Wave channels in an opaque plasma

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(Submitted July 3, 1974)
Zh. Eksp. Teor. Fiz. **68**, 1288–1298 (April 1975)

We have studied the structure and determined the parameters of wave channels produced in an opaque plasma by *TE* and *TM* type waves. We have considered the stability of plane waveguides and found the way the characteristics of waveguides change in a medium with weak dissipation.

PACS numbers: 52.40.F

An important feature of the self-interaction of waves in a plasma is the possibility that strong waves can propagate in a medium which is opaque (in the linear approximation). It is well known that waves with a frequency which is smaller than the plasma frequency ($\omega < \omega_{pe}$) can not propagate through a plasma, but if the wave intensity is sufficiently large, "bleeching" of the medium occurs. This bleeching can, in particular, be connected with the formation of a wave channel[1,2] in the plasma under the influence of the field. The wave self-channeling effect is different in principle from the quasi-optical self-focusing in an opaque medium where the non-linearity is not the cause of the wave propagation, but only leads to refraction and to the removal of the diffraction divergence of the beam. The theory of waveguide propagation in a supercritical plasma has as yet not been developed satisfactorily. In the present paper we consider several features of this effect by studying the structure of wave channels which are uniform (along the direction of propagation). We show that the field distribution in such channels is stable against perturbations of their transverse structure, and we analyze the effect on the characteristics of waveguides with weak absorption.

1. The simplest problem is the study of the structure of uniform channels formed in a plasma by monochromatic TE-type waves for which the electric field vector is perpendicular to the gradient of ϵ: $\mathbf{E} \perp \nabla\epsilon$. In that case the equation for $\mathbf{E}$ has the form

$$\nabla^2 \mathbf{E} + k_0^2 \varepsilon(|\mathbf{E}|^2)\mathbf{E} = 0. \quad (1)$$

A number of authors have studied[1,3,4] the solution of Eq. (1) in the form of uniform beams. However, we shall discuss the case of TE waveguides in an opaque plasma in detail, as this problem has not been discussed in detail in the literature.

A two-dimensional travelling TE wave of the form

$$\mathbf{E} = \mathbf{y}_0 E(x) e^{-ihx}$$

is the simplest example of such a field. We consider here for the sake of simplicity only solutions for the case of a cubic non-linearity $\epsilon = \epsilon_0 + \epsilon' |E|^2$, $\epsilon_0 = 1 - \omega_p^2/\omega^2$, $\epsilon' = \omega_p^2/\omega^2 E_p^2$; we can then write the equation for $E(x)$ in the form

$$\frac{d^2u}{dy^2} + u^3 - u = 0, \quad (2)$$

if we introduce the non-dimensional variables

$$y = (h^2 - k_0^2\varepsilon_0)^{1/2}x, \quad u = \frac{E\omega_p}{E_p c (h^2 - k_0^2\varepsilon_0)^{1/2}}, \quad k_0 = \frac{\omega}{c}. \quad (3)$$

E_p is here a field which is characteristic for the non-linear effects.

Using a solution of Eq. (2) which satisfies the condition that the field vanish at infinity, $u = \sqrt{2}/\cosh y$,[1,3] we can write for the field distribution in a single wave channel:

$$E = E_m/\mathrm{ch}\left(\frac{1}{\sqrt{2}} \frac{\omega_p}{c} \frac{E_m}{E_p} x\right) \quad (4)$$

E_m is the maximum field in the waveguide, the characteristic equation for the propagation constant is

$$h^2 = k_0^2(\varepsilon_0 + \omega_p^2 E_m^2 / 2\omega^2 E_p^2) \quad (5)$$

and the magnitude of the running power (per unit length) transferred in a uniform beam is

$$P_{\text{unif}} = \frac{c^2 E_m E_p}{2\sqrt{2}\pi\omega_p}\left(\varepsilon_0 + \frac{\omega_p^2 E_m^2}{2\omega^2 E_p^2}\right)^{1/2}. \quad (6)$$

Equations (4) to (6) are valid both for a dilute ($\omega > \omega_p$, $\epsilon_0 > 0$) and for a dense supercritical plasma ($\omega < \omega_p$, $\epsilon_0 < 0$). For the latter we find from (5) that electromagnetic waves can propagate in the plasma only provided the amplitude of the field at the maximum exceeds a threshold value which is determined from the condition that $h^2 > 0$:

$$E_m^2 > 2E_p^2(\omega_p^2 - \omega^2)/\omega_p^2. \quad (7)$$

One can study similarly a cylindrical wave channel formed in the plasma when waves with an azimuthal electric field,

$$\mathbf{E}_0 = \boldsymbol{\varphi}_0 E_\varphi(r) e^{-ihz} \quad (8)$$

propagate. In terms of the same dimensionless variables (3) the equation for the field has the form

$$\frac{d^2u}{dy^2} + \frac{1}{y}\frac{du}{dy} + u^3 - u - \frac{u}{y^2} = 0. \quad (9)$$

A solitary solution ($u \to 0$ as $y \to \infty$) of this equation, obtained through numerical integration is shown in Fig. 1. The maximum value $u_m = 1.74$ is reached for $y = u^* = 1.57$. The characteristic equation for the propagation constant becomes

$$h^2 = k_0^2(\varepsilon_0 + \omega_p^2 E_m^2/\omega^2 E_p^2 u_m^2)^{1/2}, \quad (10)$$

and the power of a uniform beam is equal to

$$P_{\text{unif}} = \frac{c^3 E_p^2 P^*}{8\omega_p^2}\left(\varepsilon_0 + \frac{\omega_p^2 E_m^2}{\omega^2 E_p^2 u_m^2}\right)^{1/2}, \quad (11)$$

$$P^* = \int_0^\infty u^2 y\, dy = 7.69.$$

It follows, in particular, from (10) that the following condition must be satisfied

$$E_m^2 > 3.04 E_p^2(\omega_p^2 - \omega^2)/\omega_p^2. \quad (12)$$

for the formation of a wave channel in a supercritical plasma. If the amplitude E_m appreciably exceeds its critical value we have for P_{unif}

$$P_{\text{unif}} \approx c^3 E_p E_m P^*/8\omega_p \omega u_m. \quad (13)$$

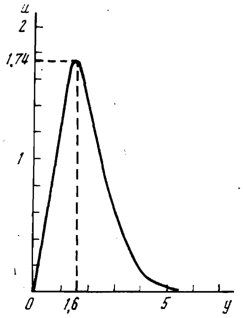


FIG. 1. Structure of the field of a uniform axially symmetric TE-type beam.

It is interesting that P_{unif} increases in proportion to E_m when the maximum amplitude increases.

2. The study of the propagation of TM-type waves in an initially opaque plasma ($\epsilon_0 < 0$) turns out to be a more complicated problem, owing to the existence of singularities in the solution in the region of plasma resonance, $\epsilon = 0$. We shall use the example of a two-dimensional TM wave which has the following components of the electric field

$$E_x \to E_x(x)e^{-ihz},\quad E_z(x)\to iE_z(x)e^{-ihz} \tag{14}$$

to elucidate the nature of the difficulties which arise. The corresponding set of field equations is of the form

$$k_0^2(\varepsilon(|E|^2)-h^2)E_x - h\frac{dE_z}{dx}=0,$$
$$\frac{d^2E_z}{dx^2}+k_p^2\varepsilon E_z + h\frac{dE_x}{dx}=0, \tag{15}$$

where $|E|^2 = E_x^2 + E_z^2$.

We have analyzed the solutions of (15) earlier[2] and shown, in particular, that there exist field distributions of the solitary beam type. The field structure in those is such that on the beam axis ($x = 0$) the longitudinal component $E_z = 0$, while the transverse component reaches its maximum value $E_x = E_m$. In an opaque plasma ($\epsilon_0 < 0$) the condition for the propagation of a wave is the requirement that the dielectric permittivity ϵ is positive on the channel axis, $\epsilon(E_m^2) > 0$. Hence we can determine the threshold field strength for self-channeling; for instance, in the cubic non-linearity approximation

$$E_m^2 \geqslant E_p^2(\omega_p^2-\omega^2)/\omega_p^2. \tag{16}$$

In the channel $\epsilon(x)$ is thus an alternating function and inevitably possesses a transition through the point $\epsilon = 0$. It is well known from the linear theory that the transverse field component E_x which is directed along the gradient of $\epsilon(x)$ must in that point have a quasi-static singularity of the kind $E_x \propto 1/\epsilon$, and the existence of this singularity contradicts the functional dependence $\epsilon(|E|^2)$. Mathematically this manifests itself in that the solution in the region $\epsilon = 0$ becomes non-unique. It follows, in particular, that it is impossible to construct continuous solutions of the equations and that it is necessary to introduce discontinuities (jumps).

We consider in detail the phase plane of the set of Eqs. (15) after writing this set in terms of dimensionless variables

$$\gamma = h/k_0|\varepsilon_0|^{1/2},\quad \mathscr{E}=E/E_p|\varepsilon_0|^{1/2},\quad x_H = k_0|\varepsilon_0|^{1/2}x$$

for the cubic non-linearity approximation, $\epsilon = \epsilon_0 + |E|^2/E_p^2$, which makes sense for the case $|\epsilon_0| \ll 1$. As a result we get the set of equations

$$(\varepsilon-\gamma^2)\mathscr{E}_x-\gamma\frac{d\mathscr{E}_z}{dx}=0,\quad \frac{d^2\mathscr{E}_z}{dx^2}+\varepsilon\mathscr{E}_z+\gamma\frac{d\mathscr{E}_z}{dx}=0;$$
$$\varepsilon=-1+\mathscr{E}^2,\quad \mathscr{E}^2=\mathscr{E}_x^2+\mathscr{E}_z^2. \tag{17}$$

The integral of this set

$$\mathscr{H}=\left(\frac{d\mathscr{E}_z}{dr}\right)^2+\int_0^{\mathscr{E}^2}\varepsilon d\mathscr{E}^2-\gamma^2\mathscr{E}_x^2 \tag{18}$$

can by means of the first Eq. (17) be written in the form

$$\gamma^2\mathscr{H}=\varepsilon^2\mathscr{E}_x^2-{}^1/_2\gamma^2(4\varepsilon\mathscr{E}_x^2-\mathscr{E}^2+1). \tag{19}$$

Equation (19) enables us to reduce the solution of the initial set (17) to the construction of trajectories in the $\mathscr{E}_x$, $\mathscr{E}_z$ phase plane. To determine the functions $\mathscr{E}_x(x)$, $\mathscr{E}_z(x)$ it is, furthermore, convenient to use a modified set of equations which enables us to determine the direction of motion along the integral curves (phase trajectories). This set of equations has the form

$$\frac{d\mathscr{E}_x}{dx}=-\frac{2(\varepsilon-\gamma^2)\mathscr{E}_x^2+\gamma^2\varepsilon}{\gamma^2(\varepsilon+2\mathscr{E}_x^2)}\mathscr{E}_z,$$
$$\frac{d\mathscr{E}_z}{dx}=\frac{\varepsilon-\gamma^2}{\gamma}\mathscr{E}_x. \tag{20}$$

We show the $\mathscr{E}_x$, $\mathscr{E}_z$ phase plane in Fig. 2. There is a saddle-point equilibrium state (0, 0) which corresponds to the value $\mathscr{H} = 0$ ($\mathscr{E}_x$, $\mathscr{E}_z \to 0$ as $x \to \pm\infty$). The integral curve corresponding to localized solution consists in that case of two sections of closed branches: On one of them there are the turning point singularities where the direction of motion changes its sign. It is just in those points that one must construct the discontinuity[1)] passing through the point $\epsilon = 0$ and joining the solutions in the regions $\epsilon > 0$ and $\epsilon < 0$. The coordinate of the turning point is determined from the condition

$$\varepsilon+2\frac{\partial\varepsilon}{\partial|\mathscr{E}|^2}\mathscr{E}_x^2=\varepsilon+2\mathscr{E}_x^2=0, \tag{21}$$

which is the same as the analogous condition for the coordinate of the jump in the quasi-static problem.[2)] The fields to the left and to the right of the jump must be connected by means of the boundary conditions – the requirement that the tangential component $\mathscr{E}_z$ of the electric field and the normal component D_x of the induction be continuous:

$$\mathscr{E}_{z+}=\mathscr{E}_{z-},\quad \varepsilon_+\mathscr{E}_{x+}=\varepsilon_-\mathscr{E}_{x-}. \tag{22}$$

The + sign indicates here the values of the various quantities at the jump in the region $\epsilon > 0$ and the – sign those in the region $\epsilon < 0$.

To find the quantity ϵ_- (the coordinate of the turning point in the localized field distribution) we must simultaneously solve Eqs. (21) and (9) with the condition $\mathscr{H} = 0$. As a result we have for ϵ_- the equation

$$|\varepsilon_-|^3-3\gamma^2|\varepsilon_-|^2+\gamma^2=0, \tag{23}$$

from which it follows, in particular, that $|\epsilon_-|^2 < 1/3$.

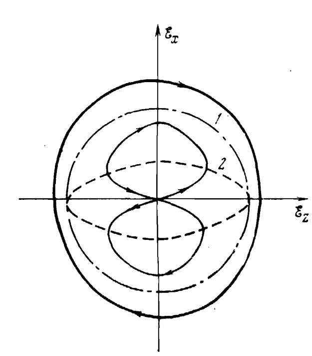


FIG. 2. The phase plane for the equations for TM waves. 1 (dash-dot): $\epsilon = 0$; 2 (dotted): $\epsilon + 2(\partial\epsilon/\partial|\mathscr{E}|^2)|\mathscr{E}_x|^2 = 0$.

Knowing the quantity ϵ_- as a function of γ we find from (22)

$$\mathscr{E}_{x-}^2=-{}^1/_2\varepsilon_-,\quad \mathscr{E}_z^2=1+{}^3/_2\varepsilon_-. \tag{24}$$

The values of the field components at the other side of the jump (in the region $\epsilon > 0$) can be found by means of the boundary conditions and Eqs. (24). Solving the appropriate algebraic equations we get

$$\mathscr{E}_{x+}^2=-2\varepsilon_-,\quad \varepsilon_+=-\varepsilon_-/2. \tag{24'}$$

We must emphasize that when we pass through the jump the magnitude of the integral[3)] $\mathcal{H}$ (in the region $\epsilon < 0$, $\mathscr{H}=0$) changes, i.e., a transition takes place to another integral curve which does not correspond to the separatrix. From (19) we get

$$\mathscr{H}_+=-{}^{27}/_4\varepsilon_-^2. \tag{25}$$

The values of the amplitudes of the field at the jump which we have just found enable us to construct the field distribution in the wave channel. One can show that when one moves away from the point $x = -\infty$ the jump can be realized either from the second quadrant of the phase plane into the first one, or from the fourth quadrant into the third one. We show in Figs. 3 and 4 as examples possible amplitude distributions for the γ values of 0.1 and 3. In Fig. 5 we show the γ-dependence of the power transmitted by the TM wave in an opaque plasma. For a comparison we have drawn in Fig. 5 the way γ depends on

$$P=P_{\text{unif}}\frac{8\pi\omega_p^2}{c^2\omega E_p^2|\varepsilon_0|}$$

in a transparent plasma with the same absolute magnitude of ϵ_0. It is clear that the presence of a jump affects the structure of the waveguide field weakly, if the maximum field E_m on the axis is well above the threshold field (16).

Introducing the variables $v = \mathscr{E}/\gamma$, $\eta = \gamma x$ and assuming that the condition $\gamma \gg 1$ is satisfied, we get instead of the set (17)

$$(v^2-1)v_x-\frac{dv_z}{d\eta}=0,\quad \frac{d^2v_z}{d\eta^2}+v^2v_z+\frac{dv_x}{d\eta}=0. \tag{26}$$

We give in Fig. 6 the functions $v_x(\eta)$, $v_z(\eta)$ obtained through a numerical integration of (26). This solution enables us also to determine the magnitude of the channeled power (per unit length)

$$P_{\text{unif}}=c^2E_m^2P_2/12\pi\omega_p,\quad P_2=\int_{-\infty}^{\infty}(v_x^2+v_z^2)v_x^2\,d\eta=5.06 \tag{27}$$

and the effective channel width

$$a_2=\frac{1{,}2\eta_2^*c}{\omega_p}\left(\frac{E_m}{E_p}\right)^{-1},\quad \eta_2^*=\left(\int_0^\infty \eta^2v^2\,dx\Big/\int_0^\infty v^2\,dx\right)^{1/2}=2.6. \tag{28}$$

We have here taken into account that

$$\frac{E_m}{E_p}=\gamma\frac{\omega}{\omega_p}\sqrt{3/2}.$$

We can similarly consider a cylindrical waveguide for large values of γ. In that case also $a_3 \propto \gamma^{-1}$, but $P_3 \propto \gamma \propto 1/a_3$.

The self-channeling of TM-type waves is accompanied by the dissipation of their energy. The absorption is in principle different from zero even in a plasma without collisions in which it is determined by the linear and non-linear effects of the transformation of an electromagnetic wave into a plasma wave in the plasma resonance region. The magnitude of the absorption depends on the parameters of the jump in the dielectric permittivity and the amplitude of the normal component

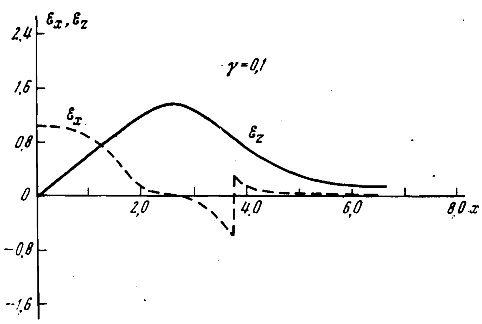


FIG. 3

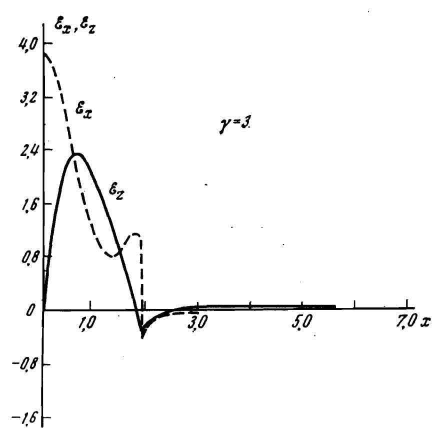


FIG. 4

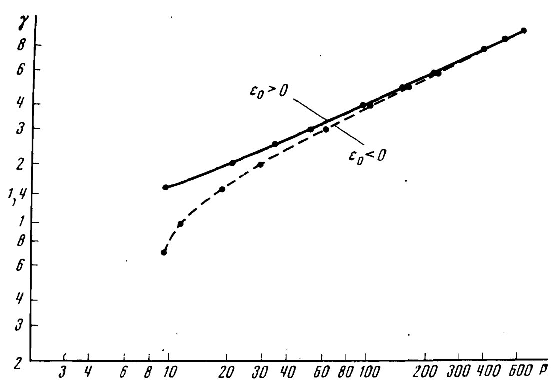
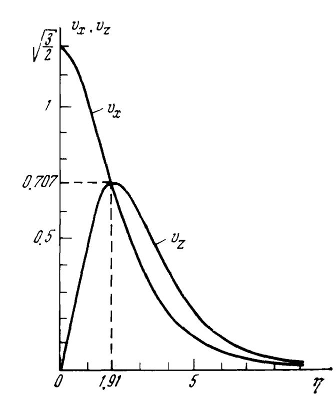


FIG. 5

FIG. 6. Structure of the field of a solitary TM wave for the case when $|\epsilon_0|/\gamma \ll 1$.



of the electric induction at the jump. The evaluation of the loss is a separate problem so that we shall without going into details give as an example an expression for the energy flux of a plasma wave excited at the transition as a result of the linear transformation. Using the relations from a paper by Gil'denburg[5] we find that there appears an energy flux per unit area of the jump

$$\delta S=(^3/_2)^{2.5}|\varepsilon_-|^{1/2}v_{Te}E_p^2/8\pi. \tag{29}$$

The quantity ϵ_- depends on the propagation constant γ and is given by Eq. (23). One can easily show that if the maximum field amplitude E_m in the channel is well above the threshold value for wave propagation, the losses are weak and can not appreciably affect the local structure of the field in the channel.

3. Above we considered the structure of self-channelized TE- and TM-type waves which were uniform (in the direction of propagation), in the plasma neglecting absorption. We study now the effect of small perturbations on the spatial deformation of the field distributions which we have found. The problem of the stability of uniform waves with respect to perturbations of the transverse spatial structure will be analyzed by taking TE-type waveguides as an example. To do this we use the Lagrangian formulation of the problem.[6] We can obtain the equations for the waveguide fields by minimizing the functional

$$\mathscr{L}=\int\left(|\mathrm{rot}\,\mathbf{E}|^2-k_0^2\int\limits_0^{|E|^2}\varepsilon d|E|^2\right)d\mathbf{r}. \tag{30}$$

Looking for the solution which is close to the waveguide solution in the form $\mathbf{E}=\mathbf{y}_0E(x,z)e^{i\gamma z}$ and neglecting higher derivatives we get from (30)

$$\mathscr{L}=\int\left\{\left|\frac{\partial E}{\partial x}\right|^2-i\gamma\left(\frac{\partial E}{\partial z}E^*-\frac{\partial E^*}{\partial z}E\right)+\gamma^2|E|^2-k_0^2\int\limits_0^{|E|^2}\varepsilon d|E|^2\right\}dx\,dz. \tag{31}$$

The Lagrangian (31) corresponds to the transition from the exact wave equation to a parabolic equation for the field amplitude E(x, z). In the case of strong non-linearity when the non-linear correction to the dielectric permittivity ϵ_0 is of the order of ϵ_0 such a transition is valid only for fields with a structure close to that of the waveguide field. We can thus use the Lagrangian (31) only for the study of the stability of waveguide fields with respect to small smooth perturbations of their dimensions.

Using the standard procedure we find from (31) the conservation laws which for spatially localized field distributions in a medium with a dielectric permittivity $\epsilon(|E|^2)$ have the form

$$\int\limits_{-\infty}^{\infty}E_yH_x^*=\gamma\int\limits_{-\infty}^{\infty}|E|^2dx=P, \tag{32}$$

$$\int\limits_{-\infty}^{\infty}\left\{\left|\frac{\partial E}{\partial x}\right|^2+\gamma^2|E|^2-k_0^2\int\limits_0^{|E|^2}\varepsilon d|E|^2\right\}dx=\tilde{\mathscr{H}}. \tag{33}$$

One can show that for given power P the functional $\tilde{\mathscr{H}}$ is bounded from below and reaches its minimum value for the case of a uniform beam. Using that fact we can show that the solutions corresponding to a uniform channel are stable against small perturbations of their transverse structure. The equation for fields close to the waveguide fields, corresponding to the Lagrangian (31), has the form

$$-2i\gamma\frac{\partial E}{\partial z}+\frac{\partial^2E}{\partial x^2}+[k_0^2\varepsilon(|E|^2)-\gamma^2]E=0. \tag{34}$$

We write the solution of (34) in the form[4)]

$$E(x,z)=\left[E_p(x)+\frac{\partial E_p(x)}{\partial P}\delta P+\delta E_p(x,z)\right]e^{-i\varphi(z)}. \tag{35}$$

$E_p(x)$ is here the unperturbed waveguide distribution with channelized power P; $\delta E_p(x, z)$ is that part of the perturbation which does not change the power;

$$\int\limits_{-\infty}^{\infty}E_p(x)(\delta E_p+\delta E_p^*)dx=0; \tag{36}$$

$\delta P(z)$ is the deviation from P caused by the total perturbation

$$\delta P(z)=\int\limits_{-\infty}^{\infty}E_p(x)[\delta E+\delta E^*(x,z)]dx. \tag{37}$$

The factor $e^{-i\varphi(z)}$ is introduced to correct the phase of the solution, as without it phase instability may occur (a monotonic drift of the phase of the perturbed solution from that of the unperturbed one).

Linearizing (34) we get the equation for the perturbation:

$$-2i\gamma\frac{\partial E_p}{\partial P}\frac{\partial}{\partial z}\delta P+2\gamma\frac{\partial\varphi}{\partial z}E_p-2\gamma\frac{\partial\gamma}{\partial P}\delta PE_p(x)+\frac{\partial^2\delta E_p}{\partial x^2}$$
$$+(k^2\varepsilon-\gamma^2)\delta E_p+k_0^2\frac{\partial\varepsilon}{\partial E_p^2}E_p^2(\delta E_p+\delta E_p^*)-2i\gamma\frac{\partial\delta E_p}{\partial z}=0. \tag{38}$$

Hence it follows, from (36), that we have a condition for the conservation of the perturbation of the power, $d\delta P/dz = 0$. Assuming that the phase satisfies the equation

$$\frac{d\varphi}{dz}=\frac{d\gamma}{dP}\delta P, \tag{39}$$

we have for δE_p the equation

$$-2i\gamma\frac{\partial\delta E_p}{\partial z}+\frac{\partial^2\delta E_p}{\partial x^2}+(k_0^2\varepsilon-\gamma^2)\delta E_p+k_0^2\frac{\partial\varepsilon}{\partial E_p^2}E_p^2(\delta E_p+\delta E_p^*)=0, \tag{40}$$

the integral of which is the same as the second variation of the functional (33). Since, as we have already noted, for fixed P the functional $\tilde{\mathscr{H}}$ is bounded from below and is minimized by uniform beams, the second variation of this functional for $\delta E_p(x, z)$ is positive. Hence, the small perturbations considered lead only to an increase in $\tilde{\mathscr{H}}$ and the wave channels are stable against such perturbations.

We note that the presence of stability can be interpreted as a consequence of the fact that the power transferred in a uniform channel is inversely proportional to its width. In an opaque plasma such a functional dependence exists also for axially symmetric waves. The conclusion about the stability can thus, in contrast to quasi-optical beams, be extended also to the case of three-dimensional channels. The same conclusion is apparently valid for TM waveguides, at least for wide waveguides for which the plasma resonance region is positioned at the periphery and therefore can not appreciably affect its characteristics.

One can similarly consider the problem of the stability of the longitudinal structure of the waveguides. It turns out that in that case, as for the usual self-focusing in a transparent medium,[7] the waveguide distributions are unstable against a longitudinal modulation. The development of the instability does not destroy the self-channeling of the waves, but merely leads to a corrugation of the waveguide—a division of it into bunches with the ratio of their longitudinal and transverse sizes dependent on the nature of the relaxation process for the non-linearity. For instance, in the case of a strictional non-linearity the maximum growth rate corresponds to perturbations for which $L_\parallel/L_\perp \sim c/v_S \gg 1$, where v_S is the sound velocity.

4. The stability of uniform beams against small perturbations of their transverse structure enables us to study the effect of weak absorption on wave channels in an opaque plasma. We may assume that the distributions of the waveguide field have locally the same structure as in a plasma without absorption, and as the dissipation progresses the field becomes subdivided (practically without loss of energy to reflection into non-waveguide fields, since the deviations from the uniform solutions which arise due to the dissipation do not build up. Since the field distribution in a uniform channel is determined by a single parameter – the propagation constant γ (or the amplitude E_m of the field at its maximum) – the criterion for the validity for such an energy approach (of the kind of the geometric optics approximation) is given by the requirement that $d\gamma^{-1}/dz \ll 1$.

Finally, if q is the volume density of the losses, the equation for the change in the total energy flux has the well known form

$$dP(\gamma, z)/dz = -Q(\gamma, z), \tag{41}$$

$P(\gamma, z)$ is the power transferred in a uniform channel, and

$$Q(\gamma, z) = \int q(\gamma, z)\, dS_\perp.$$

Generally speaking, it is necessary to write down the analogous conservation equation also for the field momentum – the quantity $\mathscr{H}$ which in a medium without absorption is also an integral of the equations. However, we may assume that when the waveguide is restructured in a medium with absorption the quantity $\tilde{\mathscr{H}}$ is not conserved, and that it diminished, while the surplus of $\tilde{\mathscr{H}}$ (by analogy with the propagation of solitons[5)]) is connected with the emission of the non-soliton part when the soliton is restructured while the energy flux in the non-soliton part is negligibly small.

As an example we consider the rearrangement of a "narrow" TE waveguide with $h(z)/k_0\epsilon^{1/2} \gg 1$. According to (6)

$$P(z) = \frac{\omega}{8\pi}\frac{\gamma^2\omega^2}{k_0^2\omega_p^2}|\varepsilon_0|\int_{-\infty}^{\infty} v^2\, d\eta\, E_p^2,$$

$$Q(z) = \frac{\nu}{4\pi}\frac{\gamma\omega^2}{k_0\omega_p^2}\sqrt{|\varepsilon_0|}\int_{-\infty}^{\infty} v^2\, d\eta\, E_p^2; \tag{42}$$

$$\int_{-\infty}^{\infty} v^2\, d\eta = 4.0.$$

Hence we have after simple transformations

$$h(z) = h(0)(1 - z/z_0), \quad z_0 = \omega h(0)/\nu k_0^2. \tag{43}$$

Hence

$$P(z) = P(0)(1 - z/z_0)^2, \tag{44}$$

which differs appreciably from the exponential damping of a wide (quasi-optical) wave beam in a transparent medium. Such a difference is cause by the rearrangement of the structure of the waveguide fields which occurs as their energy is dissipated.

In an opaque plasma it thus turns out to be possible to have waveguide propagation of strong electromagnetic waves. The structure of the waveguide field distributions depends strongly on the nature of the polarization of the electric field and turns out to have a jump in the case of TM-type waves in the plasma resonance region $\epsilon = 0$. An important feature of the waveguide propagation in an opaque plasma which distinguishes it from the quasi-optical self-focusing of waves in weakly nonlinear transparent media is the stability of the waveguides against small perturbations of their transverse structure. However, to elucidate the possibility to realize the self-channeling effects of waves in an opaque plasma it is necessary to study non-stationary processes of the formation of waveguides and wave fields which are non-uniform (in the direction of propagation) in a dense plasma. Such a study also enables us to explain the results of experiments on the bleaching of a dense supercritical plasma under the influence of strong electromagnetic radiation.[9-11]

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1)It is impossible to determine the coordinate of the discontinuity uniquely in the framework of the phenomenological equations and it turns out to be necessary to consider a micro-model.

2)This agreement is natural as the field in the vicinity of the transition has the quasi-static structure and $dE_x/dx \to \infty$ in the point determined by (21).

3)One can show that it is impossible to construct a discontinuous solution if one requires $\mathcal{H}$, $\mathcal{E}_z$, and $\epsilon\mathcal{E}_x$ to be continuous.

4)We do not lose generality by not considering here perturbations which lead to a change in the direction of the waveguide axis, as this direction is conserved according to the integrals of Eq. (31).

5)For non-linear waves described by the Korteweg-de Vries equation numerical calculations have confirmed the possibility to use the energy principle to describe the propagation of solitons in weakly non-uniform and dissipative media. [8]

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Translated by D. ter Haar
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