Absorption of high-intensity light by free carriers in dielectrics

An. V. Vinogradov

P. N. Lebedev Physics Institute, USSR Academy of Sciences
(Submitted July 29, 1974)
Zh. Eksp. Teor. Fiz. 68, 1091–1098 (March 1975)

A formula is derived for the rate at which the energy of a conduction electron increases in a high-frequency electric field. It is shown that the strong field affects the frequency of the electron-phonon collisions, and an additional dependence on field strength appears as a result in the formula for the rate of energy increase.

1. One of the proposed mechanisms of damage to transparent dielectrics by high-power laser pulses is electronic cascade ionization[1]. The task of the theory is in this case, as a rule, to calculate the cascade development constant. Under certain additional assumptions, this rate is inversely proportional to the time during which one electron acquires, on the average, an energy equal to the ionization potential. This time can be obtained by integrating the equation for the average rate of energy acquisition by a conduction electron in a high-frequency electric field

$$\frac{d\mathscr{E}}{dt}=\frac{e^2E^2}{2m\Omega^2}\nu_{eff}, \tag{1}$$

where E and Ω are the intensity and circular frequency of the electric field, e is the electron charge, m is the effective mass of the conduction electron, and ν_{eff} is the effective frequency of the electron-phonon collisions.

In the microscopic derivation of (1) it is customary to use the condition that the field be weak[2,3]1). In a weak field, the effective collision frequency is a function of $\mathscr{E}$ and Ω. The reason is that, as first noted by Holstein[4], at an initial electron energy $\mathscr{E} \ll \hbar\Omega$ the value of ν_{eff} is determined by the final energy of the electron in the elementary act of absorption of a light quantum, i.e., by the quantity $\hbar\Omega$. For example, in the case of scattering by longitudinal acoustic phonons, the collision frequency ν depends on the electron energy like $\mathscr{E}^{1/2}$, and ν_{eff} turns out to be proportional to $(\hbar\Omega)^{1/2}$. As a result, the total dependence on the frequency in (1) turns out to be of the form $\Omega^{-1.5}$.[2] Scattering by polar optical phonons leads in (1) to a complete frequency dependence proportional to $\Omega^{-2.5}$.[3]. To the contrary, when $\mathscr{E} \gg \hbar\Omega$, the value of ν_{eff} is determined by the energy of the electron, and $\nu_{eff} \sim \nu$.

In a strong electric field, the energy of electron oscillation under the influence of a field

$$e^2E^2/2m\Omega^2$$

can exceed the kinetic energy of the electron. If the energy of the oscillations exceeds also the energy of the light quantum2), then one can naturally expect the collision frequency to be determined by the energy of the electron oscillations in the field of the wave. Then, in the case of scattering by acoustic phonons, the complete field dependence in (1) is like E^3, and in the case of scattering by polar optical phonons—like E. Analogous considerations as applied to a plasma were first advanced by Silin[7].

The purpose of the present paper is to obtain formulas for the average rate of energy acquisition by a conduction electron in a high-frequency electric field without any assumptions whatever that the field is small. To solve this problem (Sec. 2) we use the Boltzmann classical equation. The entire calculation in Sec. 2 is performed for the cases of circular and linear polarization, and for electron interaction with longitudinal acoustic and polar optical phonons. We note that in the final formulas (25)—(30) we obtain an explicit dependence on the parameter $x = (2m\mathscr{E})^{1/2}\Omega/eE$, In this lies the principal difference between our results and those of Silin[7], who obtained such a dependence at $x \ll 1$ and at $x \gg 1$. In Sec. 3 we consider the justification of the results of Sec. 2 from the point of view of quantum mechanics. Criteria for the applicability of the classical description are obtained.

Within the framework of quantum mechanics, the question of the absorption of high-intensity light by free carriers in semiconductors was considered earlier by Pazdzerskiĭ[8]3), who calculated numerically the dependence of the absorption coefficient on the field intensity. The qualitative dependence of the absorption coefficient on the field intensity, obtained in that paper, agrees with the analytic formulas of the present paper4).

2. The classical kinetic equation for the conduction electrons in a strong homogeneous electric field is

$$\begin{aligned}\frac{\partial f(\mathbf{p},t)}{\partial t}+e\mathbf{E}(t)\nabla_{\mathbf{p}}f(\mathbf{p},t)=\frac{2\pi}{\hbar}\sum_{s,\mathbf{k}}\frac{|C_{s,\mathbf{k}}|^2}{V}\{f(\mathbf{p}+k,t)\cdot\\ \cdot[\bar{n}_{s,\mathbf{k}}\delta(\mathscr{E}_\mathbf{p}-\mathscr{E}_{\mathbf{p}+\mathbf{k}}-\hbar\omega_{s,\mathbf{k}})+(\bar{n}_{s,\mathbf{k}}+1)\delta(\mathscr{E}_\mathbf{p}-\mathscr{E}_{\mathbf{p}+\mathbf{k}}+\hbar\omega_{s,\mathbf{k}})]\\ -f(\mathbf{p},t)[\bar{n}_{s,\mathbf{k}}\delta(\mathscr{E}_\mathbf{p}-\mathscr{E}_{\mathbf{p}+\mathbf{k}}+\hbar\omega_{s,\mathbf{k}})+(\bar{n}_{s,\mathbf{k}}+1)\delta(\mathscr{E}_\mathbf{p}-\mathscr{E}_{\mathbf{p}+\mathbf{k}}-\hbar\omega_{s,\mathbf{k}})]\}.\end{aligned} \tag{2}$$

Here f(p, t) is the distribution function of the electron kinematic momenta, $\omega_{s,k}$ is the frequency of a phonon of momentum k belonging to the s-th phonon branch, $\bar{n}_{s,k}$ are the equilibrium phonon occupation numbers, $C_{s,k}/V^{1/2}$ is the matrix element of the electron-phonon interaction, and V is the volume of the system under consideration.

We consider the interaction of the electrons with the branch of the longitudinal acoustic phonons and with the polar optical branch. For these two cases we have respectively[11]

$$\frac{|C_{1,\mathbf{k}}|^2}{V}=\frac{G^2\hbar\omega_\mathbf{k}}{2V\rho u^2}, \tag{3}$$

$$\frac{|C_{2,\mathbf{k}}|^2}{V}=\frac{2\pi e^2\hbar\omega_0\hbar^2}{Vk^2}\left[\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}\right], \tag{4}$$

where G is the deformation potential, ρ is the density of the crystal, u is the velocity of the longitudinal waves, and ϵ_0 and ϵ_∞ are the static and high-frequency dielectric constants. The dependence of the conduction-electron energy on the momentum will be assumed to be quadratic:

$$\mathscr{E}_p = p^2/2m. \tag{5}$$

We take the electric field in the form

$$\mathbf{E}(t) = {}^1/_2(\mathbf{E}e^{i\Omega t} + \mathbf{E}^* e^{-i\Omega t}). \tag{6}$$

We change over in the kinetic equation (2) from the distribution function with respect to the kinematic momenta f(p, t) to the distribution function with respect to the canonical momenta

$$F(\mathbf{P},t) = f\left(\mathbf{P} - \frac{e}{c}\mathbf{A}(t), t\right), \tag{7}$$

where

$$\mathbf{A}(t) = -\frac{c}{2i\Omega}(\mathbf{E}e^{i\Omega t} - \mathbf{E}^* e^{-i\Omega t}). \tag{8}$$

The kinetic equation (2) takes the form

$$\begin{aligned}\frac{\partial F(\mathbf{P},t)}{\partial t} = \frac{2\pi}{\hbar}\sum_{s,\mathbf{k}}\frac{|C_{s,\mathbf{k}}|^2}{V}\Big\{F(\mathbf{P}+\mathbf{k},t)\Big[\bar{n}_{s,\mathbf{k}}\delta\Big(\mathscr{E}_{\mathbf{P}} - \mathscr{E}_{\mathbf{P}+\mathbf{k}} - \hbar\omega_{s,\mathbf{k}} + \frac{e\mathbf{A}(t)\mathbf{k}}{cm}\Big)\\ + (\bar{n}_{s,\mathbf{k}}+1)\delta\Big(\mathscr{E}_{\mathbf{P}} - \mathscr{E}_{\mathbf{P}+\mathbf{k}} + \hbar\omega_{s,\mathbf{k}} + \frac{e\mathbf{A}(t)\mathbf{k}}{cm}\Big)\Big]\\ - F(\mathbf{P},t)\Big[\bar{n}_{s,\mathbf{k}}\delta\Big(\mathscr{E}_{\mathbf{P}} - \mathscr{E}_{\mathbf{P}+\mathbf{k}} + \hbar\omega_{s,\mathbf{k}} + \frac{e\mathbf{A}(t)\mathbf{k}}{cm}\Big)\\ + (\bar{n}_{s,\mathbf{k}}+1)\delta\Big(\mathscr{E}_{\mathbf{P}} - \mathscr{E}_{\mathbf{P}+\mathbf{k}} - \hbar\omega_{s,\mathbf{k}} + \frac{e\mathbf{A}(t)\mathbf{k}}{cm}\Big)\Big]\Big\}.\end{aligned} \tag{9}$$

We seek the distribution function F(P, t) in the form

$$F(\mathbf{P},t) = \sum_n e^{in\Omega t} g_n(\mathbf{P},t). \tag{10}$$

Substituting (10) in (9) and using the formula

$$2\pi\delta\left(\mathscr{E}_{\mathbf{P}} - \mathscr{E}_{\mathbf{P}+\mathbf{k}} \pm \hbar\omega_{s,\mathbf{k}} + \frac{e\mathbf{A}(t)\mathbf{k}}{cm}\right) = \frac{1}{\hbar\Omega}\sum_n e^{in\Omega t}\int_{-\infty}^{+\infty} dx \exp\{ia^{\pm}x\} L_n(\mathbf{k},x), \tag{11}$$

where we have put

$$a^{\pm} = (\mathscr{E}_{\mathbf{P}} - \mathscr{E}_{\mathbf{P}+\mathbf{k}} \pm \hbar\omega_{s,\mathbf{k}})/\hbar\Omega, \tag{12}$$

$$L_n(\mathbf{k},x) = \frac{1}{2\pi}\int_{-\pi}^{+\pi}\exp\left\{-in\Omega t + ix\frac{e\mathbf{A}(t)\mathbf{k}}{cm\hbar\Omega}\right\} d\Omega t, \tag{13}$$

we obtain the following system of equations for the harmonics of the distribution function:

$$\begin{aligned} in\Omega g_n(\mathbf{P},t) + \frac{\partial g_n(\mathbf{P},t)}{\partial t}\\ = \frac{1}{\hbar}\frac{1}{\hbar\Omega}\sum_{s,\mathbf{k}}\frac{|C_{s,\mathbf{k}}|^2}{V}\sum_{n'}\int_{-\infty}^{+\infty} dx\, L_{n-n'}(\mathbf{k},x)\\ \times\{g_{n'}(\mathbf{P}+\mathbf{k},t)[\bar{n}_{s,\mathbf{k}}\exp(ia^-x) + (\bar{n}_{s,\mathbf{k}}+1)\exp(ia^+x)]\\ - g_{n'}(\mathbf{P},t)[\bar{n}_{s,\mathbf{k}}\exp(ia^+x) + (\bar{n}_{s,\mathbf{k}}+1)\exp(ia^-x)]\}.\end{aligned} \tag{14}$$

In the case of high frequencies, when Ω is much higher than the electron-phonon collision frequency, we can neglect in the left-hand side of (14) the term $\partial g_n(\mathbf{P}, t)/\partial t$ (except for the case n = 0). Then all the harmonics of the distribution function with n ≠ 0 are directly expressed in terms of $g_0(\mathbf{P}, t)$. The concrete form of the function $g_0(\mathbf{P}, t)$ will be of no importance to us henceforth. We shall assume, however, that the distribution function varies little over the period of the field.

Let us examine the energy dissipation. The usual formula for the dissipated power is

$$\frac{dW}{dt} = \frac{1}{2}\frac{e}{m}\sum_{\mathbf{P}}\mathbf{P}[\mathbf{E}^* g_1(\mathbf{P},t) + \mathbf{E}g_{-1}(\mathbf{P},t)]. \tag{15}$$

Substituting in (15) the expressions for $g_1(\mathbf{P}, t)$ and $g_{-1}(\mathbf{P}, t)$ from (14) we obtain after a number of elementary transformations, which reduce to a change of variables,

$$\begin{aligned}\frac{dW}{dt} = \frac{1}{(\hbar\Omega)^2}\frac{1}{2i}\frac{e}{m}\sum_{\mathbf{P},s,\mathbf{k}}\frac{|C_{s,\mathbf{k}}|^2}{V}\int_{-\infty}^{+\infty} dx[\mathbf{k}\mathbf{E}^* L_1(\mathbf{k},x) - \mathbf{k}\mathbf{E}L_{-1}(\mathbf{k},x)]\cdot\\ \cdot[\bar{n}_{s,\mathbf{k}}\exp(ia^+x) + (\bar{n}_{s,\mathbf{k}}+1)\exp(ia^-x)]g_0(\mathbf{P},t).\end{aligned} \tag{16}$$

Using (8) and (13), we can easily prove the following equalities:

$$\frac{1}{2}\frac{e}{m\hbar\Omega^2}[\mathbf{E}^*\mathbf{k}L_1(\mathbf{k},x) - \mathbf{E}\mathbf{k}L_{-1}(\mathbf{k},x)] = \frac{\partial L_0(\mathbf{k},x)}{\partial x}, \tag{17}$$

$$\begin{gathered} L_0(\mathbf{k},x) = J_0(eE_{1,2}kx/m\hbar\Omega^2);\\ E_{1,2} = [(\mathbf{E}_1\mathbf{k})^2 + (\mathbf{E}_2\mathbf{k})^2]^{1/2}/k,\\ \mathbf{E}_1 = (\mathbf{E}+\mathbf{E}^*)/2,\quad \mathbf{E}_2 = (\mathbf{E}-\mathbf{E}^*)/2i,\end{gathered} \tag{18}$$

where $J_0(x)$ is a Bessel function.

Substituting (17) and (18) in (16) and integrating by parts, we obtain

$$\begin{aligned}\frac{dW}{dt} = -\frac{1}{\hbar}\sum_{\mathbf{P},s,\mathbf{k}}\frac{|C_{s,\mathbf{k}}|^2}{V}\int_{-\infty}^{+\infty} dx\, J_0\left(\frac{eE_{1,2}kx}{m\hbar\Omega^2}\right)\\ \times[\bar{n}_{s,\mathbf{k}}a^-\exp(ia^-x) + (\bar{n}_{s,\mathbf{k}}+1)a^+\exp(ia^+x)]g_0(\mathbf{P},t).\end{aligned} \tag{19}$$

We neglect in (19) the phonon energy and integrate with respect to x. Then

$$\begin{aligned}\frac{dW}{dt} = \frac{2}{\hbar}\sum_{\mathbf{P},s,\mathbf{k}}\frac{|C_{s,\mathbf{k}}|^2}{V}(2n_{s,\mathbf{k}}+1)g_0(\mathbf{P},t)(\mathscr{E}_{\mathbf{P}+\mathbf{k}} - \mathscr{E}_{\mathbf{P}})\\ \times\left\{\left\langle\left(\frac{eE_{1,2}k}{m\Omega}\right)^2 - (\mathscr{E}_{\mathbf{P}+\mathbf{k}} - \mathscr{E}_{\mathbf{P}})^2\right\rangle\right\}^{-1/2}.\end{aligned} \tag{20}$$

The angle brackets ⟨...⟩ in (20) under the square root sign denote that the range of variables which contributes to (20) is defined by the inequality

$$(eE_{1,2}k/m\Omega)^2 > (\mathscr{E}_{\mathbf{P}+\mathbf{k}} - \mathscr{E}_{\mathbf{P}})^2. \tag{21}$$

The angle brackets ⟨...⟩ in the subsequent formulas have a similar meaning. The inequality (21) has a simple physical meaning. It imposes a limitation on the change of the electron energy under the influence of the field in the elementary scattering act.

The general expression for the dissipated power can also be represented in the form

$$\frac{dW}{dt} = \sum_{\mathbf{P}}\frac{d\mathscr{E}_{\mathbf{P}}}{dt}g_0(\mathbf{P},t), \tag{22}$$

where $d\mathscr{E}_p/dt$ is the rate at which one electron with momentum P acquires energy. Comparing (20) and (22), we can easily write out a formula for $d\mathscr{E}_p/dt$. The subsequent calculations will be made for the rate $d\mathscr{E}/dt$ at which an electron with specified energy acquires energy. The transition from the rate of energy acquisition by an electron with given momentum to the rate of energy acquisition by an electron with given energy is by means of the formula

$$\frac{d\mathscr{E}}{dt} = \sum_{\mathbf{P}}\frac{d\mathscr{E}_{\mathbf{P}}}{dt}\delta(\mathscr{E}_{\mathbf{P}} - \mathscr{E})\Big/\sum_{\mathbf{P}}\delta(\mathscr{E}_{\mathbf{P}} - \mathscr{E}). \tag{23}$$

Substituting in (23) the explicit expression for $d\mathscr{E}_p/dt$ and integrating, we obtain

$$\begin{aligned}\frac{d\mathscr{E}}{dt} = \frac{1}{2\hbar(2m\mathscr{E})^{1/2}}\sum_{s,\mathbf{k}}\frac{|C_{s,\mathbf{k}}|^2}{V}(2\bar{n}_{s,\mathbf{k}}+1)\left\{\left[\left\langle\left(\frac{2eE_{1,2}}{\Omega}\right)^2\right.\right.\right.\\ \left.\left.\left. - (k - 2\sqrt{2m\mathscr{E}})^2\right\rangle\right]^{1/2} - \left[\left\langle\left(\frac{2eE_{1,2}}{\Omega}\right)^2 - (k + 2\sqrt{2m\mathscr{E}})^2\right\rangle\right]^{1/2}\right\}.\end{aligned} \tag{24}$$

We consider two particular cases of linear and circular polarization:

a) Case of Circular Polarization

In the case of circular polarization we have $E_1 = E_2 = E$ and $E_1 \perp E_2$. We change over in (24) from summation over k to integration, and introduce spherical coordinates with a polar axis perpendicular to the polarization plane. We assume that the acoustic branch satisfies the condition $k_0T \gg \hbar\omega_k$, where k_0 is Boltzmann's constant and T is the absolute temperature. The integral that appears in (24) can be calculated. We present the result of the calculations in the form

$$\frac{d\mathscr{E}}{dt}=\left(\frac{eE}{\Omega}\right)^3\frac{2G^2k_0T}{\pi^2\hbar^4\rho u^2}\Phi_1\left(\frac{\Omega\sqrt{2m\mathscr{E}}}{eE}\right)+\left(\frac{eE}{\Omega}\right)\frac{e^2\omega_0(\varepsilon_0-\varepsilon_\infty)}{\pi\hbar\varepsilon_0\varepsilon_\infty}\operatorname{cth}\frac{\hbar\omega_0}{2k_0T}\Phi_2\left(\frac{\Omega\sqrt{2m\mathscr{E}}}{eE}\right), \tag{25}$$

where

$$\Phi_1(x)=\begin{cases}{}^1/_2\pi(1+{}^2/_3x^2-{}^1/_{15}x^4), & x\leqslant 1\\ {}^2/_3\pi(x+1/5x), & x\geqslant 1\end{cases}. \tag{26}$$

$$\Phi_2(x)=\begin{cases}\pi(1-{}^1/_3x^2), & x\leqslant 1\\ 2\pi/3x, & x\geqslant 1\end{cases}. \tag{27}$$

b) Case of Linear polarization

In the case of linear polarization we can put $E_2 = 0$ and $E_1 = E$. Then the integration in (23) can be carried out in spherical coordinates with a polar axis that coincides with the direction of the vector E. We represent the result in a form analogous to (24):

$$\frac{d\mathscr{E}}{dt}=\left(\frac{eE}{\Omega}\right)^3\frac{2G^2k_0T}{\pi^2\hbar^4\rho u^2}\Psi_1\left(\frac{\Omega\sqrt{2m\mathscr{E}}}{eE}\right)+\left(\frac{eE}{\Omega}\right)\frac{e^2\omega_0(\varepsilon_0-\varepsilon_\infty)}{\pi\hbar\varepsilon_0\varepsilon_\infty}\operatorname{cth}\frac{\hbar\omega_0}{2k_0T}\Psi_2\left(\frac{\Omega\sqrt{2m\mathscr{E}}}{eE}\right), \tag{28}$$

where

$$\Psi_1(x)=\frac{17}{30}\sqrt{1-x^2}+\frac{1}{10}\frac{\arcsin x}{x}+\frac{4}{15}x^2\sqrt{1-x^2}+\frac{2}{3}x\arcsin x-\frac{1}{15}x^4\operatorname{arch}\frac{1}{x}\quad \text{if}\quad x\leqslant 1,$$
$$\Psi_1(x)=\pi x/3+\pi/20x\quad\text{if}\quad x\geqslant 1; \tag{29}$$

$$\Psi_2(x)=\frac{4}{3}\sqrt{1-x^2}+\frac{2}{3}\frac{\arcsin x}{x}+\frac{2}{3}x^2\operatorname{arch}\frac{1}{x}\quad\text{if}\quad x\leqslant 1,$$
$$\Psi_2(x)=\pi/3x\quad\text{if}\quad x\geqslant 1. \tag{30}$$

In the case $x \ll 1$ we have

$$\Psi_1(x)={}^2/_3+{}^2/_3x^2+\ldots,\quad \Psi_2(x)=2-{}^2/_3x^2\ln(2/x)-{}^5/_9x^2+\ldots.$$

3. In analogy with the derivation of (19) and (20) from the classical kinetic equation, we can obtain from the quantum kinetic equation[12] (case of linear polarization)

$$\frac{dW}{dt}=-\frac{1}{\hbar}\sum_{P,s,k}\frac{|C_{s,k}|^2}{V}\int_{-\infty}^{+\infty}dx\,J_0\left(\frac{e\mathbf{E}\mathbf{k}}{m\hbar\Omega^2}2\sin\frac{x}{2}\right)\cdot[\bar{n}_{s,k}a^-\exp(ia^-x)+(\bar{n}_{s,k}+1)a^+\exp(ia^+x)]g_0(\mathbf{P}) \tag{31}$$

$$\approx\frac{2\pi}{\hbar}\sum_{P,s,k}\frac{|C_{s,k}|^2}{V}(2\bar{n}_{s,k}+1)g_0(\mathbf{P})(\mathscr{E}_{P+k}-\mathscr{E}_P)\times\sum_{n=-\infty}^{n=+\infty}J_n^2\left(\frac{e\mathbf{E}\mathbf{k}}{m\hbar\Omega^2}\right)\delta(\mathscr{E}_P-\mathscr{E}_{P+k}+n\hbar\Omega). \tag{32}$$

On going from (31) to (32) we have neglected the phonon energy and integrated with respect to x.

The quantum character of (31) and (32) follows directly from (32), namely, the change in the electron energy is due to the emission and absorption of light quanta. We investigate in (32) the transition to the classical limit, putting formally $\hbar \to 0$. In the limit as $\hbar \to 0$, the argument of the Bessel function in (32) assumes larger absolute values. The behavior of $J_\nu^2(x)$ at $|x| \gg 1$ depends on the ratio $|\nu|/|x|$ and can be described with the aid of the first terms of the Debye asymptotic expansions[13] 5):

$$J_\nu^2(x)=\frac{1}{2\pi(\nu^2-x^2)^{1/2}}\exp\left\{2(\nu^2-x^2)^{1/2}-2|\nu|\operatorname{arch}\left|\frac{\nu}{x}\right|\right\} \tag{33}$$

$$\text{if}\quad |\nu|>|x| \tag{33a}$$

and

$$J_\nu^2(x)=\frac{2}{\pi(x^2-\nu^2)^{1/2}}\cos^2\left[(x^2-\nu^2)^{1/2}-|\nu|\arccos\left|\frac{\nu}{x}\right|-\frac{\pi}{4}\right] \tag{34}$$

$$\text{if}\quad |\nu|<|x|. \tag{34a}$$

We note that the Debye asymptotic formulas were already used on going to the classical limit in the formula for the cross section of the stimulated bremsstrahlung effect[14].

As seen from (33), at $|\nu| > |x|$ the function $J_\nu^2(x)$ decreases exponentially with increasing ν. Therefore we can neglect in (32) the contribution from the region (33a). The inequality (33a), upon substitution of the values of ν and x from (32), coincides with inequality (21), and consequently determines the classically admissible change of the electron energy in the stimulated bremsstrahlung effect. It is easily seen that if we substitute in (32) the averaged asymptotic form (34) and neglect in (32) the discreteness of the index of the Bessel function, then (32) goes over into the classical formula (20).

Let us formulate now criteria for the applicability of the classical approximation. It follows from the inequality (21) that at $P > eE/\Omega$ we have $k \sim 2P$, and that at $P \ll eE/\Omega$ we have $k \sim 2eE/\Omega$. Then, starting from the requirement that the argument of the Bessel function in (32) be large in absolute magnitude, we obtain two sufficient criteria for the applicability of the classical approximation:

$$2e^2E^2/\hbar m\Omega^3\gg 1\quad\text{if}\quad P\ll eE/\Omega; \tag{35}$$

$$2eEP/\hbar m\Omega^2\gg 1\quad\text{if}\quad P\gg eE/\Omega. \tag{36}$$

The author thanks L. V. Keldysh for valuable advice and discussions.

1)In [2,3] they calculated the coefficient of light absorption by free carriers.

2)This situation obtained, for example, in experiment on the damage to transparent dielectrics by laser pulses of picosecond duration [5,6].

3)In addition to the cited reference, there are also papers by Buĭmistrov and Oleĭnik [9] and Dzhaksimov [10]. The result of Buĭmistrov and Oleĭnik is in error, for a reason indicated in [15]. The calculations of Dzhaksimov were not sufficiently correctly performed and the results do not lend themselves to a lucid physical interpretation.

4)In the cited paper, the absorption coefficient was calculated under the assumption that during the time of laser-pulse duration one can neglect the heating of the free carriers. Under this assumption we obtain from formulas (25) and (28) the absorption coefficient, by putting in them $\mathscr{E} = 0$ and multiplying by $N8\pi/ncE^2\eta$, where c is the speed of light, n is the refractive index, N is the concentration of the free carriers, $\eta = 1$ for linear polarization, and $\eta = 2$ for circular polarization.

5)We recognize that for interger values of ν we have

$$J_\nu^2(x)=J_{|\nu|}^2(|x|).$$

[1] N. Bloembergen, Kvantovaya elektronika No. 4, 786 (1974) [Sov. J. Quant. Electr. 4, 483 (1974)].
[2] H. Y. Fan. Rept. Progr. Phys., 19, 107 (1956).
[3] S. Visvanathan, Phys. Rev., 120, 376 (1960).
[4] T. Holstein, Phys. Rev., 96, 535 (1954).
[5] S. A. Belozerov, G. M. Zverev, V. S. Naumov, and V. A. Pashkov, Zh. Eksp. Teor. Fiz. 62, 294 (1972) [Sov. Phys.-JETP 35, 158 (1972)].
[6] V. M. Baev, A. N. Savchenko, and E. A. Sviridenkov, Zh. Eksp. Teor. Fiz. 66, 913 (1974) [Sov. Phys.-JETP 39, 443 (1974)].
[7] V. P. Silin, Zh. Eksp. Teor. Fiz. 47, 2254 (1964) [Sov. Phys.-JETP 20, 1510 (1965)].
[8] V. A. Pazdzerskii, Fiz. Tekh. Poluprov. 6, 758 (1972) [Sov. Phys.-Semicond. 6, 658 (1972)].
[9] V. M. Buĭmistrov and V. P. Oleĭnik, Fiz. Tekh. Poluprov. 1, 85 (1967) [Sov. Phys.-Semicond. 1, 65 (1967)].
[10] E. Dzhaksimov, Fiz. Tverd. Tela 15, 644 (1973) [Sov. Phys.-Solid State 15, 456 (1973)].
[11] E. M. Conwell. High Field Transport in Semiconductors, Acad. Press, N. Y. and London, 1967.
[12] E. M. Épshteĭn, Fiz. Tverd. Tela 11, 2732 (1969) [Sov. Phys.-Solid State 11, 2213 (1970)].
[13] G. N. Watson, Theory of Bessel Functions [Russ. transl.], Vol. 1, IIL (1949), pp. 267–271.
[14] N. M. Kroll and K. M. Watson, Phys. Rev., A8, 804 (1973).
[15] V. P. Oleinik, Ukr. Fiz. Zh. 16, 1777 (1971).

Translated by J. G. Adashko
116