General type of solutions of the Landau–Lifshitz equations

V. M. Eleonskiĭ, N. N. Kirova, and V. M. Petrov

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A general type of spatially one-dimensional solutions of the Landau–Lifshitz equations is investigated with two angular degrees of freedom of the magnetic-moment vector taken into account. It is shown that besides the well known solutions corresponding to the Bloch or Néel domain structures, a number of solutions exist corresponding to domain structures with two angular variables. The qualitative analysis of the problem, confirmed by numerical calculations, indicates the existence in the spatially one-dimensional case of domain structure with a change of the plane of rotation of the magnetic-moment vector. It is shown that among the solutions of the Landau–Lifshitz equations are contained solutions corresponding to isolated magnetic domains with several nodal lines and a change of the plane of rotation of the magnetic-moment vector which is localized in space. In a zero external field such solutions correspond to domain walls, and also simple waves of magnetization characterizable by two angular degrees of freedom are related to these solutions.

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1. In connection with the investigation of domain structures and, in particular, domain walls on the basis of the Landau-Lifshitz equations,[1] the model of a planar Bloch (or Néel) domain wall is widely used, where according to this model the magnetic-moment vector, which is undergoing rotation, remains, upon a change of the spatial variable, in a plane which is parallel (or perpendicular) to the plane of the domain wall.

However, although the spatially one-dimensional solutions corresponding to such a model are important, yet they are only particular solutions of the Landau-Lifshitz equations. A more general type of solutions is associated with those distributions of the magnetization for which a plane of rotation of the magnetic-moment vector, which remains constant in space, does not exist. In other words, such solutions are characterized by two angular degrees of freedom—in contrast to the Bloch or Néel solutions, which correspond to a single degree of freedom for the magnetic-moment vector.

The relative difference between the energies of Bloch and Néel domain walls, for which the angle between the constant planes of rotation of the magnetic-moment vector is equal to $\pi/2$, is determined by the characteristic parameter of the magnetic medium

$$\varepsilon=2\pi M_s^2/K, \tag{1.1}$$

where M_s is the saturation magnetization, K denotes the uniaxial anisotropy energy, and is small if the parameter $\epsilon \ll 1$. One can conjecture that for magnetic media with a low saturation magnetization and a high energy of anisotropy, the realization of domain structures with two angular degrees of freedom is possible, one of which determines the change of the plane of rotation of the magnetic-moment vector.

Below it is shown on the basis of an analysis of the Landau-Lifshitz equations that, in the spatially one-dimensional case (all quantities depend on only the variable x) a sequence of solutions, corresponding to magnetic domains with two degrees of freedom, exists in the presence of an external magnetic field. The distribution in space of the projection m_z of the magnetic-moment vector on the direction of the external field h_z is shown in Fig. 1 a for one of these solutions. For comparison the distribution of m_z for a magnetic domain with one angular degree of freedom, namely, a domain of the Bloch type, is depicted by the dotted line on the same figure, In the last case $m_x \equiv 0$ and the rotation of the magnetic-moment vector occurs in the plane of the domain wall (i.e., in the plane $\varphi = \pi/2$, if the direction of the unit magnetic-moment vector is defined by the angles θ and φ of the spherical coordinate system with axis directed along the axis of anisotropy). In contrast to a domain of the Bloch type, the found solution is characterized by the presence of two regions of inverted magnetization. The projection of the corresponding integral curve on the (θ, φ) plane, depicted in Fig. 1 b, shows that asymptotically as $x \to \pm\infty$ the magnetic-moment vector reaches equilibrium positions corresponding to uniform magnetization along the external field, undergoing rotations in the "Bloch" planes $\varphi = \pi/2$ and $\varphi = 3\pi/2$. Such behavior of the solutions guarantees the vanishing of the internal fields of demagnetization in the region of establishment of uniform magnetization.

We note that, in spite of the existence of a solution

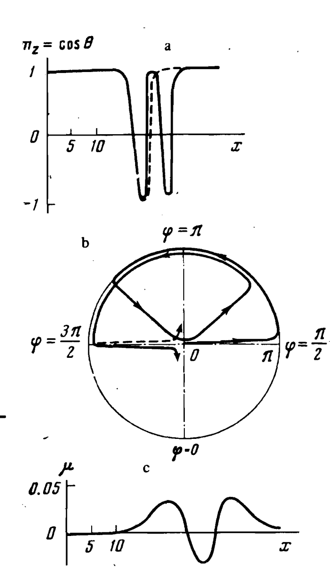

FIG. 1

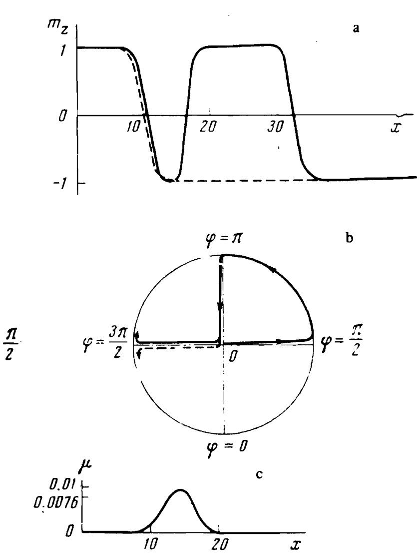

FIG. 2

FIG. 1. a—The distribution of m_z in space for a domain with two angular degrees of freedom ($h_z = 0.01$; $\epsilon = 0.1$); b—the projection of the integral curve of an isolated domain on the plane (θ, φ); c—the distribution of the torque μ in space for an isolated domain.

FIG. 2. a—The spatial distribution of m_z for a domain wall with two angular degrees of freedom ($h_z = 0$; $\epsilon = 0.1$); b—the projection of the integral curve of the domain wall on the plane (θ, φ); c—the distribution of the torque μ in space for a domain wall.

corresponding to a domain wall of the Néel type with one degree of freedom, solutions of the Landau-Lifshitz equations do not exist corresponding to magnetic domains with two angular degrees of freedom, in which the magnetic-moment vector would be found in the "Néel" planes $\varphi = 0$, $\varphi = \pi$ upon attainment of the region of uniform magnetization. From an examination of Fig. 1 b it follows that if the external domain walls are distributed in Bloch planes, then the internal domain walls, yielding a layer with a magnetic moment directed along the external field, are located in planes $\varphi \lesssim \pi/2 + \pi/4$ and $\varphi \gtrsim 3\pi/2 - \pi/4$, which are separated from the Néel plane $\varphi = \pi$. Such an arrangement of the internal domain walls leads to demagnetization fields that are small in comparison with a Néel domain wall. Finally, the distribution of the torque $\mu = \sin^2\theta\ (d\phi/dx)$ for a magnetic domain with two angular degrees of freedom is shown in Fig. 1 c. The magnitude of the torque is such that, at any rate, $\mu^2 < \epsilon$.

One of the solutions of the Landau-Lifshitz equations in the absence of an external field, corresponding to a domain wall with two angular degrees of freedom, is depicted in Fig. 2. Notably, the distribution in space of the component m_z of the magnetic moment is shown in Fig. 2 a. For comparison, the distribution of m_z in a domain wall of the Bloch type is indicated by the dotted curve on the same figure. In contrast to a Bloch wall, in the solution cited above with two angular degrees of freedom there are, as is clear from Fig. 2 b, two "external" domain boundaries of Bloch type and an "internal" domain boundary of the Néel type. The spatial distribution of the torque $\mu = \sin^2\theta\ (d\varphi/dx)$ in the domain wall is depicted in Fig. 2 c. We note that the torque tends to zero as $x \to \pm\infty$.

It should be pointed out that the existence of magnetic domains and domain walls with two angular degrees of freedom, such as those described above, is an exact and rigorous consequence of the Landau-Lifshitz equations. In what follows the existence of other types of solutions (in particular, periodic solutions) with two angular degrees of freedom will be indicated. However, the latter are of direct physical interest only to the extent of their proximity to detached solutions (i.e., to solutions of the domain or domain-wall type).

In[2] it is mentioned that a systematic transition from a stationary domain structure of the Bloch type to the corresponding simple, magnetic-moment wave (i.e., to a wave propagating with constant velocity and without a change of shape) leads in the general case to distributions of the magnetic moment which depend on two angular degrees of freedom. However, for simple waves propagating in the plane orthogonal to the axis of anisotropy, only a particular, exact solution, corresponding to a simple wave with one angular degree of freedom, is obtained in[2]. In this case, although the position of the plane of rotation of the magnetic moment is constant in space, yet it is determined by the velocity of the simple wave and coincides with the position of the Bloch plane only in the limit when the velocity is equal to zero.

The analysis of the Landau-Lifshitz equations set forth below allows one to reach a number of conclusions about the properties of simple waves of magnetization allowing for two angular degrees of freedom. Such a possibility is due to the fact that, for a nondissipative magnetic medium the Landau-Lifshitz equations admit the existence of a first integral even in the case of simple waves.

In the absence of an external field, the velocity of a simple wave corresponding to a moving domain wall with two angular degrees of freedom is related to the asymptotic positions $\varphi_\pm \equiv \varphi(\pm\infty)$ of the plane of rotation of the magnetic moment by the same equation which was also found in[2], notably

$$u^2 = \frac{\varepsilon^2 \cos^2\varphi_\pm \sin^2\varphi_\pm}{1+\varepsilon\cos^2\varphi_\pm}. \quad (1.2)$$

Here u denotes the velocity of the simple wave in units of the characteristic velocity, which is equal to $2\gamma\sqrt{AK/M_S}$, where γ is the gyromagnetic ratio, A is the exchange energy constant, and the angles $\varphi_\pm$ are either equal or differ by π. It is obvious that the continuous spectrum (1.2) of the velocities is bounded from above by the value $u_{max} = \sqrt{1+\epsilon} - 1$, just as in the case of simple waves of magnetization with a single angular degree of freedom, when $\varphi_+ = \varphi_- = \varphi(x) \equiv \text{const}$. We note that the agreement of the velocity spectrum for simple waves of magnetization with one and two angular degrees of freedom is intrinsically related to the nondissipative nature of the magnetic medium and does not take place when attenuation and external fields are taken into consideration.

In our opinion, the investigation of general spatially one-dimensional solutions of the Landau-Lifshitz equations allowing for two angular degrees of freedom, indicating the possibility of the existence of new, unusual types of domain structures and the simple waves associated with them, is of obvious physical interest.

The investigations[3-5] carried out in recent years of magnetic comains in rare-earth iron-garnets of complicated composition, for which the parameter $\epsilon \lesssim 0.1$, showed that the structure of the domain wall may be of a simple Bloch type as well as a complex type, represented by (upon displacement along the wall) a periodic structure of Bloch and Néel segments, separated by transition layers. Although such complicated domain walls are spatially non-one-dimensional structures with two angular degrees of freedom, it should be noted that a number of characteristic properties such as, how the plane of rotation of the magnetic-moment vector is turned or the existence of a periodic structure of Bloch and Néel domain walls already arises in connection with the analysis of the general, one-dimensional solutions of the Landau-Lifshitz equations. One can advance the hypothesis that the general type of spatially one-dimensional solutions of the Landau-Lifshitz equations allowing for two angular degrees of freedom appears to be useful in connection with the analysis of the properties of such complicated domain structures.

2. In the spatially one-dimensional case, the Landau-Lifshitz equations for a magnetic, nondissipative medium with uniaxial anisotropy can be written in the form

$$\begin{gathered}\frac{d^2\vartheta}{dx^2} - \left(1 + \frac{\mu^2}{\sin^4\vartheta} + \varepsilon\cos^2\varphi\right)\sin\vartheta\cos\vartheta - h_z\sin\vartheta + \frac{u\mu}{\sin\vartheta} = 0,\\ \frac{d\mu}{dx} + u\sin\vartheta\frac{d\vartheta}{dx} + \varepsilon\sin^2\vartheta\cos\varphi\sin\varphi = 0, \qquad \mu = \frac{d\varphi}{dx}\sin^2\vartheta,\end{gathered} \quad (2.1)$$

Here θ and φ are the angular variables of the magnetic-moment vector, which depend on $x - ut$ in the case of a simple, wave-type solution; $h_z = H_z M_S/2K$ where H_z is the external magnetic field, parallel to the axis of anisotropy; finally the spatial variable is expressed in units of the characteristic thickness of the Bloch domain wall, which is equal to $(A/K)^{1/2}$.

An important property of the system of equations (2.1) is the existence of the first integral

$$\left(\frac{d\vartheta}{dx}\right)^2+\frac{\mu^2}{\sin^2\vartheta}-(1+\varepsilon\cos^2\varphi)\sin^2\vartheta+2h_z\cos\vartheta=\mathscr{H}, \qquad (2.2)$$

which does not explicitly depend on the velocity of the simple wave, and the obvious inequality related to it

$$\left(\frac{d\vartheta}{dx}\right)^2=\mathscr{H}-\frac{\mu^2}{\sin^2\vartheta}+(1+\varepsilon\cos^2\varphi)\sin^2\vartheta-2h_z\cos\vartheta\geqslant 0. \qquad (2.3)$$

The Landau-Lifshitz equations (2.1) correspond to a nondissipative dynamical system with two angular degrees of freedom, and the four-dimensional phase space ($d\theta/dx$; θ; μ, φ) or ($d\theta/dx$, θ; $d\varphi/dx$, φ) is the corresponding phase space. However, owing to the existence of the first integral (2.2) an analysis of the behavior of the integral curves in the three-dimensional space (θ, φ, μ) or (θ, φ, $d\varphi/dx$) is useful in connection with the investigation of the solutions of Eqs. (2.1).

In fact, in the space (θ, φ, μ) the inequality (2.3) singles out the inside of the cylinder associated with

$$0\leqslant\vartheta\leqslant\pi,\ 0\leqslant\varphi\leqslant 2\pi;\ -\infty<\mu<+\infty$$

as a closed system in which all integral curves with a given value of the first integral $\mathscr{H}$ are located.

For u = 0 the system of Eqs. (2.1) possesses the following states of equilibrium ($d\theta/dx \equiv \mu = 0$):

$$\vartheta=0,\ \varphi=n\pi;\ \vartheta=0,\ \varphi=(2n+1)\pi/2 \qquad (2.4)$$

where n is an integer, corresponding to uniform magnetization along the external field. Here the value of the constant first integral is $\mathscr{H} = +2h_z$. Therefore, for $\mathscr{H} = +2h_z$ all the solutions of the detached type, corresponding to isolated magnetic domains, will be located inside the boundary surface $P(\theta, \varphi, \mu; 2h_z)$, which is defined by the equation

$$\left(\frac{d\vartheta}{dx}\right)^2\equiv P^2=2h_z-\frac{\mu^2}{\sin^2\vartheta}+(1+\varepsilon\cos^2\varphi)\sin^2\vartheta-2h_z\cos\vartheta=0, \qquad (2.5)$$

However, solutions of other types (for example, corresponding to a periodic domain structure with interchange of Bloch and Néel domain walls) will also be located inside the boundary surface (2.5), whose structure is depicted in Fig. 3.

Since $P \equiv d\theta/dx \neq 0$ inside the allowed region defined by the inequality (2.3), and P vanishes only on the boundary surface $P(\theta, \varphi, \mu; \mathscr{H}) = 0$, it follows that upon an increase of the spatial variable any integral curve in the space (θ, φ, μ) must inevitably be carried out to the boundary surface P = 0 and after contact with it again departs into the depths of the allowed region.

It may happen that the integral curve, which emerges from the equilibrium position corresponding to uniform magnetization as $x \to -\infty$ again enters (as $x \to +\infty$) the same equilibrium position after making a finite number of contacts with the boundary surface P = 0. Such detached curves correspond to isolated magnetic domains, characterized in the general case by two angular degrees of freedom. Integral curves corresponding to isolated magnetic domains with a single angular degree of freedom are located in the intersection of the allowed region $P \geq 0$ by the plane $\mu = 0$. Notably, isolated Bloch domains and Néel domains correspond, respectively, to motion along the rays $\varphi = (2n + 1)\pi/2$ and $\varphi = n\pi$ for $0 \leq \theta \leq \pi$.

In order to realize the possibility of the existence of

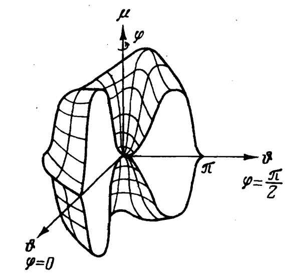

FIG. 3

isolated comains with two angular degrees of freedom, it is necessary that the equilibrium positions (2.4) be the source of a set of emerging (for $x \to -\infty$) and incoming (for $x \to +\infty$) integral curves. In this connection only those integral curves, which correspond to $\mu \neq 0$ in the neighborhood of the singular point ($\mu = 0$, $\theta = 0$) lead to distributions of the magnetic moment with two angular degrees of freedom. In order to investigate the asymptotic behavior of the solutions near the singular point, let us change from the space (θ, φ, μ) to the space (θ, φ, $d\varphi/dx$). For $\theta \ll 1$ and the constant first integral $\mathscr{H} = +2h_z$, by using expression (2.2) we find that

$$P\equiv\frac{d\vartheta}{dx}=\pm\left[1+h_z+\varepsilon\cos^2\varphi-\left(\frac{d\varphi}{dx}\right)^2\right]^{1/2}\vartheta. \qquad (2.6)$$

In this connection the second of the equations in the system (2.1) leads to the equation

$$\frac{d^2\varphi}{dx^2}+2(\operatorname{sign}P)\left[1+h_z+\varepsilon\cos^2\varphi-\left(\frac{d\varphi}{dx}\right)^2\right]^{1/2}\frac{d\varphi}{dx}+\varepsilon\cos\varphi\sin\varphi=0. \qquad (2.7)$$

Analysis of this equation showed that, in the vicinity of the equilibrium positions $\theta = 0$, $\varphi = (2n + 1)\pi/2$, where n is an integer, the incoming and outgoing integral curves are located in the detached planes

$$d\varphi/dx=\pm[(1+h_z+\varepsilon)^{1/2}-(1+h_z)^{1/2}][\varphi-(2n+1)\pi/2]. \qquad (2.8)$$

In the space (θ, φ, $d\varphi/dx$) the "incoming" and "outgoing" curves themselves are determined by the intersection of the detached planes (2.8) with the one-parameter family of surfaces

$$\varphi=(2n+1)\frac{\pi}{2}+C\vartheta^a, \quad a\equiv 1-\left(\frac{1+h_z}{1+h_z+\varepsilon}\right)^{1/2}. \qquad (2.9)$$

where C is the only free parameter in the problem. In the space (θ, φ, μ) the detached planes (2.8) correspond to the detached surfaces

$$\mu=\pm[(1+h_z+\varepsilon)^{1/2}-(1+h_z)^{1/2}]\vartheta^2[\varphi-(2n+1)\pi/2]. \qquad (2.10)$$

The origin of the solutions, corresponding to isolated domains with two angular degrees of freedom, results from the fact that after a finite number of contacts with the boundary surface the integral curve emerging (as $x \to -\infty$) from the singular point may again pass into the singular point (as $x \to +\infty$). The actual realization of such an event is possible only at a definite—characteristic—value of the parameter C. Therefore, out of the continuous set of values of the parameter C the detached solutions correspond to a denumerable set, which may be ordered according to the number of contacts of the integral curve with the boundary surface between emergence and entrance to the singular point. If $h_z = 0$, then a second position of equilibrium $\theta = \pi$ appears, which leads to the possibility of the existence of domain walls separating the regions of uniform magnetization $\theta = 0$ for $x \to -\infty$ and $\theta = \pi$ for $x \to +\infty$. For $h_z = 0$ the system

(2.1) is invariant under the substitution $\theta \to \pi - \theta$; therefore, the asymptotic behavior near $\theta = \pi$ can be obtained by putting $h_z = 0$ in Eqs. (2.8), (2.9), and (2.10) and by replacing θ by $\pi - \theta$. The determination of solutions of the domain-boundary type with two degrees of freedom, possessing nodal lines, is completely analogous to the problem of isolated domains in an external field.

The analogy between the analysis of the detached solutions expounded above and the well known, so-called "billiards problems"[6] should be noted. In particular, for an arbitrarily chosen value of the parameter C, one would expect with a probability close to unity that the integral curve emerging from the singular point as $x \to -\infty$ will undergo a certain motion inside the boundary surface even for an unlimited increase in the number of contacts and, consequently, it does not return to the singular point as $x \to +\infty$. This type of solutions corresponds to a transition from the region of uniform magnetization along the external field (for $x \to -\infty$) to a domain structure characterized by two angular degrees of freedom (for $x \to +\infty$). Finally, for the same value $\mathscr{H} = +2h_z$ of the constant first integral, solutions are possible for which the integral curves do not reach the singular point either as $x \to -\infty$ or as $x \to +\infty$, notwithstanding the unlimited number of contacts with the boundary surface. A domain structure, characterized by two angular degrees of freedom and occupying all of space, corresponds to this type of solutions.

Let us turn our attention to the fact that the existence of the three types of solutions enumerated above for a given value of the constant first integral is due to taking account of two angular degrees of freedom in the Landau-Lifshitz equations.

The existence of two degenerate solutions with one angular degree of freedom, corresponding to the domain structures of Bloch and Néel, indicates that under certain conditions the last of the above enumerated types of solutions with two degrees of freedom corresponds to spatially periodic stratification of the medium's magnetization into mutually alternating Bloch and Néel domains which are almost isolated. In this connection the change in the plane of rotation of the magnetic-moment vector takes place near the singular point, i.e., in the region with an almost uniform distribution of the magnetization along the external field.

It was shown in[7] that one more first integral $\mu \equiv \mu_0 = \text{const}$ arises as $\epsilon \to 0$, and the system of Landau-Lifshitz equations is completely integrable for $\epsilon = 0$. For $\epsilon \neq 0$, by multiplying the second of Eqs. (2.1) by $\mu = \sin^2\theta\,(d\varphi/dx)$ and integrating the obtained relationship along an integral curve passing through a certain point $(\theta_0, \varphi_0, \mu_0)$, which is located inside the boundary surface, we find that

$$\mu^2 - \mu_0^2 = -2\varepsilon \int_{\varphi_0}^{\varphi} d\varphi \sin\varphi \cos\varphi \sin^4\vartheta(\varphi). \qquad (2.11)$$

One can show that the absolute magnitude of the integral on the right hand side of Eq. (2.11) does not exceed unity, and consequently the following inequalitites are valid:

$$\mu_0^2 - \varepsilon \leqslant \mu^2 \leqslant \mu_0^2 + \varepsilon. \qquad (2.12)$$

If it is known that the condition $\mu_0^2 > \epsilon$ is satisfied at a certain point x_0 of space, then for all $x \geq x_0$ the integral curve will be contained inside the layer $(\pm\sqrt{\mu_0^2 + \epsilon}, \pm\sqrt{\mu_0^2 - \epsilon})$ of the space (θ, φ, μ), which is bounded in θ and φ by the boundary surface (Fig. 3). The position of the layer with respect to the plane $\mu = 0$ is determined by the sign of μ_0. Therefore, in this case the quantity μ preserves its sign for all $x \geq x_0$ and cannot vanish.

If the condition $\mu_0^2 < \epsilon$ is satisfied at a certain point x_0 of space, then for all $x \geq x_0$ the integral curve will be contained inside the layer $(+\sqrt{\epsilon + \mu_0^2}, -\sqrt{\epsilon + \mu_0^2})$ of space (θ, φ, μ), which is bounded in θ and φ by the boundary surface (Fig. 3) and symmetrically situated relative to the plane $\mu = 0$. In this case a change in the sign of μ is possible for $x > x_0$, and also entrance of the integral curve into the singular point $(\mu = 0, \theta = 0)$ corresponding to uniform magnetization along the external field. Therefore, the inequality $\mu^2 < \epsilon$ is valid for the detached solutions corresponding to isolated domains.

3. A number of numerical integrations of the system (2.1) were carried out in order to confirm the results expounded above of the qualitative analysis of the solutions of the Landau-Lifshitz equations allowing for two angular degrees of freedom.

The dependence $\varphi = \varphi(\theta)$, obtained as the result of numberical integration for values of the parameters $\epsilon = 0.1$ and $h_z = 0.01$, is presented in Fig. 4. The point associated with the values $\theta_0 = 0.785$, $\varphi_0 = 0.785$, and $\mu_0 = 0.526$, located on the boundary surface $P(\theta, \varphi, \mu; \mathscr{H}) = 0$ associated with the value $\mathscr{H} = +2h_z$ of the constant first integral, was taken as the initial point of integration. In this case the inequality $\mu_0^2 > \epsilon$ is satisfied, and the motion of the magnetic-moment vector associated with an increase of the spatial variable corresponds to a nutation around the value $\theta = \pi/2$ and a precession around the axis of anisotropy.

The dependence $\mu = \mu(x)$, reflecting the oscillations of μ around the initial value μ_0 with a characteristic amplitude satisfying the inequalities (2.12), is depicted in Fig. 5. For $\mu_0^2 > \epsilon$ and $\epsilon \ll 1$ both the qualitative analysis and the numerical calculation indicate that the solutions of the degenerate problem which we previously investigated,[7] when $\epsilon \to 0$, are a good approximation to the exact problem.

Numerical calculations were performed on an electronic computer to verify the conclusion reached above about the existence of isolated magnetic domains with two angular degrees of freedom. A method of determining detached integral curves for dynamical systems with two degrees of freedom, possessing a first integral,[8] formed the basis of the calculations. As a result of the calculations a solution is found, corresponding to three contacts by the detached integral curve with the boundary surface (2.5), and the eigenvalue of the parameter for the problem turns out to be

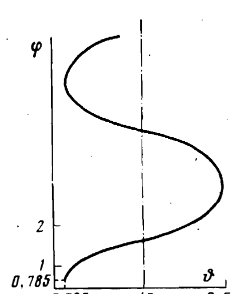

FIG. 4

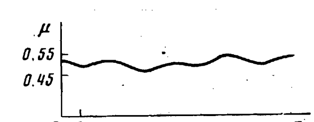

FIG. 5

$$0.10 < C < 0.11.$$

The spatial variation of the projection of the magnetic moment on the direction of the external field in such an isolated domain is depicted by the solid line on Fig. 1 a. In contrast to an isolated domain of the Bloch type, the found distribution of the magnetic moment is characterized by the presence of two regions of inverted magnetization, separated by a layer which is magnetized almost precisely along the external field. The ring (or hollow bubble) isolated domains with cylindrical geometry, observed in orthoferrites and rare-earth iron-garnets,[9,10] are the analog of such an isolated domain.

The solutions of the domain-wall type for $h_z = 0$ and $\epsilon = 0.1$, depicted in Fig. 2, correspond to an eigenvalue of the parameter $0.030 < C < 0.035$.

On the basis of qualitative and numerical analysis of the problem, one can conclude that solutions of the Landau-Lifshitz equations of the domain-wall type without nodal lines and isolated domains with one nodal line do not exist, except for degenerate solutions with a single angular degree of freedom (solutions of the Bloch and Nêel types).

In conclusion we note that investigations of spatially one-dimensional solutions of the Landau-Lifshitz equations allowing for two angular degrees of freedom indicate the possibility of the existence of a whole series of new types of domain structures, characterized by a change in the position of the plane of rotation of the magnetic-moment vector in space, and such investigations are of interest in connection with investigations of new types of domain structures and domain walls in magnetic materials with high anisotropy energies and low saturation magnetizations, and also in connection with investigations of the effects of constant and high-frequency fields on domain structure.

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