π condensate and scattering of pions by nuclei

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The possibility of observing π condensate in nuclei by means of π-meson experiments is considered. For this purpose the nuclear-matter density-modulation amplitude due to the π condensate is calculated as well as the pion polarization operator (with frequencies $\omega \gtrsim 1$). The contribution of N^*-isobar pole to the indicated quantities is consistently taken into account. It is shown that the main effect of the π condensate is density modulation of the nuclear matter. The amplitude for scattering of pions by nuclei is then calculated and the manifestiations of the π condensate in pion scattering are analyzed.

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1. INTRODUCTION

It is shown by A. B. Midgal[1-3] that a π condensate is produced in nuclear matter starting with a certain density n_c. It was found in[4] that in sufficiently heavy nuclei, at a short distance from the surface, a planar layered condensate structure is produced (the same as in an infinite medium). It was noted[2-4] that the π condensate leads to modulation of the density of the nuclear matter. From the results of experiments on electron scattering by nuclei follows the existence of a periodic structure of nuclear matter, and this seems to confirm the assumed presence of the π condensate in the nuclei[4,5]. Other explanations of this structure encounter certain difficulties[5].

A program of research on the scattering of pions by nuclei, using meson factories that provide intense meson beams, is in its initial stage of realization. Since this should increase appreciably the measurement accuracy, it is very important to investigate in greater detail the singularities of the behavior of the pions in nuclei, particularly the influence of the π condensate on pion scattering and other observable properties (see also[4,5] on this subject).

In this article we calculate the density modulation of nuclear matter in the presence of a π condensate. We then calculate the pion polarization operator, which is subsequently used to find the pion-nucleus scattering amplitude. The expression obtained for the polarization operator is applicable also in the theory of pionic atoms, where it makes it possible to take into account effects connected with the condensate.

At low energies, pion-nucleon scattering proceeds mainly via two channels, resonant and pole. Accordingly, the polariation operator of the pions has two terms, $\Pi = \Pi_R + \Pi_p$.

We present first expressions for Π without allowance for the condensate field. As a rule, the contribution of the pole channel of the πN interaction to Π (in the considered energy region) was calculated[6-8] using ordinary (nonrelativistic) Green's functions of the nucleons in the medium (see, e.g.,[9]). However, inasmuch as the condition $\omega \ll k$ is not satisfied for pions, where ω and $\mathbf{k}$ are the frequency and momentum of the pion ($\omega \gtrsim 1$), such a calculation is no longer valid. We calculate next the polarization operator in the presence of π condensate.

We shall show that whereas in vacuum the pole channel contributes mainly to the P-wave scattering of the pion by the nucleon (in an isotropically symmetrical medium the contribution of the S scattering for all pions is practically equal to zero[3]), the density modulation produced by the π condensate in a medium causes the polarization operator to contain a D-wave admixture. Indeed, in a medium the frequency and momentum of the pion are not on the mass shell ($q^2 = \omega^2 - \mathbf{k}^2 \neq m_\pi^2$), and after $\mathbf{k}$ is replaced by $-i\nabla$ a "splitting" m_π^2 begins to manifest itself in an inhomogeneous medium ($n(\mathbf{r}) \neq \text{const}$) even in the lowest-order approximation. A D-wave admixture appears also in the resonant channel and is due to the scatter of the nucleon momentum up to the Fermi boundary.

Finally, we consider the manifestation of the π condensate in the scattering of pions by nuclei under the condition $kR \gg 1$, where R is the radius of the nucleus, when the quasiclassical approximation can be used (with the exception of the pion-energy region near the resonance maximum). Owing to a number of unique effects, experiments on the scattering of pions by nuclei provide a good method of verifying whether the nuclei contain a layered structure due to the pion condensate.

We shall show that the principal correction to the scattering amplitude appears at pion energies above resonance at momentum transfers $q \sim 2k_0$, where k_0 is the momentum of the condensate field (cf.[3]), and examine qualitatively the behavior of the scattering amplitude in this angle region. For simplicity we consider henceforth a neutral pion field. We use pionic units throughout ($\hbar = c = m_\pi = 1$).

2. THE N* ISOBAR IN NUCLEAR MATTER

It is known that when a pion moves in nuclear matter, its frequency and momentum are not on the mass shell (i.e., $q^0 = \omega^2 - \mathbf{k}^2 \neq m_\pi^2$) (see, e.g.,[6-8,10]). This is particularly significant in the analysis of effects connected with the π condensate, the frequency of which is $\omega = 0$ and the momentum $k_0 \approx p_F$. We therefore consider briefly the corresponding behavior of the elastic resonant πN scattering amplitude.

To determine the behavior of the scattering amplitude in vacuum off the mass shell, we can use the well known expression of scattering theory[11,12].

$$f(\mathbf{q}, \mathbf{q}'; \omega) = f^{(0)}(\mathbf{q}, \mathbf{q}'; \omega) + 4\pi \sum_k \frac{f^+(\mathbf{q}; k) f(k; \mathbf{q}')}{\omega_k^2 - \omega^2 - i\delta}. \quad (1)$$

Here $f^{(0)}$ is the scattering amplitude in the Born approximation, and the summation over k extends over the intermediate states. At low energies, the N scattering occurs principally in the P wave (the amplitude of the S-wave scattering is practically equal to zero). This means that

$$f(\mathbf{q}, \mathbf{q}'; \omega) = \mathbf{q}\mathbf{q}' h(\omega, \mathbf{q}). \quad (2)$$

Substituting this expression in (1) and summing over k with allowance for the fact that $\omega_k = (1 + \mathbf{k}^2)^{1/2}$ (cf. the Bethe-Salpeter[3] and Chew-Low[12] equations), we obtain

$$\mathrm{Im}[h(\omega, q)]^{-1} \approx \mathrm{const} \cdot (\omega^2 - 1)^{3/2} \Theta(\omega^2 - 1).$$

The function $\Theta(\omega^2 - 1)$ appeared after circling around the pole $\omega_k = \omega + i\delta$ on account of the condition $\omega_k \geq 1$. Further calculations with concrete forms of the function $h(\omega, \mathbf{q})$ were performed many times[8,12,13]. The result is

$$h(\omega, \mathbf{q}) \equiv h(\omega) \approx \frac{a_0}{\omega_R - \omega - i\gamma(\omega)}, \quad (3)$$

where $\gamma(\omega) = \gamma_0(\omega^2 - 1)^{3/2} \Theta(\omega - 1)$, and $\omega_R \approx 2.4$ is the frequency corresponding to the maximum of the resonance in the πN scattering. The parameters a_0 and γ_0 ($a_0 \approx \gamma_0$) depend little on ω and can be determined from comparison with the experimentally obtained scattering amplitude[3,13]. As indicated in[2,3], the mass of the N* isobar and the πNN* matrix in the medium differ little from the corresponding vacuum quantities. Thus, we can assume that

$$\mathscr{L}'_{\pi NN^*} = f^* \psi_N^+ \mathbf{O} \psi_{N^*} \cdot \nabla \varphi + \text{h.c.}, \quad f^* \sim 1.3, \quad \mathbf{O}^2 = 1,$$
$$G_0^*(p) = \frac{1}{\varepsilon - \varepsilon_p^* + i\gamma(\omega)}. \quad (4)$$

We shall henceforth assume that $\epsilon_p^* \approx \omega_R + \epsilon_p$, and $\epsilon_p = p^2/2m$; $\gamma(\omega)$ was defined above, and the constant f^* was taken in[3] (in the notation of that paper, $f^{*2} = 2 \times 4\pi a \sim 1.6$; cf. also[10]). Inasmuch as the characteristic parameter of the variation of the πNN* vertex is a mass on the order of the nucleon mass, when ω changes from $\omega \sim 1$ to $\omega = 0$ we can neglect the change of this vertex ($m \gg 1$)[3]. We analogously neglect the change of the mass of the isobar as $\omega \to 0$.

Finally, owing to the existence in the medium of low-lying excitations, the damping of the isobar $\gamma(\omega)$ will differ somewhat from its vacuum value. However, experiments on pion scattering by nuclei show that, this difference can be neglected (at $\omega \gtrsim 1$) at least with the exception of the region near the maximum of the resonance.

3. MODULATION OF NUCLEAR-MATTER DENSITY

Assume that a π condensate was produced in nuclear matter. Physically this means that starting with a certain nucleon density n_c it becomes energywise profitable for the system to acquire a periodic static pionic field of the type

$$\varphi_0 = a \sin \mathbf{k}_0 \mathbf{r}. \quad (5)$$

In an infinite system we have $a^2 = 4 |\tilde{\omega}^2(\mathbf{k}_0)| / 3\lambda$[2], and $\omega(\mathbf{k})$ is given by

$$\tilde{\omega}^2(\mathbf{k}) = 1 + \mathbf{k}^2 + \Pi(0, \mathbf{k}) \approx \tilde{\omega}^2(\mathbf{k}_0) + \alpha(k^2 - k_0^2)^2, \quad \alpha > 0$$

(at k close to k_0). Here $\Pi(\omega, \mathbf{k})$ is the polarization operator of the pions, and k_0 corresponds to the position of the minimum of the function $\tilde{\omega}^2(\mathbf{k})$. At a density n close to the critical n_c ($n > n_c$) we have

$$\tilde{\omega}^2(\mathbf{k}_0) \approx \nu(n_c - n) < 0.$$

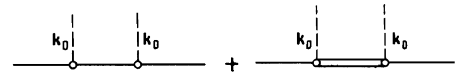
In a finite system, the amplitude a decreases from its value in the volume to zero in a transition layer $\delta \sim 1/|\tilde{\omega}(\mathbf{k}_0)|$[4].

The condensate field (6) leads to modulation of the density of the nuclear matter[2-5]. Appropriate calculations in the Thomas-Fermi approximation make it possible to obtain readily the amplitude of the modulation in the limit of small $k_0 \ll 2p_F$.[14] Let us calculate the magnitude of the modulation without assuming that k_0 is small.

We consider densities n close enough to critical. Then the amplitude a is small and we can seek the magnitude of the modulation density in the form of an expansion in powers of a. The nucleon density is

$$n(x) = -iG_{\alpha\alpha}(x, x_1), \quad \mathbf{r} = \mathbf{r}_1, \quad t = t_1 - 0.$$

In our approximation in terms of the amplitude a, the contribution to the mass operator of the nucleons is made by the diagrams



(6)

The first diagram corresponds to the pole channel of the πN scattering, and the second to the resonant channel. The dashed, solid, and double lines denote the condensate pion, the nucleon, and the N* isobar, respectively.

The first diagram leads to the expression

$$\delta n_N(x) = -\frac{i}{2} f^2 k_0^2 a^2 \int \frac{d^4 p}{(2\pi)^4} G_0(p) G_0(p + k_0) G_0(p + 2k_0) \cos 2\mathbf{k}_0\mathbf{r}.$$

The second diagram leads to

$$\delta n^*(x) = -\frac{i}{2} f^{*2} k_0^2 a^2 \int \frac{d^4 p}{(2\pi)^4} G_0(p) G_0^*(p + k_0) G_0(p + 2k_0) \cos 2\mathbf{k}_0\mathbf{r}.$$

In these relations f and f* are the πNN and πNN* coupling constants, $f = g/2m \approx 1$.[3] The Green's function of the nucleon $G_0(p)$ is given by[9,15]

$$G_{0\alpha\beta}(p) = \delta_{\alpha\beta} \left[\frac{1 - n_p}{\varepsilon - \varepsilon_p + i\delta} + \frac{n_p}{\varepsilon - \varepsilon_p - i\delta} \right]. \quad (7)$$

Substituting $G_0(p)$ and $G_0^*(p)$ in δn, we obtain after calculating the obtained integrals

$$n(x) = n_0(1 + \xi^2 \cos 2\mathbf{k}_0\mathbf{r}), \quad (8)$$

where $\xi^2 = \xi_N^2 + \xi^{*2}$, with

$$\xi_N^2 = \frac{3}{4} \frac{f^2 a^2 p_F^2}{\varepsilon_F^2} \left[\Phi' \left(\frac{k_0}{2p_F} \right) - \Phi \left(\frac{k_0}{p_F} \right) \right]$$
$$\xi^{*2} = \frac{3}{4} \frac{f^{*2} a^2 k_0^2}{\varepsilon_F^2} \frac{\varepsilon_F}{\omega_R} \Phi \left(\frac{k_0}{p_F} \right).$$

The function $\Phi(z)$ is equal to[2,15]

$$\Phi(z) = \frac{1}{2} + \frac{1 - z^2}{4z} \ln \left| \frac{z + 1}{z - 1} \right|, \quad (9)$$

and the momentum p_F is connected with n_0 by the relation $n_0 = p_F^3 / 3\pi^2$. Allowance for the nucleon correlations reduces to multiplication of ξ_N^2 by $[1 + g^- \Phi(k_0/2p_F)]^{-2}$, where g^- is the spin-spin interaction constant of the nucleons ($g^- \approx 1.6$).[13,15] As a result, at a nuclear density $n_0 \approx 0.39$ we obtain, with the nucleon correlations taken into account, $\xi^2 \approx 4a^2$. We note that the first and second diagrams of (6) make approximately numerical contributions to δn.

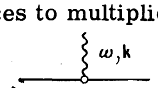
4. PION POLARIZATION OPERATOR WITHOUT ALLOWANCE FOR THE CONDENSATE

We proceed now to calculate the polarization operator of the pions for the scattering problem (i.e., at frequencies $\omega \sim 1$). We consider first the contribution of the pole channel of πN scattering to Π. It should be noted first that expression (7) for the Green's function $G_0(p)$ is valid if the characteristic frequencies of the problem satisfy the condition $\omega \ll k$, $\omega < \epsilon_F$. This condition is satisfied for most excitations of interest (since the velocity on the Fermi surface—the characteristic velocity—is $v_F \ll 1$), and the use of expression (7) does not result in significant errors. On the other hand, in the pion-scattering problem the frequency and momentum satisfy the inverse relation $\omega \gtrsim k$, $\omega \gtrsim 1 \gg \epsilon_F$. Accordingly, to find the contribution of the pole channel to the polarization operator it is necessary to add in (7) one more term corresponding to the relativistic generalization of $G_0(p)$ (other corrections lead to different renormalizations):

$$G_0(p)=\frac{2m}{(\varepsilon+m)^2-(\varepsilon_p+m)^2+i\delta}+2\pi i\delta(\varepsilon-\varepsilon_p)\approx\frac{1-n_p}{\varepsilon-\varepsilon_p+i\delta}+\frac{n_p}{\varepsilon-\varepsilon_p-i\delta}-\frac{1}{\varepsilon+\varepsilon_p+2m-i\delta}, \quad (7')$$

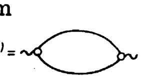
where the nucleons are assumed as before to be nonrelativistic, $\epsilon_p = p^2/2m$.

As will be shown later on (see formula (10)), owing to the introduced term we are now able (since the mass m has been cancelled out) to take into account, besides $k^2/2m$, the term $\omega^2/2m$, which is no longer small. Expression (7′) enables us to treat in a unified manner both the scattering problem and the Fermi-liquid effects, i.e., to obtain an interpolation expression for the polarization operator in different branches of the pion spectrum (the spin-sound; the solution corresponding to instability with respect to condensate formation; the pion spectrum in the scattering problem). Indeed, on the one hand, the parameters of the Green's function of the quasinucleon differ little from the parameters of the Green's function of the nucleon[3,15], and on the other hand, to calculate the interaction of the pions with the nucleons in the nucleus at $\omega \gtrsim 1 \gg \epsilon_F$ we can use the gas approximation, introducing into G constants corresponding to the nucleons in vacuum. Of course, the contribution of the antinucleon pole in the effect corresponding to the condition for the applicability of the theory of the Fermi liquid need not be taken into account. Furthermore, it will be significant only for pions of frequency $\omega \gtrsim 1$. Allowance for the nucleon correlations, as shown in[15], reduces to multiplication of each vertex



by a factor $[1 + g^- \Phi_1(\omega, k)]^{-1}$. As $\omega \to 0$ we have $\Phi_1(k, \omega) \to \Phi(k/2p_F) \sim 1$, and the constant is $g^- \approx 1.6$. Thus, this correction is significant for vertices corresponding to the interaction of nucleons with the condensate. At $\omega \gg kv_F$ we have $\Phi_1(k, \omega) \sim (kv_F/\omega)^2 \ll 1$, and g^- already seems to coincide with its vacuum value $g^-_{vac} \approx 0.8$[3,15]. Consequently, allowance for the nucleon correlations in vertices with $\omega \sim 1$ leads to very insignificant corrections.

The contribution of the pole channel of the πN interaction to the polarization operator of the pions is determined by the diagram

$\Pi_p^{(0)} =$ 

or ($q = (\omega, \mathbf{k})$)

$$\Pi_P^{(0)}(q)=-if^2k^2\int\frac{d^4p}{(2\pi)^4}G_0(p)G_0(p-q)=2f^2k^2\int\frac{d^3p}{(2\pi)^3}n_p$$

$$\times\left\{\frac{1}{\varepsilon_p-\varepsilon_{p-k}+\omega+\omega^2/2m}+\frac{1}{\varepsilon_p-\varepsilon_{p-k}+\omega+\omega^2/2m}\right\}=-f^2k^2\frac{mp_F}{\pi^2}\frac{p_F}{k}\{a_1\Phi(a_1)+a_2\Phi(a_2)\}, \quad (10)$$

where $a_{1,2} = (k^2 - \omega^2 \pm 2m\omega)/2kp_F$, and the function $\Phi(a)$ is defined in (9). We have described the vacuum term due to the antinucleon pole in (7′).

In the limit $\omega \gg kv_F \sim \epsilon_F$ we have

$$\Pi_P^{(0)}(q)\approx-\frac{f^2k^2q^2}{m\omega^2}n_0, \quad (10')$$

where $q^2 = \omega^2 - k^2$. Since the contribution of the nucleon correlations is small at $\omega \gg kv_F$, expression (10′) describes the pole channel of the polarization operator in this approximation. Cancellation of the quantity ω in the numerator (10′) is typical of charged pions only in a medium with Z = N, and for π^0 mesons at arbitrary Z and N. In an isotopically asymmetrical medium no such cancellation takes place for charged ions, and the correction $\omega^2/2m$ turns out to be insignificant (for $\pi^\pm$ mesons in an isotopically asymmetrical medium we have $\Pi^{(\pm)}(-\omega) = \Pi^{(\pm)}(\omega) \neq \Pi^{\pm}(\omega)$[3]).

It was shown in[7] that in the gas approximation in the pion-nucleon interaction

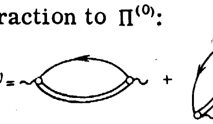
$$\Pi=-4\pi nF(0), \quad (11)$$

F(0) is the amplitude of zero-angle πN scattering in vacuum. If we substitute here the term F_p, which describes the pole channel of the πN amplitude[13]:

$$F_P=-\frac{g^2k^2}{16\pi m^2}\left[\frac{2m}{m_\pi^2-2m\omega}+\frac{2m}{m_\pi^2+2m\omega}\right]\approx\frac{f^2m_\pi^2k^2}{4\pi m\omega^2},$$

then we immediately obtain the relation (10′) at $q^2 = m_\pi^2$ (i.e., if the pion is on the mass shell).

We now obtain the contribution of the resonant channel of the πN interaction to $\Pi^{(0)}$:

$\Pi_R^{(0)} =$ 

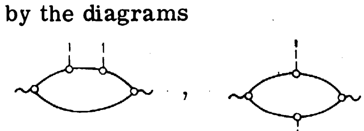
Using (4) and (7′), we obtain far from $\omega = \omega_R$

$$\Pi_R^{(0)}(q)=2f^{*2}k^2\int\frac{d^3p}{(2\pi)^3}\left\{\frac{n_p}{\omega+\varepsilon_p-\varepsilon^*_{p+k}+i\gamma(\omega)}+\frac{n_p}{-\omega+\varepsilon_p-\varepsilon^*_{p+k}+i\gamma(\omega)}\right\}\approx\frac{f^{*2}k^2n_0}{\omega-\omega_R+i\gamma(\omega)}+\frac{f^{*2}k^2n_0}{-\omega-\omega_R+i\gamma(\omega)} \quad (12)$$

where $\gamma(\omega)$ is defined in (3). We have neglected the influence of the scatter of the nucleon velocity up to v_F in the denominator of (12), and the corresponding correction in $\gamma(\omega)$, since $\epsilon_F \ll 1$. The factor 2 is the result of summation over the spin indices. We note that (11) makes it possible in this case to obtain for $\Pi_R^{(0)}$ an expression that is suitable also off the mass shell (far from $\omega = \omega_R$).

5. PION POLARIZATION OPERATOR IN THE PRESENCE OF π CONDENSATE

We calculate first the contribution of the pole channel of the πN interaction to Π. In the approximation lowest in the condensate-field amplitude, the contribution to Π_p is made by the diagrams


(13)

1 2

or, in analytic form

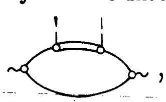
$$\delta\Pi_P^{(1)}=-\frac{1}{4}f^4a^2k_0^2e^{-i\mathbf{kr}}\mathbf{k}\nabla e^{i\mathbf{kr}}\int\frac{d^4p}{(2\pi)^4}\{G_0(p)G_0(p+k_0)G_0(p+2k_0)$$
$$\times[(G_0(p+q)+G_0(p-q_1))e^{-2i\mathbf{k}_0\mathbf{r}}+(G_0(p+q_1)+G_0(p-q))e^{2i\mathbf{k}_0\mathbf{r}}]$$
$$+G_0^2(p)[G_0(p+k_0)+G_0(p-k_0)][G_0(p+q)+G_0(p-q)]\},\quad(14)$$
$$\delta\Pi_P^{(2)}=-\frac{1}{4}f^4a^2(2k_{0i}k_{0k}-k_0^2\delta_{ik})e^{-i\mathbf{kr}}k_i\nabla_k e^{i\mathbf{kr}}\int\frac{d^4p}{(2\pi)^4}\{G_0(p)G_0(p+k_0)$$
$$\times G_0(p+q)G_0(p+q-k_0)e^{-2i\mathbf{k}_0\mathbf{r}}+G_0(p)G_0(p-k_0)G_0(p+q)G_0(p+q+k_0)$$
$$\cdot e^{2i\mathbf{k}_0\mathbf{r}}+2G_0(p)G_0(p-k_0)G_0(p+q)G_0(p+q-k_0)\},$$

where we put $q_1 = (\omega, -\mathbf{k})$ (rather than $q = (\omega, \mathbf{k})$). We have expressed $\delta\Pi_p$ in the coordinate representation in accordance with diagrams (13) and then changed over in the obtained integral to the momentum variables. It is easily seen that $\delta\Pi_p^{(1)}$ describes principally scattering by density modulation (cf. the expression for $\delta n_N(x)$, whereas $\delta\Pi_p^{(2)}$ does not reduce to interaction only with the modulation density due to the condensate. Diagrams containing no condensate field contribute in this approximation to the phenomenological parameters of the theory (such as the nucleon mass, the residue of the Green's function, etc.).

The integrals in $\delta\Pi_p$ can be calculated in the $\omega \gtrsim 1$ limit. As a result we obtain, taking (10′) into account, the following expression for the polarization operator:

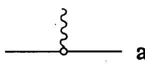
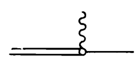
$$\Pi_P\approx-\frac{f^2}{m\omega^2}k_iq_\alpha n(\mathbf{r})q_\alpha k_i+\frac{2f^4a^2}{m\omega^2}\frac{mp_F}{\pi^2}\frac{\Phi(k_0/2p_F)}{[1+g^-\Phi(k_0/2p_F)]^2}\times(1+\cos2\mathbf{k}_0\mathbf{r})([\mathbf{k}\times\mathbf{k}_0])^2.\quad(15)$$

The subscript i runs here through the values 1, 2, and 3, while α runs from 0 to 3 ($q_\alpha q_\alpha = \omega^2 - k^2$). When the factors are arranged in order, it must be borne in mind that $q_\alpha = i\partial_\alpha$ and $\mathbf{k} = -i\nabla$. The expression for $n(\mathbf{r})$ was obtained in Sec. 3 (in addition to $\delta n_N(\mathbf{r})$, one should include in $n(\mathbf{r})$ also $\delta n^*(\mathbf{r})$, for in addition to the diagrams (13) it is also necessary to take into account the diagram


(16)

which corresponds to the second term in (16)). We have taken the nuclear correlations into account in (15) by multiplying by the square of the factor $[1 + g^-\Phi(k_0/2p_F)]^{-1}$.

We proceed now to calculate the contribution made to Π by the resonant channel of the πN interaction. To this end it is necessary to replace in all possible manners the nucleon lines in (13) by the N*-isobar lines, bearing in mind that there are only vertices

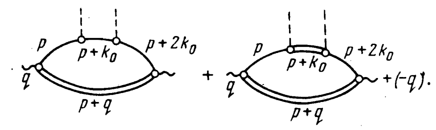
 and 

and similar vertices with a condensate pion. Accordingly, $\delta\Pi_R$ is determined by diagrams with only one and two N*-isobar lines. The analytic expressions for $\delta\Pi_R$ will be similar to expression (14) for $\delta\Pi_p$, in which one or two functions $G_0(p)$ are replaced by $G_0^*(p)$.

We consider the case of pions with energies far from the resonant ω_R. Then the expression for $\delta\Pi_R$ can be represented as an expansion in powers of the parameters

$$\frac{\varepsilon_F}{\omega_R},\quad\frac{kv_F}{\omega},\quad\frac{\varepsilon_F}{|\omega-\omega_R|}\ll1,\quad(17)$$

the last parameter being connected with allowance for the scatter of the nucleon momenta up to the Fermi boundary in the N*-isobar pole. As a result, the principal terms in $\delta\Pi_R$ take the form



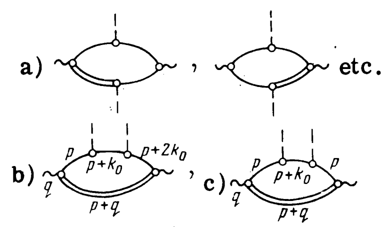
(18)

It is easily seen that it is precisely these diagrams which determine the correction to the density in (12). Consequently, in the lowest-order approximation in the parameters (17), Π_R can be expressed in the form

$$\Pi_R\approx\frac{f^{*2}\mathbf{k}n(\mathbf{r})\mathbf{k}}{\omega-\omega_R+i\gamma(\omega)}+\frac{f^{*2}\mathbf{k}n(\mathbf{r})\mathbf{k}}{-\omega-\omega_R+i\gamma(\omega)}.\quad(19)$$

Here $\mathbf{k} = -i\nabla$ and $n(\mathbf{r})$ is given by (8). This means that in the approximation considered here the π condensate manifests itself only via the nuclear-matter density modulation it produces.

The terms of next order in the parameters (17) are determined from the diagrams



(20)

We omit the intermediate calculations and present the results.

The diagrams (20a) correspond to the expression

$$\frac{f^{*2}k_iA_{ik}(\mathbf{r})k_k}{\omega-\omega_R+i\gamma(\omega)}+\frac{f^{*2}k_iA_{ik}(\mathbf{r})k_k}{-\omega-\omega_R+i\gamma(\omega)},\quad(21)$$

and the diagrams (20b) and (20c) correspond to

$$\frac{f^{*2}\mathbf{k}B_\omega(\mathbf{r})\mathbf{k}}{\omega-\omega_R+i\gamma(\omega)}+\frac{f^{*2}k_ik_kC_\omega(\mathbf{r})k_ik_k}{\omega-\omega_R+i\gamma(\omega)}+(-\omega).\quad(22)$$

Here

$$A_{ik}=-2\beta_{ik}(1+\cos2\mathbf{k}_0\mathbf{r})\frac{\Phi(k_0/2p_F)}{1+g^-\Phi(k_0/2p_F)},$$
$$B_\omega=\beta\frac{\omega_R}{\omega-\omega_R+i\gamma}(1+\cos2\mathbf{k}_0\mathbf{r})\frac{\Phi(k_0/2p_F)}{[1+g^-\Phi(k_0/2p_F)]^2},$$
$$C_\omega=\frac{\beta}{k_0^2}\frac{\omega_R}{\omega-\omega_R+i\gamma}\cos2\mathbf{k}_0\mathbf{r}\frac{\Phi(k_0/2p_F)-\Phi(k_0/p_F)}{[1+g^-\Phi(k_0/2p_F)]^2},$$
$$\beta_{ik}=-\frac{1}{4}f^2a^2\frac{mp_F}{\pi^2}\frac{2k_{0i}k_{0k}-k_0^2\delta_{ik}}{\omega_R},\quad\beta=\beta_{ii}.$$

The factor $[1 + g^-\Phi]^{-1}$ takes into account the nucleon correlations; in (21) and (22) we have $\mathbf{k} = -\nabla$. We did not correct the πNN* vertex, since it has a complicated structure and is less sensitive to the medium[3]. It should be noted that the corrections (21) and (22) at $|\omega - \omega_R| \gg \epsilon_F$ are of the same order as the corrections in (15), so that far from $\omega = \omega_R$ they must be taken into account.

As seen from the results, an admixture of the D-wave type, connected with the distribution of the nucleon momenta up to the Fermi boundary, appears in the polarization spectrum of the pions, We note that whereas the correction (21) is connected only with a condensate, the correction (22) is connected also with the influence of the Fermi distribution on the isobar pole. At low energies these terms are of the same order, while near resonance the terms (22) alter the resonant-type structure determined by (19) more strongly. The parameter

$\epsilon_F/|\omega - \omega_R| \sim 1$, and consequently the corrections from the diagrams (20b) and (20c) will be of the same order as the corrections (18) due to the density modulation.) Finally, from (15), (19), (21), and (22) it follows that at $k \gg k_0$, i.e., if the wavelength of the incident pion is small in comparison with the wavelength of the condensate, and consequently with the wavelength of the density modulation, the influence of the π condensate on the scattering reduces to effects of modulation of the density of matter (even when the corrections are taken into account using the parameters (17)).

Thus, an optical potential with Kisslinger parametrization (see, e.g.,[16,17]) describes satisfactorily the polarization operator of the pions in the presence of a π condensate at high energies ($k \gg k_0$) and at low energies (in the approximation lowest in (17)), with the exception of the region near $\omega \sim \omega_R$.

6. PION SCATTERING BY NUCLEI

We consider now the main features of pion scattering by nuclei at energies far from resonance ($|\omega - \omega_R| > \epsilon_F$). As already noted, the principal manifestation of the π condensate in this region reduces to modulation of the density of nuclear matter, and we therefore confine ourselves to expression (19) for the pion polarization operator. Since the layered structure of nuclear matter should become more strongly manifest in experiments with oriented nuclei, we consider this case first.

We put

$$\Pi(\mathbf{r},\mathbf{k}) \approx h(\omega)\mathbf{k}n(\mathbf{r})\mathbf{k}, \quad h(\omega) \approx f'^2 \frac{2\omega_R}{\omega^2-(\omega_R-i\gamma)^2}. \tag{23}$$

The scattering problem for pions is formulated in the following manner: inside the nucleus we have

$$\Delta\varphi+(\omega^2-1)\varphi=\Pi(\mathbf{r},-i\nabla)\varphi, \tag{24}$$

and outside the nucleus

$$\Delta\varphi+(\omega^2-1)\varphi=0. \tag{24'}$$

We introduce the function $u(\mathbf{r})$:

$$\varphi(\mathbf{r})=u(\mathbf{r})/[1+n(\mathbf{r})h(\omega)]^{1/2}.$$

Then Eq. (24) can be rewritten in the form

$$\Delta u+[k^2-V(\mathbf{r})]u=0, \tag{25}$$

where $k^2 = \omega^2 - 1$, and the effective potential

$$V(\mathbf{r})=k^2\frac{n(\mathbf{r})h}{1+n(\mathbf{r})h}+\frac{\Delta[1+n(\mathbf{r})h]^{1/2}}{[1+n(\mathbf{r})h]^{1/2}} \tag{26}$$

is a periodic function inside the nucleus. Far from the maximum of the resonance we have $V < k^2$ (since $n_0h < 1$). Then in the limit $kR \gg 1$ the scattering amplitude is determined by the expression

$$F(\mathbf{q};V)=\frac{ik}{2\pi}\int e^{-i\mathbf{q}\boldsymbol{\rho}}\left[1-\exp\left(\frac{1}{2ik}\int_{-\infty}^{+\infty}V(\mathbf{r})dz\right)\right]d^2\rho, \tag{27}$$

where $\mathbf{q} = \mathbf{k}' - \mathbf{k}$, the z axis being chosen along the vector $\mathbf{k}$.

The potential $V(\mathbf{r})$ can be represented in the form

$$V(\mathbf{r})=V_0(r)+\xi^2V_1(\mathbf{r}),$$

where the parameter ξ^2, which determines the density modulation amplitude in (8), turns out to be $\xi^2 \approx 0.08$.[5] It follows therefore that in (27) we can carry out an expansion, which is suitable for practically all nuclei, of the exponential in powers of ξ:

$$F(\mathbf{q};V)=F(\mathbf{q};V_0)+\delta F(\mathbf{q}), \tag{28}$$

The amplitude $F(\mathbf{q}; V_0)$ being determined in accordance with (27) from the potential $V_0(r)$. We note that $V_0(r)$ is given by

$$V_0(r)=k^2\frac{n_0(r)h(\omega)}{1+n_0(r)h(\omega)}.$$

Substituting here $h(\omega)$ from (23), we obtain immediately a well known expression for the shift of the resonant frequency in the medium[10]:

$$\omega_R'=\omega_R-f'^2n_0(r).$$

To determine the amplitude $\delta F(\mathbf{q})$, we proceed in the following manner. The potential $V_1(\mathbf{r})$ is equal to ($\delta n_1 = -4k_0^2n_1$)

$$V_1(\mathbf{r})=\left(1-\frac{V_0(r)+2k_0^2}{k^2}\right)V_0(r)\cos 2\mathbf{k}_0\mathbf{r}\equiv V_2(r)\cos 2\mathbf{k}_0\mathbf{r}. \tag{29}$$

Accordingly, the amplitude $\delta F(\mathbf{q})$ turns out to be

$$\delta F(\mathbf{q})=-\frac{1}{4\pi}\xi^2\int e^{-i\mathbf{q}\boldsymbol{\rho}}e^{-i\chi_0(\rho)}V_2(r)\cos 2\mathbf{k}_0\mathbf{r}\,d^2\rho\,dz, \tag{30}$$

with the phase $\chi_0(\boldsymbol{\rho})$ equal to

$$\chi_0(\rho)=\frac{1}{2k}\int_{-\infty}^{+\infty}V_0(r)dz\approx\frac{V_0}{k}(R^2-\rho^2)^{1/2}.$$

Integrating with respect to z in (30), we obtain ultimately $\delta F(\mathbf{q})$ in the form of the sum

$$\delta F(\mathbf{q})=\frac{\xi^2V_2}{8\mathbf{k}\mathbf{k}_0}\{F(\mathbf{q}-2\mathbf{k}_0;V_+)-F(\mathbf{q}-2\mathbf{k}_0;V_-)+F(\mathbf{q}+2\mathbf{k}_0;V_+)-F(\mathbf{q}+2\mathbf{k}_0;V_-)\}, \tag{31}$$

where $V_\pm = V_0 \pm 2\mathbf{k}\cdot\mathbf{k}_0$, and the amplitude $F(\boldsymbol{\kappa}, V_\pm)$ corresponds to scattering by a spherically symmetrical (with respect to the coordinates) potential $V_\pm$ and is determined from formula (27). We note that actually $\mathbf{q} \pm 2\mathbf{k}_0$ contains, according to (27), not $\mathbf{k}_0$ but $\mathbf{k}_{0\perp} = \mathbf{k}_0 - (\mathbf{k}\cdot\mathbf{k}_0)\mathbf{k}/k^2$.

Let us examine briefly the singularities of the amplitude (31). It can be shown that in the angle range satisfying the condition $qR \ll (V_{\text{eff}}R/k)^2$ the amplitude $F(q; V)$ takes the form ($\alpha = VR/k \gg 1$)

$$F(q;V)\approx ikR^2\left(\frac{J_1(qR)}{qR}+\frac{J_0(qR)}{\alpha^2}\right)+kR^2\frac{\alpha}{\beta^2}e^{-i\beta}\left(1-\frac{i}{\beta}\right), \tag{32}$$

where $\beta = [(qR)^2 + \alpha^2]^{1/2}$.

Since $F(q; V)$ has (at $kR \gg 1$) a sharp maximum at $\mathbf{q} = 0$, the function (31) should have extrema at $\mathbf{q} \sim \pm 2\mathbf{k}_0$. We consider $\mathbf{q}$ close to $2\mathbf{k}_0$. Then the function (31) can be rewritten in the form

$$\delta F(q)=\frac{1}{2i}\xi^2kR^2\frac{\alpha_2\alpha_0^2}{\beta_0^3}\left\{\frac{\sin(\beta_+-\beta_-)/2}{2(\alpha_0/\beta_0)\mathbf{n}\mathbf{k}_0R}+i\frac{\beta_0}{\alpha_0^2}\left(1-2\frac{\alpha_0^2}{\beta_0^2}\right)\cdot\cos\frac{\beta_+-\beta_-}{2}\right\}\exp\left\{-i\frac{\beta_++\beta_-}{2}\right\}, \tag{33}$$

where $\beta_{\pm,0} = [(\mathbf{q} - 2\mathbf{k}_0)^2R^2 + \alpha_{\pm,0}^2]^{1/2}$, $\alpha_{\pm,0,2} = k^{-1}RV_{\pm,0,2}$, $\mathbf{n} = \mathbf{k}/k$. The expression (33) is valid under the condition $\mathbf{k}\cdot\mathbf{k}_0 < V_0$. As a function of the angle between $\mathbf{k}$ and $\mathbf{k}_0$, the function $\delta F(\mathbf{q})$ attains its maximum as $\mathbf{k}\cdot\mathbf{k}_0 \to 0$ (i.e., $\mathbf{k} \perp \mathbf{k}_0$). In this case

$$\delta F(\mathbf{q})\approx\frac{1}{2i}\xi^2kR^2\frac{\alpha_2\alpha_0^2}{\beta_0^3}e^{-i\beta_0}. \tag{34}$$

Since $V_2 \sim V_0$, the amplitude of this maximum as $\mathbf{q} \to 2\mathbf{k}_0$ turns out to be $\sim \frac{1}{2}\xi^2\exp\{k^{-1}R\,\mathrm{Im}\,V_0\}kR^2$. Were we to have $\mathrm{Im}\,V_0 = 0$, then the width of this maximum would be determined by the condition $|\mathbf{q} - 2\mathbf{k}_0| < |V_0/k|$ and would amount to several degrees (up to

15°) regardless of the nuclear radius. Since Im $V_0 \neq 0$, the edges of this "flat top" can rise somewhat ($|\mathrm{Im}\,\beta|$ is maximal at $q = 2k_0$ and decreases with increasing $|\mathbf{q} - 2\mathbf{k}_0|$).

Observation of these singularities of the angular distribution is facilitated by the fact that at such large angles ($q \sim 2k_0$) the first term of (28) has already decreased greatly, whereas $\delta F(\mathbf{q})$ is maximal and influences strongly the course of the cross section (for certain angles $\delta F(\mathbf{q})$ is even larger than $F_0(\mathbf{q})$).

At not too large $\mathbf{k} \cdot \mathbf{k}_0 \neq 0$, the picture remains qualitatively the same. It is interesting to note that according to (27), (31), and (32), were it possible to realize the case $\alpha_+ \ll 1$ and $\alpha_- \ll 1$, the amplitude of $\delta F(\mathbf{q})$ as a function of $\mathbf{k} \cdot \mathbf{k}_0$ would reach a maximum also at $2\mathbf{k} \cdot \mathbf{k}_0 = \pm V_0$ (one of the terms in (31) vanishes as $\alpha \to 0$, and the second remains on the order of the black body amplitude $\xi^2 F_{b.b}$). Actually, it is always possible to satisfy the condition $2\mathbf{k} \cdot \mathbf{k}_0 = \pm \mathrm{Re}\, V_0$. The corresponding term in $\delta F(\mathbf{q})$ passes through Re F = 0, and consequently the scattering cross section has an additional energy resonance (with the exception of $\omega \sim \omega_R$). Finally, the amplitude of $\delta F(\mathbf{q})$ decreases strongly (by a factor $k_0 R$) with decreasing angle between $\mathbf{k}$ and $\mathbf{k}_0$.

Thus, by comparing the cross section for scattering by oriented nuclei at different angles $\mathbf{k}$, it would be possible to observe the periodic structure that the π condensate produces in the nuclear matter. In addition, it would be similarly possible, in principle, to observe also the corrections (15), (21), and (22) in the polarization operator.

In experiments on non-oriented nuclei, an averaging over the directions of the vector $\mathbf{k}_0$ takes place in (31) (see[5]), and this leads to a weakening of the role of the corresponding correction to $F(\mathbf{q})$ (for $|\mathbf{q}| \sim 2k_0$): a density distribution of the form

$$n(r) = n_0\left(1 + \xi^2 \frac{\sin 2k_0 r}{2k_0 r}\right).$$

will be observed.

Finally, we indicate two other possibilities of observing the π condensate in nuclei.

As noted in[4], the π condensate leads to deformation of the nuclei, and the direction of the layers is connected with the direction of the nuclear spin. Therefore, if the π condensate does not manifest itself in the density modulation of the nuclear matter, the deformation can be observed in the scattering of different particles and in other effects.

Another possibility is provided by pionic atoms. Inasmuch as the binding energy of the pion in these atoms is of the order of several dozen keV,[18] the condition $\epsilon \gg \epsilon_F$ ($\omega \sim 1$) is satisfied as before. In this connection, if the condensate in the nucleus is of the form (12), then the expression obtained for the pion polarization operator (15), (19), (21), and (22) remains valid. It is easy to estimate that allowance for the condensate in this case introduces into the energy levels a correction amounting to several hundred electron volts. Since experiments with pionic atoms are carried out with very high accuracy (up to several electron volts[18]), the effects connected with the π condensate must be taken into account.

As indicated in[14], there are grounds for the existence in nuclear matter of a three-dimensional condensate field in the form

$$\varphi \sim \sin k_0 x + \sin k_0 y + \sin k_0 z. \tag{35}$$

It can be shown that to take this into account in the expressions derived above for the nuclear-matter density modulation, for the polarization operator, and for the pion scattering amplitude, it suffices to present these quantities in the form of three independent terms for each field component (35) (which differ only in the direction of $\mathbf{k}_0$).

Finally, we note two important circumstances.

1. According to the resonant diagram (6), the π condensate gives rise to a constant shift of the nucleon density $\Delta n \approx -0.4 n_0 a^2$. The point is that the π condensate is a superposition of nucleon-hole and isobar-hole pairs, so that some of the nucleons go over into the N^* isobar that is bound into the condensate.

2. It is indicated in[14] that it is possible to realize a condensate that does not lead to density modulation. Allowance for such a field reduces to discarding the terms proportional to $\cos 2\mathbf{k}_0 \cdot \mathbf{r}$ from the obtained expressions.

Note added in proof (12 May 1975).

1. Relations (7′), (10), and (10′) point to the importance of relativistic corrections in Π_p at $\omega \gtrsim 1$. In the exact calculation, it is necessary to take into account also other corrections that are not considred in the text.

2. Since pion absorption is large in the region of interest to us, the main effects in the scattering are due to a narrow layer near the surface of the nucleus, where the role of the condensate is small but the diffuseness of the nuclear edge is appreciable. Therefore all the results are only qualitative in character. The correction (34) amounts to ~10%.

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