[6] In the dirty limit (mean free path $l \ll a$) we obtain the Ambegoakar-Baratov formula for the critical current,[7] and in a pure superconductor ($l \gg \xi_0$) the critical current is determined by the concentration of the superconducting electrons and does not depend on l (i.e., on the resistance in the normal state).

In Sec. 3 we investigate the properties of bridges of length $d > \xi$. In the latter case there arises a situation that recalls the Parks-Little effect[6], or the current states in a superconducting ring in the field of a vector potential. We call this the case of macroscopic quantization, since it is connected with discrete quantum states of the superconducting condensate in a region bounded by the geometric dimension d of the filament. The evolution of such states determines the possible manifestation of the nonstationary Josephson effect in the system. We analyze criteria that make it possible to discriminate between different branches of the dependence of the current on the phase and to predict the behavior of the system in the dynamic regime, i.e., in the presence of an accelerating field.

We shall show that the amplitude of the nonstationary Josephson current decreases with length d like 1/d at $a \ll d \ll \xi$ and $d \gg \xi$, and that in the latter case this amplitude is much smaller than the critical current I_c of the bridge. An important effect is exerted on the appearance of current oscillations in long bridges (of the Josephson type) by fluctuations of the order parameters, but these are not considered in this paper.

2. CRITICAL CURRENTS IN SHORT BRIDGES

In this section we consider bridges whose length is small in comparison with $\xi(T)$, but at the same time is large in comparison with the transverse dimensions (a). Weak superconducting bridges can be simulated in various ways, say in the form of a hyperboloid of revolution (Fig. 1) or in the form of a filament (channel) that joins two superconducting half-spaces (massive "shores") (Fig. 1b). It is clear that the qualitative features of the situation do not depend on the concrete geometric form and are determined only by the length of the bridge 2d and its radius a. An important role is played by the dimensionality of the problem. We consider three-dimensional bridges, in contrast to one-dimensional weak con-

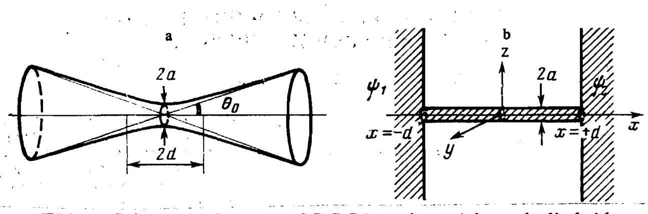


FIG. 1. Schematic diagram of S-C-S junction: a) hyperbolic bridge, b) rectangular bridge.

nections in film bridge-type junctions. For the latter it is impossible to obtain the simple boundary condition at the point of contact with the bulky superconductor, a boundary condition that is derived below in the case of a three-dimensional constriction (S-C-S junction).

Proceeding to the investigation of the three-dimensional bridge, we use the Ginzburg-Landau scheme. In terms of the normalized variables, the Ginzburg-Landau equations take the form

$$\nabla^2\psi+\psi(1-|\psi|^2)=0, \quad (\mathbf{n}\nabla\psi)_S=0, \tag{1}$$

where S denotes the surface of the superconductor, n is a local normal to S, and the unit distance scale in (1) corresponds to the coherence length ξ.

The current flowing through the bridge takes the form (the integration is over the cross section)

$$I=I_0\int ds\,\mathrm{Im}(\psi^*\nabla\psi), \tag{2}$$

where I_0 is a dimensional parameter equal to

$$I_0=2e\hbar\xi|\psi_0|^2/m=N_s e\hbar\xi/2m, \tag{3}$$

N_S is the concentration of the superconducting electrons. Within the limits of applicability of the Ginzburg-Landau theory, the geometric parameters of the bridge a and d are assumed to be large in comparison with the coherence length $\xi(0)$ of the superconductor at T = 0.

In the case of the bridge shown in Fig. 1b, under the conditions $a \ll \xi(T)$ and $d \gg a$ inside the filament, we can solve the one-dimensional Ginzburg-Landau equations, and the shores of the bridge are equivalent here to certain boundary conditions at the points $x = \pm d$.

To determine the boundary condition at the point of contact between a thin filament and a bulky superconducting shore, we must solve Eq. (1) for a half-space with a pointlike singularity corresponding to injection of a current of finite value I into the superconductor (Fig. 2a). One might think that if the filament radius $a \ll 1$ (or if $a \ll \xi$ in dimensional units) the situation shown in Fig. 2a does not differ from spherically symmetrical outflow of current from a sphere of radius a, as shown in Fig. 2b (the arrows show the current flow lines). This situation leads to the following equations (in spherical coordinates):

$$\frac{1}{a^2}\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{dF}{d\rho}\right)-\frac{j^2}{\rho^4F^3}+F(1-F^2)=0, \tag{4}$$

$$j=\frac{1}{a}\rho^2F^2\frac{d\chi}{d\rho}, \tag{5}$$

where F and χ are the modulus and the phase of the order parameter ψ, $j = I/4\pi a^2$ is the current per unit area and measured in units of I_0, while the radius vector ρ is measured in units of a. The boundary conditions for Eq. (4) are

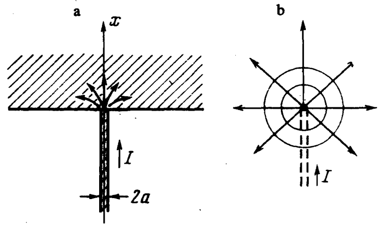


FIG. 2

$$F(\rho)=1 \text{ as } \rho\to\infty,\ dF(\rho)/d\rho=0 \text{ at } \rho=1, \tag{6}$$

where $\rho = 1$ is the radius of the source (the black spot in Fig. 2b) in units of a. The boundary condition at the point $\rho = 1$ corresponds to matching to the solution of the Ginzburg-Landau equation in the channel region in which the current is much smaller than 1/a. As will be shown later on, the critical current of the bridge is of the order of $\sim 1/d$ and satisfies this criterion at $d \gg a$.

Let us find the solution of the boundary-value problem (4), (6), letting $a \to 0$, but leaving the product aj finite. This assumption is subsequently confirmed, inasmuch as a calculation shows that the maximum value of the current that the aperture is capable of passing is of the order of 1/a.

At $a \ll 1$, the equation

$$\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{dF}{d\rho}\right)-\frac{v^2}{4F^3\rho^4}=0, \qquad v=2aj,$$
$$F'(1)=0, \quad F(1)=F_0 \tag{7}$$

can be easily integrated. The solution takes the form

$$F=[F_0^2+(v/2F_0)^2(1-1/\rho)^2]^{1/2}. \tag{8}$$

To determine the constant F_0 we proceed as follows: We consider the exact equation (4). The solution (8) is valid in the range of variation of the variable $\rho \ll 1/a$, in which the discarded term $a^2F(1-F^2)$ is small in comparison with the term $v^2/4\rho^4F^3$.

In the region $\rho \gg 1/\sqrt{a}$, we solve the linearized equation for the quantity $f = 1 - F$:

$$-\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{df}{d\rho}\right)+2a^2f=\frac{v^2}{4\rho^4} \qquad f(\infty)=0, \tag{9}$$

from which we have

$$f(\rho)=\frac{v^2}{8\cdot2^{1/2}a\rho}\left\{\exp(-2^{1/2}a\rho)\int_C^\rho\frac{\exp(2^{1/2}a\rho_1)}{\rho_1^3}d\rho_1\right.$$
$$\left.-\exp(2^{1/2}a\rho)\int_\infty^\rho\frac{\exp(-2^{1/2}a\rho_1)}{\rho_1^3}d\rho_1\right\}. \tag{10}$$

The regions of applicability of the solutions (8) and (10) overlap, since $1/a \gg 1/\sqrt{a}$. Considering the asymptotic forms of the solution (8) at $\rho \gg 1/\sqrt{a}$ and of the solution (10) at $\rho \ll 1/a$, we obtain from the conditions that they match the values of F_0 and C. For F_0 we get

$$F_0^2={}^1/_2(1+\sqrt{1-v^2}). \tag{11}$$

Using (5) we obtain analogously for the phase χ:

$$\chi(\rho=1)=\chi(\rho=\infty)-\mathrm{arctg}(1/F_0^2-1)^{1/2}. \tag{12}$$

It follows from (11) and (12) that at $j \ll 1/a$ the values of F and χ at the point $\rho = 1$ coincide (accurate to terms $\sim(aj)^2$) with their values at $\rho = \infty$. This means that the boundary condition at the point of contact between the bulky superconductor and the channel reduces to continuity of the ψ-function. We note that in the two-dimensional case this condition no longer holds, since the two-dimensional equation analogous to (4) has no bounded solutions as $a \to 0$.

It is curious to note that the solution (8) makes it possible to find the dependence of the current on the phase for a channel of infinitesimally small length ($d \ll a$). In each of the half-spaces we solve the equation of the spherical outflow (7); the boundary condition

$F' = 0$ at the point where the half-spaces are jointed is in this case the natural consequence of the symmetry of the problem. Assuming $\chi(1) = 0$ and calculating the phase at infinity ($\rho = \infty$), we identify it with $\varphi/2$, where φ is the total phase difference. As a result we find

$$\varphi = 2\operatorname{arctg}(1/F_0^2 - 1)^{1/2}. \qquad (13)$$

It follows therefore that $\varphi = \sin^{-1} v$ or

$$j = j_c \sin\varphi, \quad j_c = 1/2a. \qquad (14)$$

We proceed further to the study of the current states in a bridge of length $d \gg a$. In the case of the bridge having the geometry of Fig. 1b we get, taking the obtained boundary conditions into account, the system of equations

$$\psi'' + \psi(1 - |\psi|^2) = 0; \qquad (15)$$

$$\psi = e^{-i\varphi/2} \text{ at } x = -d, \quad \psi = e^{i\varphi/2} \text{ at } x = d, \qquad (16)$$

where φ is the total phase difference on the bridge. The solution at $d \ll 1$ takes the form

$$\psi(x) = \cos\frac{\varphi}{2} + i\frac{x}{d}\sin\frac{\varphi}{2}. \qquad (17)$$

We then have for the current

$$I = I_0 S \operatorname{Im}\left(\psi^* \frac{d\psi}{dx}\right) = \frac{I_0}{2d}\pi a^2 \sin\varphi.$$

Returning to the dimensional variables, we find that the critical current of the bridge is

$$I_c = N_s e\hbar\pi a^2/4md. \qquad (18)$$

This formula agrees with (14) at $d \sim a$.

For a dirty bridge with a mean free path smaller than the constriction radius, expressing I_c in terms of the normal-state resistance R_N, we obtain

$$I_c = \pi\Delta^2/4eR_N T_c, \quad l \ll \xi_0, \quad l \ll a, \qquad (19)$$

which agrees with the usual Josephson expression for the weak-coupling critical current as $T \to T_c$.[4-7]

We note that (18) holds true in both the dirty and the pure limits, and in each limiting case it is only necessary to substitute the corresponding expression for N_s. Thus, using the expression for N_s of an alloy with a mean free path $l \ll \xi_0$,[9] we obtain formula (18). For a pure superconductor we have $N_s = 2(1 - T/T_c)N$ as $T \to T_c$; for $l \gg \xi_0$ we can represent the critical current in the form

$$I_c \sim \pi\Delta^2\xi_0/4eR_N T_c l, \quad a \gg l \gg \xi_0, \qquad (20)$$

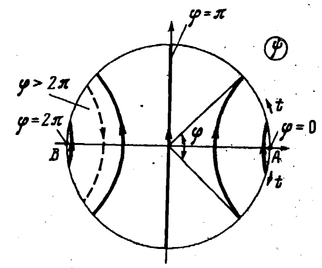
from which we see that I_c is in this case smaller by a factor ξ_0/l than the value given by the Aslamazov-Larkin theory,[4] and does not depend on the mean free path l, since $R_N \sim)/l$.

Formula (18) has a lucid physical meaning. The critical current density $j_c = I_c/\pi a^2$ in the constriction is equal to

$$j_c \sim \frac{N_s e\hbar}{m}(\nabla\varphi)_{max}, \qquad (21)$$

where $(\nabla\varphi)_{max} \sim 1/d$ is the maximum value of the phase gradient. Under the assumptions made, it is much larger than the critical pair-breaking momentum $\nabla\varphi \sim 1/\xi$.[6] It can thus be stated that the weak-superconductivity situation corresponds to stabilization of the current state at pair momenta appreciably larger than the critical pair-breaking momentum $1/\xi$, on account of the local inhomogeneity.

FIG. 3. Phase trajectories for a rectangular bridge.



In an inhomogeneous system, the pair momentum has no definite value, and by virtue of the uncertainty relation $\Delta p \Delta x \sim \hbar$, where $\Delta x = d$, we find that the permissible values of the momentum are at most of the order of Δp, i.e., $p \sim \hbar/d$. These values of the momentum still do not lead to complete suppression of the superconductivity, so that the critical current of the bridge can be $\gtrsim N_s e\hbar/2md$. Formula (18) does not contradict this estimate. Consequently the "weak" superconductivity critical current density in the bridge exceeds the pair-breaking current density j_d, while the small value of the total current I_c is due to the smallness of the radius a of the S-C-S junction.

In the case of a hyperbolic bridge (Fig. 1a), the role of the effective length is played by the quantity $d = a\cot\theta_0$. At $d \gg a$, in coordinates, oblate spheroidal the problem also reduces to one-dimensional. The equation for ψ takes in this case the form

$$\operatorname{ch} u \frac{d}{du}\left(\operatorname{ch} u \frac{d\psi}{du}\right) + d^2 \operatorname{ch}^4 u\, \psi(1 - |\psi|^2) = 0, \quad \psi(\pm\infty) = e^{\pm i\varphi/2}, \qquad (22)$$

where u is a coordinate that identifies the points in the direction of the length of the bridge, $-\infty < u < +\infty$. At $d \ll 1$ we can neglect the nonlinear term in (22). The correctness of this procedure can be proved in analogy with the derivation of the boundary conditions (16) in the case of a filament. As a result, accurate to numerical coefficients, the expression for the critical current of the hyperbolic bridge coincides with (18). Introducing the normal resistance, we arrive at expressions that coincide with (19) and (20).

Let us remark concerning the properties of a bridge in an electric field. If a potential difference V is applied between the superconductors 1 and 2 of Fig. 1b, then φ in (16) varies with time in accordance with the equation

$$2eV = \hbar\dot\varphi \qquad (23)$$

The behavior of the bridge in this case is illustrated by its "phase trajectory"—see Fig. 3. In the complex plane ($\operatorname{Re}\psi$, $\operatorname{Im}\psi$), the order parameter is mapped, as x varies from $-d$ to $+d$, by a point on a line that begins and ends on a unit circle of radius $|\psi| = 1$. In the course of time, this phase curve shifts to the left, and when the point B is reached it is "reflected" and begins to move in the opposite direction, oscillating between the points A and B at a frequency equal to the Josephson frequency $\omega = 2eV/\hbar$.

3. MACROSCOPIC QUANTIZATION AND CURRENT STATES IN LONG BRIDGES

In this section we analyze the properties of bridges whose length exceeds the coherence length ξ. This corresponds to the limit $d \gg 1$ in Eq. (15). Putting $\psi = Fe^{i\chi}$, we obtain an equation for F:

$$F'' - I^2/F^3 + F(1 - F^2) = 0, \qquad (24)$$

where $I = F^2\chi'$ is the conserved current, and from the boundary condition (16) it follows that

$$I\int_{-d}^{d}\frac{dx}{F^2}=\varphi-2\pi n. \tag{25}$$

We call attention to the arbitrary phase shift $-2\pi n$ in formula (25) (n is an integer), which is not fixed by the boundary conditions (the values of ψ on the shores); this is of great significance in what follows.

Equation (24) was analyzed in a number of papers ([10, 11] and others). Its solution is expressed in terms of elliptical functions and has a behavior that decreases monotonically or oscillates in space. For a long bridge, we are interested in solutions that are equal to unity at $x = \pm d$ and tends asymptotically to a certain value $F_0 < 1$ at distances that are large in comparison with unity away from the edge. There exist solutions that do not satisfy this requirement (in particular, a solution that vanishes at the center of the bridge at $j = 0$), but it can be shown that at $d \gg 1$ these solutions correspond to unstable states.

Placing the origin on the edge of the bridge and letting $d \to \infty$, we can write down the solution of interest to us in the form

$$x=\frac{1}{\sqrt{2}}\int_{z}^{1}\frac{dz}{[z(1-z)^2+cz-2I^2]^{1/2}}, \tag{26}$$

where $z = F^2$ and c is the integration constant. For x to become infinite as $z \to z_0 = F_0^2$ the radicand must have coinciding roots ($z_2 = z_3$ in the notation of[11]). From this condition we obtain the connection between I and F_0. In parametric form we have

$$F_0=(1-k^2)^{1/2},\quad I=k(1-k^2),\quad c=k^2(2-3k^2). \tag{27}$$

Solutions of this type exist at $k < 1/\sqrt{3}$.

Substituting (27) in (26), we obtain the function $F(x)$ in implicit form:

$$x=\frac{1}{\sqrt{2}}\int_{z}^{1}\frac{dz}{(z-1+k^2)(z-2k^2)^{1/2}}. \tag{28}$$

Equation (25) can be rewritten in the form

$$I\left[\frac{2d}{F_0^2}+2\int_{0}^{\infty}\left(\frac{1}{F^2(x)}-\frac{1}{F_0^2}\right)dx\right]=\varphi-2\pi n, \tag{29}$$

which yields, when (28) is taken into account,

$$2k(d-d_0(k))=\varphi-2\pi n, \tag{30}$$

$$d_0(k)=\frac{1}{\sqrt{2}}\int_{1-k^2}^{1}\frac{dz}{z(z-2k^2)^{1/2}}. \tag{31}$$

Since $d_0(k)$ is nearly equal to unity, it can be neglected in first-order approximation. At each fixed value of φ, Eq. (30) then determines a discrete set of values k:

$$k=k_n=(\varphi-2\pi n)/2d,\quad n=0,\pm1,\pm2,\ldots \tag{32}$$

and corresponding currents

$$I_n(\varphi)=k_n(1-k_n^2). \tag{33}$$

A plot of the corresponding relations is shown in Fig. 4. Figure 5 shows (apart from a constant term) the energy of the discrete states

$$E=\int_{-d}^{d}\left[\frac{1}{2}|\psi'|^2-\frac{1}{2}|\psi|^2+\frac{1}{4}|\psi|^4\right]dx, \tag{34}$$

the value of which as $d \to \infty$ is

$$E_n\sim d[k_n^2-{}^1\!/_2k_n^4-{}^1\!/_2]. \tag{35}$$

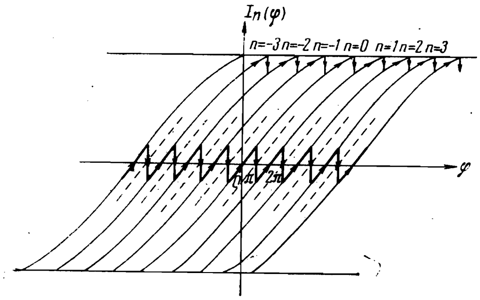


FIG. 4. Branches of the dependence of the current on the phase in a long bridge ($d \ll \xi$). The dashed lines passing through the points $\varphi = (2n + 1)\pi$ show the unstable states of the system.

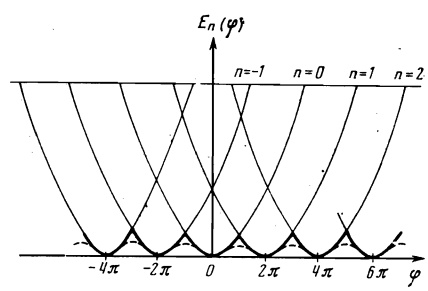


FIG. 5. Dependence of the energy on the phase for a long bridge. The thick line corresponds to the state of absolute energy minimum. The dashed line shows the function $E(\varphi)$ with account of the fluctuations.

A characteristic feature of the obtained picture is that the solutions of the system (24) are multiply-valued at given values of φ. As seen from the foregoing, the picture recalls the current states in a superconducting ring, if φ is taken to mean the integral

$$\frac{2e}{\hbar c}\oint \mathbf{A}\,dl$$

($\mathbf{A}$ is the vector potential). This of course, is not an accident and reflects the deep analogy between the coherent states of all the superconducting systems.

It is easy to illustrate the transition from the behavior described in Sec. 2 to the behavior considered in the present section. The number of solutions of the system (15) at fixed φ is of the order of $N \sim d$ ($N \sim d/\xi$ in dimensional units) and decreases with decreasing d. At $d \ll \xi$ we have a unique solution. Its limiting form in the case $d \ll \xi$ is given by (17).

The most interesting question is what the dynamic behavior of the system will be at zero voltage V. According to (23), φ increases linearly with time in this case. States with different values of n are metastable, and if their decay times (τ) are large in comparison with the Josephson period $T_D = h/2eV$, they evolve in time in accordance with the equation $\dot{n} = 0$. However, when the end of the $E_n(\varphi)$ curve is reached there must inevitably occur a transition to some other state (presumably the closest to the given one[12] $E_{n+1}(\varphi)$). Consequently, the current will depend on the time in this case, as shown by the arrows in the upper part of Fig. 4. On the other hand if $\tau \ll T_D$, then at each instant of time under the influence of the thermal fluctuations (which were not taken into account by us) the system will select the value of n corresponding to the absolute minimum of the energy, and as a result the current will depend on t as shown in the central part of Fig. 4 (solid curve).

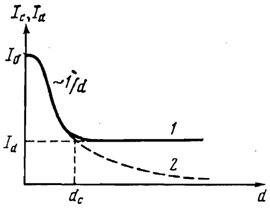


FIG. 6. Dependence of the maximum superconducting current I_c (curve 1) and of the amplitude of the alternating Josephson current I_a (curve 2) on the length of the bridge. At $d < d_c$ the values of I_c and I_a coincide. I_d is the critical pair-breaking current, and $I_0 \sim I_d\xi/a$.

4. CONCLUSIONS

Thus, our analysis shows that the current states in superconducting bridges (S-C-S junctions) are quite unique and differ from the corresponding states in other known types of weak junctions. These features reduce to the following:

1. The density of the critical current of weak superconductivity is quite large; in particular, it exceeds the pair-breaking current density. In the dirty limit (i.e., at a small electron mean free path) the value of the critical current I_c is proportional to l and can be expressed in terms of the resistance of the constriction in the normal state. At $l \ll \xi_0$, the critical current is determined not by the normal resistance but by the concentration of the superconducting carriers.

2. An essential role is played by the dimensionality of the problem. Three-dimensional constrictions can be described by the Josephson current-phase relation $I - I_c \sin\varphi$ at $d \ll \xi$. In film bridges, the phase-difference concept cannot be introduced in the usual manner as the difference between the values of $\arg\psi$ at points far from the constriction region.

3. Starting with a certain critical value of the bridge length $d = d_c \sim \xi$, the state of the system becomes ambiguous and there appear a number of branches ("bands" of the dependences of the Energy $E_n(\varphi)$ and the current $I_n(\varphi)$ on the phase φ. The physical nature of this effect consists in spatial quantization of the macroscopic wave function of the pair condensate in a channel of finite width d, the boundary conditions in which are fixed by the bulky shores. Since φ satisfies the Josephson relation $2eV = \hbar\dot{\varphi}$, the determination of the time behavior of the system reduces to an investigation of the relaxation of the discrete states.

We note that similar properties (discrete spectrum) can be observed also in specially prepared systems consisting of two Josephson (tunnel or point) junctions connected by a long ($d \gg \xi$) superconducting filament.

4. Even at $d \gg \xi$, the bridge can in principle have a Josephson current component that oscillates in time at a frequency $\omega = 2eV/\hbar$, the magnitude of which (which we shall denote I_a) is inversely proportional to the bridge length d. The dependence of the critical current I_c and of I_a on the length is qualitatively illustrated in Fig. 6. Since the Josephson regime is connected at $d \gg \xi$ with discrete transitions between different quantum states, it will be strongly acted upon by the fluctuations and can lead to large widths of the emission line. At sufficiently large d, when the level of these fluctuations becomes noticeable, the Josephson behavior vanishes.

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