Nonlinear magnetoacoustic effects

V. I. Kozub

A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences
(Submitted January 11, 1974; resubmitted June 21, 1974)
Zh. Eksp. Teor. Fiz. **68**, 1014–1031 (March 1975)

A nonlinear theory of the absorption is developed for short-wave sound whose wavelength is much smaller than the electron mean free path in a conductor with an arbitrary electron dispersion law, placed in a classically strong magnetic field of arbitrary direction. It is shown that the nonlinearity mechanism can be ascribed to sound-wave-induced distortion of the electron trajectory in sections where interaction with the wave is is effective (i.e., sections near points where the trajectory is tangent to the wave front). Expressions for the absorption coefficient are obtained and its magnetic field dependence (which, as is well known, is oscillatory for weak sound) is analyzed under conditions of strong nonlinearity. The nonlinear behavior varies greatly, depending on the specific experimental conditions. It is shown that an increase in the sound intensity leads to a decrease in the absorption coefficient and also to a change in the oscillation pattern. Various manifestations of this are the appearance of additional oscillations peaks, a decrease in the depth of modulation, or the appearance of a stronger dependence of the amplitude on the field intensity. It is found that the nonlinearity can be removed by increasing the magnetic field. Nonlinear effects in crossed magnetic and electric fields are analyzed. The possibility of experimental observation of the effects is assessed.

The present paper is devoted to the study of the absorption of high-frequency sound of high intensity in a conductor placed in a classically strong magnetic field. As is known,[1-3] in the case of sound of low amplitude the absorption coefficient has a nonmonotonic oscillatory dependence on the magnitude of the field in such a situation. We shall see that the increase in the sound intensity leads to a significant change in the picture of magnetoacoustic effects. In particular, the nonlinearity can lead to the appearance of additional oscillation maxima in the absorption.

We consider the propagation of shortwave sound ($ql \gg 1$, q is the wave vector of the sound, and l is the free path length of the electrons) in a conductor placed in a magnetic field H satisfying the conditions

$$qv_F \gg \frac{1}{T} \gg \frac{1}{\tau}, \quad \frac{\hbar}{T} \ll \Theta. \tag{1}$$

Here T is the period of motion along a trajectory in the magnetic field, v_F the velocity of the electrons at the Fermi surface, τ the relaxation time of the electrons, Θ the temperature. Let the wave vector of the sound be directed along the x axis and the vector **H** lie in the (x,z) plane. The conditions (1) allow us to speak of motion of the electron along a classical trajectory in a magnetic field, the characteristic dimension of which R is large in comparison with the wavelength of the sound. Its specific form depends on the dispersion law of the electrons and the configuration of the experiment. In particular, if the trajectories in momentum space are closed, and $\mathbf{q} \perp \mathbf{H}$, then the projection of the trajectories in coordinate space on the (x,y) plane are closed (Fig. 1). In an oblique field (α is the angle between **H** and **j**, where **j** is the unit vector of the z axis) a drift appears along x that is connected with the nonzero projection of the velocity $\mathbf{v}_H$ on the x axis (v_H is the component of the velocity along the magnetic field). In the presence of trajectories that are open in momentum space, the electron has a drift transverse to the field, i.e., drift along x can take place even at $\mathbf{q} \perp \mathbf{H}$. The drift along x naturally leads to an open character of the trajectories in the (x,y) plane (Fig. 2).

We shall be interested below in the projections of the trajectories on the (x,y) plane in a set of coordinates connected with the wave. For simplicity, we shall define as the trajectory that curve which characterizes the motion of the electron relative to the potential relief of the wave (the case in which motion in momentum space is considered will be discussed separately). We note that in a set of coordinates connected with the wave, an additional drift appears, due to the translational velocity (which is equal to the sound velocity w). Within the framework of qualitative considerations, however, we shall neglect the motion of the potential relief and use Figs. 1 and 2 as illustrations (in the quantitative theory, the indicated drift will be taken rigorously into account).

As is known, the electron interacts effectively with the wave only on trajectory intervals on which the component of the velocity along the wave vector of the sound is close to zero, i.e., near the classical turning points in the x coordinate (the intervals AA′, BB′ on Figs. 1, 2). The total contribution of the electron to the absorption is determined by the sum of the contributions of such intervals over the entire trajectory, traversed in a time $\sim\tau$, and naturally depends on the correlation of the phases of the wave on these intervals. Inasmuch as the indicated correlation is determined by the geometry of the trajectories, the sound absorption has a nonmonotonic oscillatory dependence on the magnetic field H.

If the displacement of the electron relative to the potential relief of the wave in a period T is equal to zero (Fig. 1), geometric oscillations of the absorption take place,[1] due to correlation of the phases at the points x_1 and x_2. If this displacement is different from zero (Fig. 2), then resonance magnetoacoustic oscillations take place.[2,3] They are due to the resonance dependence of the contribution of the trajectory to the absorption on the value of the shift B_x. There are generally several turning points in the interval of the trajectory

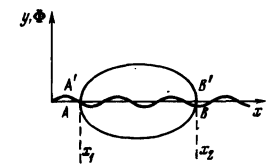

FIG. 1

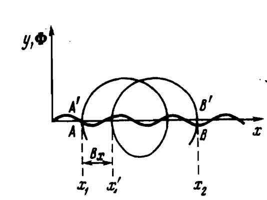

FIG. 2

which is traversed within the period. We shall denote as equivalent those points which are separated by one or several periods of the trajectory. Inasmuch as the shift B_x is not a multiple of the wavelength in the general case, the electron at the various equivalent turning points "feels" different phases of the wave (see the points x_1 and x_1' in Fig. 2). As a consequence of the summation over the periods and the corresponding summation over the phases of the turning points, the contribution of the trajectory to the absorption turns out to be small (of the order of the contribution of a single period). If the displacement is a multiple of the wavelength, the contributions of the periods are identical and the total contribution of the trajectory to the absorption increases by a factor $\sim\tau/T$.

These phenomena, which represent a convenient method for the investigation of the Fermi surface, were studied in detail both experimentally and theoretically within the framework of the theory that is linear in the intensity. However, it is known[4, 5] that in the case of short-wave sound, even for moderate intensities satisfying the condition

$$\Phi^0/\varepsilon_F \ll 1, \quad (2)$$

the so-called momentum nonlinearity, which is connected with the effect of the sound wave on the motion of particles interacting with it, becomes important. Here Φ^0 is the amplitude of the potential of the effective field of the wave, and ϵ_F is the Fermi energy. In an external magnetic field, such an effect should lead to distortion of the trajectory of the electron on intervals of effective interaction, which in turn leads to a significant anharmonism in the contribution to the electron distribution function and consequently to nonlinear effects in the absorption.

Owing to the motion along the trajectory, the magnetic field imparts a velocity $\Delta v_x \sim v(qR)^{-1/2}$ (v is the velocity along the trajectory, R the characteristic dimension of the trajectory) to the electron at a distance of the order of a wavelength near the turning point. In turn, the characteristic velocity given to the electron by the effective field of the wave is $\sim\tilde{v} = (\Phi^0/m)^{1/2}$. Therefore, the distortion of the trajectory near the turning point is determined by the parameter

$$b=[v(1/qR)^{1/2}\tilde{v}^{-1}]^2=mv\Omega/q\Phi^0,$$

Ω is the characteristic frequency of the motion along the trajectory. If $b \gg 1$, the effect of the sound wave on the trajectory can be neglected; this corresponds to the linear theory. If $b \lesssim 1$, the wave strongly distorts the trajectory near the turning points. In particular a group of entrapped particles is separated, which execute a finite motion in the potential wells of the wave. The condition $b \ll 1$ means the impossibility of removal of these electrons from the well by the magnetic field. Here the effect of the field on the motion of these particles is weak and their contribution to the absorption is calculated in analogy with the theory constructed in the absence of a magnetic field.[4] Inasmuch as the trapped electrons execute an oscillatory motion in the limits of the potential well, this contribution is determined only by the energy balance with account of scattering processes (which remove the particle from the trapped group). At moderate intensities, it is small in comparison with the contribution of the remaining—untrapped—particles.[5] (quantitative estimates are given below). In turn, the untrapped particles, which carry out motion above the potential relief of the wave, can, at $b \ll 1$, have turning points only in the neighborhood of the crests of ridges of the potential relief. This leads to a significant breakdown, at the turning points, in the phase correlation which is characteristic of the linear theory. These qualitative discussions are clarified by Fig. 3, on which is plotted the dependence of the energy of the longitudinal motion of the electron

$$E_1(x)={}^1/{}_2 mv_x{}^2+\Phi(x)$$

on the x coordinate.

The nonlinear picture of absorption in the perpendicular configuration $q \perp H$ was considered in[5] for the case of an isotropic quadratic electron spectrum. The purpose of the present research is the study of the nonlinear effects in the general case of an arbitrary configuration and arbitrary electron spectrum.

We shall see that in the regime of strong nonlinearity the increase in the sound intensity S leads to a decrease in the absorption coefficient. The qualitative reason is that the wave "synchronizes" the turning points of the electron trajectories. They all turn out to be distributed near the crests of the ridges. Therefore the contributions of the different intervals (for example, AA′ and BB′ on Figs. 1, 2) are equal in value, but opposite in sign (by virtue of the different directions of the velocity of the electron) and the total absorption decreases. The character of the falloff depends on the details of the electron spectrum. Along with the oscillating part of the absorption, which falls off rapidly with increase in intensity ($\sim S^{-3/2}$), in the case of a significant departure of the spectrum from isotropic, a nonlinear monotonic contribution can appear, which falls off more slowly ($\sim S^{-1/2}$). This in turn can lead to a decrease in the depth of modulation of the oscillation picture.

The nonlinearity also changes the very character of the oscillation picture. The picture of geometric oscillations is preserved even in the nonlinear regime, while the period does not change in comparison with the linear theory, although the shape does undergo change. Decrease in the depth of modulation of these oscillations at reasonable intensities can take place only in the case of an anisotropic spectrum and thus the conclusion of[5] that the oscillations become "smeared" for the isotropic quadratic spectrum is in error.

In the situation which corresponds to the investigation of magnetoacoustic resonance effects (Fig. 2), the distortion of the trajectory leads to a breakdown in its periodicity. As a consequence, the phase correlation at the turning points separated by a period in the case of an unperturbed trajectory is no longer determined by a simple shift equal to B_x (cf. Fig. 2), but by a more complicated dependence, which takes into account the location of the points themselves. In turn, in the nonlinear regime, the contribution of the individual turning point is no longer a harmonic function of the corresponding phase. A quantitative calculation shows that these circumstances lead to the appearance of additional res-

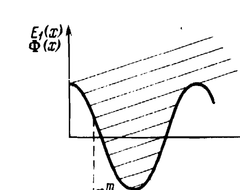

FIG. 3. Dependence of the energy of longitudinal motion of an electron on the x coordinate. The slant lines correspond to the set of curves $E_1(x)$ for different trajectories of the motion of the electron; the boundary points of the curves correspond to the classical turning points in x.

onance peaks, located between the peaks of the basic system and smaller in amplitude (at sufficient intensities, additional peaks should appear in the region of strong fields, beyond the extreme peak of the basic system).

The dependence of the nonlinearity parameter itself on the magnetic field has a significant effect on the oscillation picture. At larger field values the nonlinearity becomes significant at higher values of the sound intensity—the magnetic field "removes" the nonlinearity. This phenomenon is due to "carrying out" of the electrons from the potential wells of the wave by the magnetic field (Fig. 3). In the regime of high nonlinearity, such a circumstance leads to the result that the dependence of the height of the oscillation maxima on the field becomes much stronger.

We have seen that the nonlinearity parameter is connected both with the details of the electron spectrum and with the values of the deformation potential in the neighborhood of definite points of the Fermi surface (given by the configuration of the experiment). Therefore the experimental study of the nonlinear magnetoacoustic effects can give additional information on these important characteristics of the electron system. In particular, such information can be obtained by determining the relative height of the additional peaks in the regime of weak nonlinearity.

1. EQUATION OF THE TRAJECTORY

For the construction of the nonlinear theory of magnetoacoustic phenomena, it is first necessary to investigate the motion of the electron with an arbitrary dispersion law in a classically strong magnetic field, which satisfies the conditions (1), with account of its interaction with the sound wave. We shall describe the interaction with the deformation field of the sound wave with the help of the deformation potential, assuming the interaction energy to be equal to $\Phi_p(x) = \Lambda_{ik}(p)u_{ik}(x)$, where u_{ik} is the deformation tensor in the sound wave, and Λ_{ik} is the deformation potential tensor. On the basis of considerations analogous to[4], we can neglect the higher harmonics of the effective field if the condition (2) is satisfied, and set

$$\Phi_p(x,t)=\Phi_p^0\cos(qx-\omega t).$$

The equation of motion of the electron with allowance for its interaction with the sound takes the form

$$\frac{\partial \mathbf{p}}{\partial t}=-\frac{\partial\Phi}{\partial x}\mathbf{i}+\frac{e}{c}[\mathbf{v}\times\mathbf{H}], \tag{3}$$

i is a unit vector along the x axis. In coordinate space, it is convenient to transform to a set of coordinates connected with the wave, introducing the wave coordinate $\xi = qx-\omega t[\Phi(x,t) = \Phi(\xi)]$.

To solve the absorption problem, we first study the motion of the electron on intervals of effective interaction near the turning points ($\mathbf{q}\cdot\mathbf{v} = 0$) and then find the connection of the coordinates of the turning point with account taken of the effect of the effective field of the wave.

The characteristic width (in terms of the variable ξ) of the interval of effective interaction, in the regime of strong nonlinearity, is determined by the distance at which the electron gains a speed $\sim(\Phi_0/m)^{1/2}$ due to its turning in its trajectory in the magnetic field and thus amounts to $\sim 1/b$. Such an estimate corresponds to the qualitative considerations given above and, as we shall see further, is confirmed by the rigorous calculation. In comparison with the characteristic dimension of the trajectory $\sim qR$, this quantity has the smallness $\sim 1/bqR \sim \Phi^0/\epsilon_F \ll 1$. It is clear (starting from the character of the motion of the electron in the magnetic field) that this same parameter determines the relative dimensions of the corresponding intervals on the trajectory in momentum space. Therefore, assuming the spectrum of the electrons $\epsilon(p)$ to be a sufficiently smooth function, we can restrict ourselves, in the intervals of interest to us, to the quadratic expansion

$$\varepsilon(\mathbf{p}(\xi\sim\xi_1))=\varepsilon(\mathbf{p}(\xi_1))+(\mathbf{v}(\xi_1)\Delta\mathbf{p})+\sum_{ij}m_{ij}^{-1}(\xi_1)\Delta p_i\Delta p_j.$$

Here ξ_1 is the coordinate of the turning point. We further require

$$\alpha\ll 1 \tag{4}$$

(i.e., the situation is close to the perpendicular configuration $\mathbf{q}\perp\mathbf{H}$), which allows us to make accurate quantitative estimates by reducing the problem to one dimension.[1)] This condition, and also the smallness of v_x near the turning point lead to the estimate $\Delta p_{y,z} \ll \Delta p_x$, whence $dv_x = m_{xx}^{-1}dp_x|_{\xi\sim\xi_1}$. At the same time, by virtue of [2)] $\Delta v_y = v_y(\xi) - v_y(\xi_1)|_{\xi=\xi_1} \ll v_y$, one can set $v_y(\xi) \approx v_y(\xi_1)$. Integrating (3) over ξ in the neighborhood of the turning point, with account of the foregoing arguments, we obtain

$$\begin{aligned}\frac{v_\xi^2(\xi)}{2}&=(\xi-\xi_1)\frac{\Omega(\xi_1)v(\xi_1)}{q}+\frac{\Phi(\xi_1)}{m(\xi_1)}-\frac{\Phi(\xi)}{m(\xi_1)}\\&=\frac{\Phi^0}{m}\bigg|_{\xi_1}[(\xi-\xi_1)b_1\beta_1+\cos\xi_1-\cos\xi].\end{aligned} \tag{5}$$

Here

$$v_\xi=v_x-w,\quad \Omega=eH/m(\xi_1)c,$$
$$m(\xi_1)\equiv m_{xx}(\xi_1),\quad v(\xi_1)\equiv v_y(\xi_1),$$
$$b_1=\left|\frac{mv\Omega}{q\Phi^0}\right|_{\xi_1},\quad \beta_1=\operatorname{sign}\left(\frac{mv\Omega}{q\Phi^0}\right)\bigg|_{\xi_1}.$$

We note that near the left turning point (the point x_1 on Figs. 1 and 2) of the unperturbed trajectory we have $\Omega v > 0$; near the right turning point (the point x_2 on Figs. 1 and 2) we have $\Omega v < 0$. In contrast to[5], the quantities m, Ω, v, and Φ^0 can generally be different for different turning points. In the case $\beta_1 > 0$ and $\Phi^0/m > 0$ we have

$$v_\xi^2=2\left|\frac{\Phi^0}{m}\right|[(\xi-\xi_1)b+\cos\xi_1-\cos\xi]. \tag{6}$$

In the general case, the expression (5) reduces to (6) by means of a suitable change of the variable ξ in the interval of interest to us ($\xi\to-\xi$ or $\xi\to\xi+\pi$).

Equation (5) is in correspondence with the qualitative discussions given earlier. The effect of the wave on the trajectory is determined by the parameter b. At $b \gg 1$, the terms $\sim\Phi/m$ can be neglected, and we obtain the equation of the trajectory near the turning point in the magnetic field in the absence of the sound. At $b \lesssim 1$, the effect of the effective field of the wave is important. By virtue of the natural requirement that the left side of (5) be positive, the electrons are classified as trapped, as mentioned in the Introduction. It is easy to see that for $\beta_1 > 0$, $m > 0$, $\Phi^0 > 0$, the dependence of the one-dimensional energy E_1 on x, shown in Fig. 3, follows directly from (5) (neglecting the sound velocity w).

Inasmuch as the contribution of the trapped particles to the absorption is small in comparison with the contribution of the untrapped particles, as was noted above, we shall be further interested just in the contribution of the untrapped particles. The latter move chiefly along the ordinary trajectory in the magnetic field, experiencing the effect of the wave only over small intervals. In the regime of strong nonlinearity $b \ll 1$, the untrapped electrons can have turning points only on the crests of the potential contour of the wave. This statement also follows from (5) with account of the requirement of positiveness of the left side for different ξ, and is illustrated in Fig. 3. For the description of the location of the turning points on the "allowed" intervals, we shall reckon their ξ coordinate from the corresponding crest and denote this quantity by η. It is not difficult to obtain from (5) that the boundary value (between the untrapped and trapped particles) of the coordinate of the turning point η^m, which characterizes the width of the allowed interval, amounts to $\sim(4\pi b)^{1/2}$.

Thus, Eq. (5) describes completely the motion of the electron on those intervals of the trajectory where it interacts effectively with the sound. For calculation of the absorption of the sound, it is necessary to find the connection between the coordinates of the turning points, i.e., to investigate the general equation of the trajectory of the untrapped particles.

For the description of the location of the electron in momentum space, it is convenient to transform from p_x, p_y, p_z to coordinates ϵ, p_H, t^0 connected with the trajectory of the electron in momentum space in the absence of the sound perturbation.[6] Here ϵ is the energy of the electron on the trajectory, p_H the projection of the momentum in the direction of the magnetic field, and t^0 the time of motion from some initial point to the point with the prescribed p_x, p_y, p_z. The formulas for the transformation of the coordinates are:

$$\frac{d\mathbf{p}}{dt^0}=\frac{e}{c}[\mathbf{v}\times\mathbf{H}],\quad \mathbf{v}=\frac{\partial\varepsilon}{\partial\mathbf{p}}. \tag{7}$$

Using the equation of motion (3), we get in the new variables,

$$\frac{dt^0}{d\xi}v_\xi=-\frac{dt^0}{dp_x}\frac{\partial\Phi}{\partial\xi}+\frac{1}{q},\quad \frac{\partial\varepsilon}{\partial\xi}v_\xi=-v_x\frac{\partial\Phi}{\partial\xi},\quad \frac{\partial p_H}{\partial\xi}v_\xi=-\frac{\partial\Phi}{\partial\xi}\sin\alpha. \tag{8}$$

It is seen from (8) that, with accuracy to within terms $\sim\Phi/\epsilon$, we can neglect the change of ϵ and, with account of (4), the change in p_H also in motion along the trajectory. In other words, we can assume that ϵ and p_H are constant for the given trajectory and are its parameters (as in the absence of the effect of the sound wave).

In order to study the effect of the sound perturbation on the location of the turning points, we introduce in place of t^0 the variable X, which is the ξ coordinate of the point with the prescribed p_x, p_y, p_z on the trajectory in the absence of the sound perturbation:

$$dX=v_\xi(t^0,\varepsilon,p_H)\,dt^0.$$

Transforming the first equation in (8) with account of this, we get

$$\frac{dX}{d\xi}=1-q\frac{dt^0}{dp_x}\frac{\partial\Phi}{\partial\xi}. \tag{9}$$

We now integrate (8) along the trajectory:

$$X_2-X_1=\xi_2-\xi_1-q\int\limits_{(\xi_1)}^{(\xi_2)}\frac{dt^0}{dp_x}\frac{\partial\Phi}{\partial\xi}d\xi. \tag{9a}$$

Here 1, 2 are certain points of the trajectory; integration along ξ is carried out on a contour which represents the interval of the trajectory which joins the prescribed points. The value of dt^0/dp_x, which is determined by the dispersion law, is a smooth function of the point on the trajectory (with a characteristic scale of change in terms of ξ of the order aR); the quantity $\Phi^0_{p(t^0)}$ behaves similarly. Taking this into account, for integration along ξ we separate intervals on the integration contour which corresponds to integer periods of the wave, bounded by the crests closest to ξ_1 and ξ_3. Expressing explicitly, the contribution of the ends of the integration contour, we obtain the following expression which determines the connection between the turning points:

$$B_{12}=\xi_2-\xi_1+\beta_1\left(\frac{\cos\xi_1-1}{b_1}\right)-\beta_2\left(\frac{\cos\xi_2-1}{b_2}\right)+P^{12}. \tag{10}$$

Here B_{12} is the distance between the prescribed points in the absence of the sound perturbation. The quantity P^{12} is the integral over an integer number of periods of the wave. Its dependence on ξ_1 and ξ_2 is due only to the integrand and is smooth (with characteristic scale of change $\sim$qR) so that P^{12} can be considered constant for a change of ξ_1 of the order of unity.

As is seen from (5) and (10), the potential of the wave causes a displacement $\sim 1/b$ of individual turning points (in comparison with the unperturbed trajectory). The periodicity here is, generally speaking, disrupted. We note an important fact, however. Along with the quantities which are expressed directly in terms of the potential of the wave, and which change significantly over distances on the order of a wavelength, in our calculations there also appear quantities the dependence of which on ξ is determined only by the dependence of the parameters of the electron spectrum on the location on the trajectory in momentum space. This dependence is smooth, inasmuch as the characteristic interval of change is of the order of the dimensions of the trajectory $\sim$qR. When considering the behavior of such functions, we can neglect the distortion of the trajectory over small intervals under the action of the sound. Thus, the disruption of the periodicity, on the one hand, should lead to a nonlinear change in the picture of magnetoacoustic oscillations (inasmuch as these effects are determined directly by the periodicity of the distribution of intervals of effective interaction), and on the other hand is unimportant for smooth functions. Therefore, we shall call an interval of the trajectory between two turning points separated by a period in the absence of sound perturbation, a quasiperiod. It can be shown (by using the periodicity of the integrand) that the integral $P^{12} = 0$ for turning points separated by a quasiperiod. Therefore, (10) in such a case takes the form

$$B_\xi=\xi_2-\xi_1+\frac{\beta_1}{b_1}(\cos\xi_1-\cos\xi_2),\quad B_\xi=q(B_x-wT). \tag{11}$$

Equations (5), (10), and (11) completely describe the trajectory on the intervals of interest to us: knowing the location of one of the turning points, we can, with the help of (10) and (11), determine the location of the others. In this connection, a certain trajectory (for given ϵ, p_H) can be specified by specifying one of the turning points, for example, the right-hand one; we call this the defining point. The trajectory itself is represented as a set of quasiperiodic intervals. We introduce the enumeration of the turning points on the given trajectory: the upper index will denote the corresponding quasiperiod, the lower the position of the point on this quasiperiod, and the defining point of the trajectory corresponds to the symbol ξ_0^0.

The following method of description of the trajectory (more accurately, of the distribution of turning points of interest to us) is convenient. In the neighborhood of the turning point, we carry out a transformation of variables according to the formula

$$\vartheta=\xi+(1-\cos\xi)/b. \tag{12}$$

We call the quantity ϑ the "reduced phase." In the regime of strong nonlinearity $b \ll 1$, as we noted above, the turning points are located within narrow "allowed" intervals near the crests (Fig. 3). As is not difficult to see in (5), the change in the coordinate of the turning point in the limits of the allowed interval corresponds to a change of ϑ by 2π (or, with accuracy to terms $\sim b$, in the interval $[2\pi n, 2\pi(n+1)]$). In what follows, a continuous change of ϑ corresponds to a "jump" in the turning point to the neighboring interval. Thus, the continuous quantity ϑ describes the location of the turning points on the entire discrete set of allowed intervals. In order to express the value of the potential of the wave at the turning point in terms of ϑ, we introduce the function Z:

$$Z|_{0<\vartheta<2\pi}=\vartheta-\pi, \quad Z(\vartheta+2\pi)=Z(\vartheta). \tag{13}$$

Using the concept of the reduced coordinate η introduced previously, it is not difficult to show (by starting out from (12) and (13)) that, in the case $b \ll 1$,

$$Z(\vartheta)=\eta+\frac{1-\cos\eta}{b}-\pi\approx\frac{\eta^2(\vartheta)}{2b}-\pi. \tag{14}$$

The quantity Z, which directly determines the location of the turning point on the crest of the wave and thus the value of the effective potential at the given turning point, we call the reduced potential.

As a result, the connection between the turning points takes the simple form

$$\beta_i\vartheta_i{}^j=\beta_k\vartheta_k{}^l-B_{ji,\,lk}+P^{ji,\,lk}. \tag{15}$$

and, in particular, for turning points separated by a quasiperiod,

$$\vartheta_i{}^j=\vartheta_i{}^{j+1}-B_\xi\beta_i. \tag{16}$$

2. THE SOUND ABSORPTION COEFFICIENT

As is known, the electronic absorption coefficient of sound in conductors is expressed in terms of the reactive response of the electron system (see, for example,[7])

$$\Gamma=\frac{1}{\rho\omega^2u^2}K, \quad K=\frac{2}{2\pi(2\pi\hbar)^3}\int d^3p\int_0^{2\pi}\frac{\partial\Phi}{\partial\xi}f(\mathbf{p},\xi)\,d\xi. \tag{17}$$

Here ρ is the density of the crystal, u the displacement vector, f the electron distribution function, which must be sought from the kinetic equation

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial\mathbf{r}}+\frac{e}{c}[\mathbf{v}\times\mathbf{H}]\frac{\partial f}{\partial\mathbf{p}}-\frac{\partial\Phi}{\partial x}\frac{\partial f}{\partial p_x}+\hat{I}f=0, \tag{18}$$

where $\hat{I}$ is the collision operator. We set $f = F_0(\epsilon+\Phi(\xi)) + g$, where F is the equilibrium distribution function; it is seen that contribution to the absorption is made only by the quantity g. With account of the fact that the trajectories of the electrons are the characteristics of the kinetic equation, we obtain for the function $g(\mathbf{p}(\xi),\xi)$, defined on the trajectory,

$$\frac{d\xi}{v_\xi(\xi,\xi_0{}^0)}=\frac{dg}{g(\xi,\xi_0{}^0)/q\tau-G}, \quad G=w\frac{\partial\Phi}{\partial\xi}\frac{\partial F_0}{\partial\varepsilon}, \tag{19}$$

$v_\xi(\xi, \xi_0^0)$ is the value of the velocity of the electron, moving along the trajectory from the defining point ξ_0^0 to the point ξ. As was pointed out earlier, in the case of strong nonlinearity, we consider the untrapped particles; the contribution of the trapped particles was analyzed in[4, 5]. The validity of the approximation of the relaxation time can be established similarly.[4, 5] Integrating (19) along the trajectory, we get

$$g=\int_{(\xi_{-\infty})}^{(\xi)}d\xi'\frac{G(\xi')}{v_\xi(\xi',\xi_0{}^0)}\exp\left(\frac{1}{q\tau}\int_{(\xi)}^{(\xi')}\frac{d\xi''}{v_\xi(\xi'',\xi_0{}^0)}\right), \quad \xi_{-\infty}=\xi(t\to-\infty). \tag{20}$$

We now transform the general expression for the absorption coefficient (17). Carrying out a change of variables in momentum space, we have[2]

$$K=\frac{2}{2\pi(2\pi\hbar)^3}\frac{eH}{c}\int d\varepsilon\,dp_H\int_0^{2\pi}\frac{\partial\Phi}{\partial\xi}d\xi\int dt^0 g(\varepsilon,p_H,t^0\ \xi)+K_t, \tag{21}$$

K_t is the contribution of the trapped particles. Summation over t^0 reduces to the fact that at each point ξ summation should be carried out over all trajectories passing through the given point (i.e., the projections of which on the axis take on the corresponding value). As defining points of the trajectories, it is convenient to choose the extreme right-hand turning points, which correspond to the quasiperiodic intervals to which the point ξ belongs. Now, transforming from ξ, t^0 to the variables ξ, ξ_0^0 with account of (9), we have

$$dt^0=-\frac{1}{qv_\xi(\xi,\xi_0{}^0)}\left[1+\frac{\beta}{b}\sin\xi\Big|_{\xi_0{}^0}\right]d\xi_0{}^0. \tag{22}$$

Summation over ξ_0^0, with account of what has been said above, is done over the extreme right-hand turning points of all the quasiperiodic intervals (corresponding to different trajectories) to which the point ξ belongs. Using the requirement of the nonperiodicity of $g(\xi, \xi_0^0) = g(\xi+2\pi, \xi_0^0+2\pi)$ (which takes into account the fact that the shape of the various trajectories for given ϵ, p_H, the defining points of which are separated by an interval $2\pi n$, are the same), we transform the summation over ξ_0^0 in analogy with[5]:

$$\int_0^{2\pi}\frac{\partial\Phi}{\partial\xi}d\xi\sum_{\xi_0{}^0}g(\xi,\xi_0{}^0)=\int_{-\eta_0^m}^{0}d\xi_0{}^0 \times\int_{(T\xi_0^0)}d\xi\frac{\partial\Phi}{\partial\xi}g(\xi,\xi_0{}^0)\frac{dt^0}{d\xi_0{}^0},$$

where $(T\xi_0^0)$ denotes the taking of the integral over the quasiperiod of the trajectory to which the point ξ_0^0 belongs. $\eta^m = 2\pi$ in the case of an undistorted trajectory and $\eta^m \approx \sqrt{4\pi b}$ in the case of strong nonlinearity. As a result, the electron response takes the form

$$K=\frac{2}{2\pi(2\pi\hbar)^3}\int d\varepsilon\,dp_H\left\{-\int_{-\eta_0^m}^{0}d\xi_0{}^0\left[1+\frac{\beta}{b}\sin\xi\Big|_{\xi_0{}^0}\right]\mathscr{L}(\xi_0{}^0)\frac{w}{q}\frac{\partial F_0}{\partial\varepsilon}\right\}+K_t,$$

$$\mathscr{L}(\xi_0{}^0)=\frac{eH}{c}\int_{(T\xi_0{}^0)}\frac{\tilde{g}(\xi)\,\partial\Phi/\partial\xi}{v_\xi(\xi,\xi_0{}^0)}d\xi, \tag{23}$$

$$\tilde{g}(\xi)=\int_{(\xi_{-\infty})}^{(\xi)}d\xi'\frac{\partial\Phi/\partial\xi'}{v_\xi(\xi',\xi_0{}^0)}\exp\left[\frac{1}{q\tau}\int_{(\xi)}^{(\xi')}\frac{d\xi''}{v_\xi(\xi'',\xi_0{}^0)}\right].$$

In the case of strong nonlinearity, we transform from the variable ξ_0^0 to the already introduced reduced phase

$$K=\frac{2}{2\pi(2\pi\hbar)^3}\int d\varepsilon\, dp_H\left(-\int_0^{2\pi}d\vartheta_0{}^0\mathscr{L}(\vartheta_0{}^0)\right)\frac{w}{q}\frac{\partial F_0}{\partial\varepsilon}+K_t. \qquad (23a)$$

The integration in the argument of the exponential is connected with the contribution of the intervals of the trajectory of the order of qR. The result of this integration is thus similar to the linear theory and is a smooth function of the points ξ and ξ'. At the same time, with account of (5), we can see that the integrals of the quantities $\sim v_\xi^{-1}\partial\Phi/\partial\xi$ with respect to ξ and ξ' converge near the turning points over distances $1/b \ll qR$ in the case strong nonlinearity); in particular, this proves the assertion made previously that the intervals of effective interaction (which are indeed the ones that make a contribution to the integral) are small. For this region, the integration along the trajectory reduces to summation of the contributions of the various turning points (and for calculation of these contributions, we can neglect the change of exponential within the limits of the interval of effective interaction).

Using the expression for the velocity of the electron on the intervals of effective interaction (6) (making, if necessary, the corresponding transformation of the variable ξ), we reduce the integral over the trajectory close to the turning point to the ordinary integral over ξ. As a result, the contribution of some turning point ξ_i^j to the integration over ξ (or ξ') takes the form

$$J_i^j=\alpha_i\beta_i\int_{\eta_i^j}^{\infty}\frac{d\xi\,\partial\Phi/\partial\xi}{\{2|\Phi^0/m|[(\xi-\eta_i^j)b_i+\cos\eta_i^j-\cos\xi]\}^{1/2}}=-\alpha_i\tilde{p}_i\mathscr{F}(\eta_i^j,b_i), \qquad (24)$$

where $\alpha_i = 1$ for the boundary points of a quasiperiod, and $\alpha_i = 2$ for the remaining points; the factors β_i appear as a result of a transformation of the variable ξ; $\tilde{p}_i = \beta_i\Phi_i^0[^{1/2}|m_i/\Phi_i^0|]^{1/2}$;

$$\mathscr{F}(x,y)=\int_x^{\infty}\frac{\sin\xi\, d\xi}{[\cos x-\cos\xi+y(\xi-x)]^{1/2}}. \qquad (25)$$

The integral $\mathscr{F}$ is a function of the dimensionless parameters η_i^j and b_i. The problem is thus reduced to simple integration. In the case $b \gg 1$ we have in the lowest order of $1/b$,

$$\mathscr{F}_i^j\equiv\mathscr{F}(\eta_i^j,b_i)\approx(\pi/b_i)^{1/2}\sin(\eta_i^j+\pi/4),$$

and the results of linear theory follow from (23) and (24). In the regime of strong nonlinearity in which we are interested, we have $x \ll 1$ and $y \ll 1$ in (25). Asymptotic estimates give[3]

$$\mathscr{F}_i^j\sim 2+[\pi b_i-(\eta_i^j)^2/2]+b_i\mathscr{F}^{\log},$$
$$\mathscr{F}^{\log}\approx\frac{\sqrt{2}}{2}\left\{\ln\left[\frac{\eta_i^j+b_i)^2}{4\pi b_i}\left(\frac{4\pi b_i-(\eta_i^j)^2}{6\pi b_i-(\eta_i^j)^2}\right)\right]-2\right\}, \qquad (26)$$

or in terms of the quantities introduced by us,

$$\mathscr{F}_i^j\approx 2+(-b_i)Z(\vartheta_i^j)+b_i\mathscr{F}^{\log}$$

We now transform the expression for the function $\tilde{g}$ (23), writing down the integral over ξ' in the form of a sum of the contributions of the individual turning points. Neglecting the contribution of the initial intervals of the trajectory, which constitute a fraction of a quasiperiod, with accuracy up to terms $\sim T/\tau$ (relative to the basic effect), we get

$$-\tilde{g}=\sum_{j=0}^{\infty}\sum_{i=0}^{l}\alpha_i\tilde{p}_i\mathscr{F}_i^j\exp\left(\frac{1}{q\tau}\int_{(\xi_0{}^0)}^{(\xi_i^j)}\frac{d\xi'}{v_\xi(\xi',\xi_0{}^0)}\right).$$

Here l is the number of turning points on a quasiperiod. We expand the reduced potential in a Fourier series:

$$Z(\vartheta)=\sum_{k=1}^{\infty}\left(-\frac{1}{k}\right)\sin(k\vartheta).$$

Using this expression, we carry out summation over the index of quasiperiods with account of (16). Keeping terms which make a contribution to the absorption, with accuracy up to $\sim T/\tau$, we have

$$-\tilde{g}=\sum_{i=0}^{l}\alpha_i\tilde{p}_ib_i\sum_{k=1}\frac{\lambda_k\sin(k\vartheta_i{}^0)}{k}+\lambda_0\left[2\sum_{i=0}^{l}\alpha_i\tilde{p}_i+\mathscr{F}^{\log}(\vartheta_0{}^0)\sum_{i=0}^{l}\alpha_i\tilde{p}_ib_i\right]. \qquad (27)$$

Here

$$\lambda_k=\mathrm{Re}\left[\frac{1}{1-\exp(ikB_\xi-\gamma)}\right],\quad \gamma=\frac{1}{q\tau}\int_{(T)}\frac{d\xi}{v_\xi}=\frac{T}{\tau}. \qquad (28)$$

For k = 0, this quantity is nearly equal to τ/T and does not have a resonance dependence. At $k \neq 0$, the quantity λ_k as a function of B_ξ has at $B_\xi = 2\pi n/k$ resonance peaks of height $\sim\tau/T$ and relative width $\sim T/\tau k$.

Transforming the integration over ξ in (23) in similar fashion and further integrating, with account of the performed operations, with respect to ϑ_0^0 in (23a), we get

$$\int_0^{2\pi}d\vartheta_0{}^0\mathscr{L}(\vartheta_0{}^0)=\bar{\mathscr{L}}_R+\bar{\mathscr{L}}_A,$$
$$\bar{\mathscr{L}}_R=\sum_{k=1}\frac{\lambda_k}{k}\int_0^{2\pi}\left[\sum_{i=0}^{l}\alpha_i\chi_ib_i^{3/2}\sin(k\vartheta_i{}^0)\right]^2d\vartheta_0{}^0,$$
$$\bar{\mathscr{L}}_A=\lambda_0\int_0^{2\pi}[2\Delta(\chi b^{1/2})+\Delta(\chi b^{3/2})\mathscr{F}^{\log}(\vartheta_0{}^0)]^2\,d\vartheta_0{}^0 \qquad (29)$$
$$\Delta(\chi b^\nu)=\sum_{i=0}^{l}(\alpha_i\chi_ib_i{}^\nu),\quad \chi_i=\beta_i\Phi_i{}^0\left(q\left|\frac{m_i}{v_i}\right|\right)^{1/2}.$$

We note that the neglect (which is important for us) of the contribution of the fractional portions of quasiperiods is permissible if we can neglect the contribution of a single turning point in comparison with the quantity (29), which includes the contribution of the integer number of quasiperiods. Therefore, at $\Delta(xb^{1/2}) = 0$, the applicability of our calculations is restricted by the condition $(\tau/T)b^2 \gg 1$.

3. NONLINEAR PICTURE OF MAGNETOACOUSTIC OSCILLATIONS

In Eq. (29), which determines the sound absorption by the untrapped electrons, there are two separate terms; the resonance term $\mathscr{L}_R$, which is connected with the harmonics of the reduced potential and the resonance factors λ_k, and the nonresonant part, which is proportional to $\lambda_0 \sim \tau/T$. The contribution of the trapped particles represents the contribution to the nonresonant term. The nonlinear picture of the absorption depends on the relation between the resonant and the nonresonant parts.

We first analyze the resonant term. First we note the rapid decay of resonance absorption with increase of the intensity $\sim S^{-3/2}$, which is connected with the quantities $b_i \sim 1/\Phi_i^0$. We begin the study of the dependence on the magnetic field with the very simple case of perpendicular configuration $q \perp H$ (and $\omega \ll 1/T$) and trajectories that are closed in momentum space. This case was considered for an isotropic quadratic spectrum in[5].) The trajectory in coordinate space is closed (Fig. 1) and $B_\xi = 0$, as a consequence of which all the λ_k are equal to one another. In the simplest case, when the trajectory has two turning points and $b_1 = b_2$, $\chi_1 = -\chi_2$, we get from (29) (with account of (15)),

$$\bar{\mathscr{L}}_R \sim \int_0^{2\pi} [Z(\vartheta_1) - Z(\vartheta_2)]^2 d\vartheta_1 \sim [(B_{12}')^3 + (2\pi - B_{12}')^3],$$
$$B_{12}' \equiv B_{12} - 2\pi \left[\frac{B_{12}}{2\pi}\right], \qquad (30)$$

[ ] denotes the integer part. As is seen from (30), the contribution of the trajectory to the absorption has an oscillatory dependence on the size of the trajectory B_{12}: it is a maximum for $B_{12} = 2\pi n$ and a minimum for $B_{12} = \pi(2n+1)$. This dependence, which is preserved in the general case $(\chi_1, b_1) \neq (-\chi_2, b_2)$ leads to geometric oscillations of the absorption as also in the linear theory. The period of these oscillations is determined by the size of the extremal cross section of the Fermi surface and does not change in comparison with the linear theory. The shape of the oscillations is somewhat changed, as is seen from (30). The depth of the modulation depends on the ratio of $\bar{\mathscr{L}}_R$ to the nonresonant part $\bar{\mathscr{L}}_A$. The contribution of the nonresonance part will be analyzed in detail below. For the present, we note that in the case $\Delta(\chi b^{1/2}) = 0$, for example, for an isotropic quadratic spectrum, we can neglect the contribution of the nonresonant part and the depth of modulation $\sim (1/qR)^{1/2}$ just as in the linear theory. Thus the conclusion of[5] as to the "smearing" of the oscillations in this situation is in error already for $b \lesssim 1$ and is connected with the insufficiently detailed analysis of the expression (19) in[5].

We now consider the case of trajectories that are open in coordinate space, $B_\xi \neq 0$ (Fig. 2). First we note that $\mathscr{L}_R$ has resonant peaks at $B_\xi = 2\pi n$. These peaks correspond to the ordinary magnetoacoustic resonance.[2,3] Moreover, in contrast to the linear theory, a set of additional peaks appear for $B_\xi = \pi, \ldots, 2\pi m/n$, connected with the set of the various λ_k. It is not difficult to prove that these peaks have the relative amplitude $\sim 1/n^2$ in comparison with the basic system (for fixed b_i), inasmuch as the resonance condition is satisfied only for k that is a multiple of n. Their relative width is $\sim 1/n$.

In order to make clear the mechanism of the appearance of the additional peaks in the absorption, we return to the expressions (21)–(24). It is seen that the appearance of the terms λ_k with $k \geq 2$ in the nonlinear situation is connected with the nonharmonicity of the quantities $\mathscr{F}_i^j$ that describe the effective interaction of the electron with the sound. We estimate the parameter that determines the appearance of this nonharmonicity. For this purpose, we find the corrections to the integral $\mathscr{F}_i^j$ which take into account the finiteness of the sound amplitude in the case of relatively low intensity $b \gg 1$. With the aim of taking into account also the displacement of the turning point due to interaction with the wave, we make the change of variable $\xi \to \vartheta$ in correspondence with (12) and (16). Expanding the integrand in (24) in powers of 1/b, we get, in particular, a term corresponding to the second harmonic:

$$\sim \frac{1}{b}\left(\frac{\pi}{2}\right)^{1/2} \sin\left[2\vartheta_i^j + \frac{\pi}{4}\right].$$

Simple calculations show that the relative amplitude of the additional peak associated with it is of the order of $1/2b^2$. Thus, from the relative height of these peaks, we can assess the values of b_i and, in particular, the deformation potential, and indirectly also the intensity of the sound introduced into the crystal, and so on.

It is seen from Eq. (26) that, along with the resonant dependence on the field, the quantity $\bar{\mathscr{L}}_R$ has a monotonic, but very sharp, dependence $(\sim H^3)$ connected with the factors b_i. With increase in the magnetic field in the region of sufficiently strong fields, the condition $b \gg 1$ begins to be satisfied; from there on the absorption is described by the linear theory. Thus the strong magnetic field removes the nonlinearity. The reason is the increase in the curvature of the trajectories and the removal of the electrons from the potential wells of the wave. In this connection, we recall that the manifestation of momentum nonlinearity in the absence of a magnetic field is described by the parameter $\Phi^0(ql)^2/\epsilon$.[4] As is not difficult to see, in the situation considered, the threshold of the momentum nonlinearity exceeds by a factor $[\Omega\tau ql]^2$ in intensity the threshold of nonlinearity in the absence of the field. The strong dependence of the height of the oscillation peaks on the field significantly changes the oscillation picture. Thus, if the condition $b \ll 1$ is satisfied near the extreme peak of the linear picture $(B_\xi = 2\pi)$, additional peaks, corresponding to stronger fields $(B_\xi = \pi, \ldots, 2\pi/n)$, exceed the amplitude of the basic peak $(B_\xi = 2\pi)$.

We now analyze the nonresonant $\bar{\mathscr{L}}_A$ (which appears only in the nonlinear picture). In the case of a complicated electron spectrum, the quantity $\Delta(\chi\vartheta^{1/2})$ is generally different from zero and, as is easy to see, precisely this term gives the basic contribution (for a given p_H) to the monotonic part of the absorption coefficient. The nonresonant absorption is here proportional to b and falls off with increase in the intensity and with decrease in the field as $S^{-1/2}$ and H^2, respectively, i.e., much more slowly than the resonant term. As a result, the modulation depth of the resonant picture falls off in proportion to b^2. In particular, in the corresponding configuration, this leads to the decrease in the depth of modulation of the geometric oscillation predicted in[5]. Thus, from the change in the depth of modulation we can assess the anisotropy of the quantity $\chi b^{1/2}$ and, in particular, the anisotropy of the deformation potential.

If the same quantity $\Delta(\chi b^{1/2})$ is negligibly small for almost all the important values of p_H (for example, in the isotropic case), the nonresonant part of the electronic absorption is connected only with the contribution of the trapped particles or with the logarithmic term. Simple estimates show that for the characteristic trajectories, at moderate intensities, these contributions are small in comparison with the resonance term.[4)]

Integration over p_H is carried out as in the linear theory[2] and leads to similar conclusions on the behavior of the oscillation picture as a function of the mutual orientation of the vectors q and H and the singularities of the electron spectrum. In particular, if B_ξ does not depend on p_H (for example, in the presence of trajectories that are open in momentum space or in the case of acoustic cyclotron resonance $\omega = 2\pi n/T$), the resonance conditions are satisfied simultaneously for all electrons and the resonance peaks of $\bar{\mathscr{L}}_R$ correspond directly to the peaks of the absorption coefficient. In the case of a dependence of B_ξ on p_H (which occurs, for example, if the Fermi surface is closed and the drift is created by an oblique field), resonant behavior of the integral over p_H can be connected only with the extrema of $B_\xi(p_H)$. If there exists an internal extremum $(\partial B/\partial p_H = 0)$, then the peaks of $\bar{\mathscr{L}}_R$ correspond to the absorption peaks.

Of great interest from the viewpoint of the study of nonlinear effects is the situation in which the extremum is connected with a limiting point. Actually, the vicinity of a limiting point corresponds to small values of the velocity along the trajectory and, consequently, to a large effect of the sound wave on the trajectory. First, we shall make some estimates for the linear picture of the specified effect. According to[8] the participation of the limiting point in the magnetoacoustic resonance requires a configuration close to the perpendicular:

$$(1/ql)^{1/2}\gg\alpha\gg 1/qR.$$

Here, $\partial B_\xi/\partial p_H$ near the limiting point is different from zero in the general case and we are dealing with a limiting extremum. For convex Fermi surfaces, such an extremum is the only possible one. The vicinity of the limiting point that makes an effective contribution to the resonance effect is determined by the width of the function $\lambda(p_H)$: the characteristic angle of the corresponding segment of the Fermi surface[5)] is $\varphi_0 \sim (\alpha q l)^{-1/2}$. Estimates of the integral over p_H (of the type given in[2] for the study of the limiting extremum at $\alpha \sim 1$) give, in the linear case,

$$\Delta\Gamma_n\sim\Gamma_0\frac{\pi}{Tq\alpha}\frac{1}{v_F\max[\varphi_0,(\Delta_n/B_\xi+|\Delta_n/B_\xi|)^{1/2}]}\left[\frac{1}{2}+\frac{\operatorname{sign}B_\xi}{\pi}\operatorname{arctg}\left(\frac{\Delta_n\tau}{T}\right)\right],$$

$$\Delta_n=B_\xi-2\pi n.$$

Thus, the function $\Gamma(1/H)$ is a set of the root peaks of decreasing amplitude ($\sim\Gamma_0 n^{-1}(T_i\alpha qR/\tau)^{-1/2}$, which are limited on the high-field side by a strong discontinuity, and decay rapidly with increasing $1/H$ down to values $\sim\Gamma_0/n$. The region near the discontinuity (a sharp peak vertex of width $\sim\varphi_0^2/H_1$) corresponds to satisfaction of the resonance condition in the neighborhood of the limiting point, and the region of decay corresponds to the contribution of the intervals far from the limiting point.

The specifics of the nonlinear effects for this situation are connected with the dependence of the coefficient b on v_y and, consequently, on the proximity to the limiting point. Thus, the nonlinear picture is "scanned" on the graph of $\Gamma(1/H)$. With increase in the intensity, the nonlinearity becomes significant, beginning with the intervals corresponding to the contribution of the limiting point (and for intensities much less than those on the characteristic trajectories). This appears first in the decrease in the height of the basic peaks $\propto b^3$. Additional peaks, which arise in the region between the basic peaks, are sharp, inasmuch as they are connected only with the contribution of the vicinity of the limiting point. With further increase in the intensity,[6)] when the condition $b \lesssim 1$ begins to be satisfied for the characteristic trajectories, the oscillation effects corresponding to the limiting point actually disappear.

We note that the drift of the electrons along the ξ axis can be due (in the case of a semimetal or a semiconductor) to the presence of an external electric field in the system.[3]

In conclusion, we give estimates for the possibilities of experimental observation of the considered phenomena. They are all connected with the parameter b:

$$b_i^{-1}=\left(\frac{\Phi^0 q}{\Omega v m}\right)\bigg|_i=\frac{q\Phi_i^0 c}{eHv_i}\sim qR\left(\frac{\Phi_i^0}{\varepsilon_F}\right)\frac{v_F}{v_i}.$$

We note first that very pure materials are required with long mean free paths, which admit of large values of the parameters ql and qR. For typical metals with open trajectories, for $\epsilon_F \sim 10^{4\circ}$, $v_F/v_i \sim 10$ (i.e., when the basic contribution to the absorption is made by points near the inflection points of the trajectories) and $qR \sim 200$, the value $b \sim 1$ is achieved at sound intensities $S \sim 4\,W/cm^2$. The situation for semimetals is more favorable. For semimetals with $\epsilon_F \sim 200°$ and a constant deformation potential ~ 10 eV in the case $qR \sim 40$, the value $b \sim 1$ is achieved on the characteristic trajectories at $S \sim 4 W/cm^2$, and in the vicinity of the limiting point (setting $ql \sim 10^3$) at $S \sim 0.04\ W/cm^2$.

I take this opportunity to express my deep gratitude to Yu. M. Gal'perin, V. L. Gurevich and R. Katilyus for detailed discussions of the work and valuable advice.

APPENDIX

The quantity $\mathscr{F}(x, y)$ is a two-parameter integral (see (25)). In correspondence with our assumptions we have $x \ll 1$ and $y \ll 1$ (inasmuch as $y = b_i$ and $x = \eta_1^i$ in (24); on the other hand, $\eta \le \eta^m \approx (4\pi b)^{1/2}$). We make quantitative estimates, which have asymptotic meaning. For this purpose, we separate from the integration interval sections that are close to the crests of the wave and correspond to small values of the denominator. We take C in the interval $y^{1/2} \ll C \ll 1$; then $C^2 \gg (y, x^2/2)$. We represent the desired integral in the form

$$\mathscr{F}=\mathscr{F}_1+\mathscr{F}_2+\mathscr{F}_3,\quad \mathscr{F}_1=\int_x^C\varphi\,d\xi,$$
$$\mathscr{F}_2=\left[\int_C^{2\pi-C}+\ldots\int_{2\pi n+C}^{2\pi(n+1)-C}+\ldots\right]\varphi\,d\xi, \qquad (A.1)$$
$$\mathscr{F}_3=\left[\int_{2\pi-C}^{2\pi+C}+\ldots\int_{2\pi n-C}^{2\pi n+C}+\ldots\right]\varphi\,d\xi,$$
$$\varphi=\sin\xi/[\cos x-\cos\xi+y(\xi-x)]^{1/2}.$$

We note that because of the smallness of x we can expand cos x up to the quadratic term, and by virtue of the smallness of C we can carry out expansion in $\mathscr{F}_1$ and $\mathscr{F}_3$: $\sin\xi = \sin\eta \sim\eta$; $\cos\xi = \cos\eta \sim 1-\eta^2/2$. As a result, we have for $\mathscr{F}_1$

$$\mathscr{F}_1\approx 2\left[(1-\cos C)^{1/2}-\frac{x^2-yC}{(1-\cos C)^{1/2}}-y\sqrt{2}\ln\left(\frac{C+y+(C^2-x^2)^{1/2}}{x+y}\right)\right]. \qquad (A.2)$$

We now estimate $\mathscr{F}_2$:

$$\mathscr{F}_2=\sum_{n=0}^{\infty}\int_C^{2\pi-C}\frac{\sin\xi\,d\xi}{[\cos x-\cos\xi+y(\xi-x)+2\pi n]^{1/2}}. \qquad (A.3)$$

We now expand the integrand in powers of $y(\xi - x)$, keeping only the term giving a nonvanishing contribution;

$$\mathscr{F}_2=\sum_{n=0}^{\infty}\int_C^{2\pi-C}\frac{\sin\xi(-y(\xi-\pi))}{2(\cos x-\cos\xi+\pi y+2\pi ny)^{3/2}}. \qquad (A.4)$$

The integrand can now be summed over n. Carrying out the summations, and making use of the relative smallness of the step of the summation (by virtue of the fact that $y \ll C$), we replace it by integration with the help of the trapezoid formula. It can be shown that such a transformation corresponds to the neglect of terms $\sim b^2$. Then, carrying out integration over ξ, we finally obtain

$$\mathscr{F}_2\approx-\frac{1}{2\pi}2\xi(\cos x-\cos\xi)^{1/2}\Big|_C^{2\pi-C}+\frac{1}{\pi}\int_C^{2\pi-C}(\cos x-\cos\xi)^{1/2}d\xi, \qquad (A.5)$$

whence

$$\mathscr{F}_1+\mathscr{F}_2\approx\frac{4\sqrt{2}}{\pi}+\frac{C}{\pi(1-\cos C)^{1/2}}\left(\pi y-\frac{x^2}{2}\right)-y\sqrt{2}\ln\left(\frac{C+y+(C^2-x^2)^{1/2}}{x+y}\right).$$

For the estimate of $\mathscr{F}_3$, we perform the transformation

$$\mathcal{F}_3=\sum_{n=1}^{\infty}\int_{2\pi n-C}^{2\pi n+C}\frac{\sin\xi}{[1-\cos\xi-x^2/2+y(\xi-x)]^{1/2}}d\xi$$

$$\approx\sum_{n=1}^{\infty}\int_{-C}^{C}\frac{\xi\,d\xi}{[\xi^2/2-x^2/2+y(\xi-x)+2\pi ny]^{1/2}}. \qquad (A.6)$$

Summation over n is done in two parts $(1, [C^2/4\pi y])$, $[C^2/4\pi y]+1, \infty)$. For n within the limits of the second interval, by virtue of the condition $x^2 \ll C$, we can carry out the expansion of the integrand in powers of (ξy) up to the first nonvanishing order. We must then carry out the summation over n (replacing it by integration as above), and then integration over ξ.

In the first interval, it is convenient first to integrate over ξ, obtain the following expressions:

$$2\left(\frac{\xi^2}{2}-\frac{x^2}{2}+2\pi ny+y\xi\right)^{1/2}\Bigg|_{-C}^{C}-2\sqrt{2}\,y\ln\left|\frac{C+(C^2+4\pi ny-x^2)^{1/2}}{[2(2\pi y-{}^1/_2x^2+2\pi(n-1)y)]^{1/2}}\right| \qquad (A.7)$$

Carrying out the summation over n, we can, as before, replace it by integration. We note that in this case, generally speaking, the requirement of the relative smallness of the summation step for the denominator of the expression lying under the logarithm sign is not satisfied. However, we can show (by means of direct estimates) that the difference between the integration and summation is determined by a quantity $\sim b/24$, which is numerically small.

Finally, summing all the results, we obtain

$$\mathcal{F}_1+\mathcal{F}_2+\mathcal{F}_3\approx\left[\frac{4\sqrt{2}}{\pi}+O(C^2)\right]+\left(\pi y-\frac{x^2}{2}\right)\frac{3.5}{\pi}+y\sqrt{2}\left\{\ln\left(\frac{x+y}{\sqrt{4\pi y}}\right)\right.$$
$$\left.+\ln\left(\frac{4\pi y-x^2}{6\pi y-x^2}\right)-\frac{1}{2}\ln(4\pi y)-1\right\}+O\left(\frac{b^2}{C^2}\right), \qquad (A.8)$$

whence, neglecting terms that are symptotically small, we obtain the expression given in the text.

1)As is seen from the qualitative discussions given above, the obtained results are valid, in order of magnitude, for arbitrary α, in each case that is not too close to $\pi/2$.

2)Most trajectories correspond to $v_y \sim v_F$. For the vicinity of the reference point of the Fermi surface (associated with small v) the estimates of the validity of such a consideration are given below.

3)The methodology of these estimates is set forth in the Appendix.

4)Inasmuch as the contribution of the trapped particles $\Gamma_t \sim (q\tau\sqrt{\Phi^0/m})^{-1}\Gamma_0$, [4] it is comparable with the resonant part only when $(qRT/\tau)^2(\Phi^0/\epsilon)^{5/2} \gtrsim 1$. In turn, the logarithmic term is important at $|\ln(4\pi b)| \gg 1$, which also corresponds to very high intensities.

5)We note that the trajectories associated with the interval of this neighborhood in the immediate surroundings of the limiting point ($\Delta p_H \lesssim p_F\alpha$) do not contain (at least in the linear case) the turning point $v_x = 0$. The boundary trajectory, on which the points $v_x = 0$ first appear, requires in general special consideration, inasmuch as $v_y = 0$ at the points $v_x = 0$ on it. However, if $T/\tau\alpha^2 \gg 2\pi$, the contribution of this trajectory can be neglected in comparison with the contribution of the trajectories of the ordinary type, which satisfy (5).

6)The description of the perturbed trajectories used by us (which does not take into account the change in v_y) in the vicinity of the limiting point is valid only for $\Delta v_x \sim (\Phi^0/m)^{1/2} \ll v_y \sim v_F\varphi_0$. For $(\Phi^0/m)^{1/2} \gtrsim v_F\varphi_0$, almost all the electrons corresponding to these trajectories turn out to be trapped.

1A. B. Pippard, Proc. Roy. Soc. (London) **A257**, 165 (1960); V. L. Gurevich, Zh. Eksp. Teor. Fiz. **37**, 71, 1680 (1959) [Sov. Phys.-JETP **10**, 51, 1190 (1960)].

2E. A. Kaner, V. G. Peschanskiĭ and I. A. Privorotskiĭ, Zh. Eksp. Teor. Fiz. **40**, 214 (1961) [Sov. Phys.-JETP **13**, 147 (1961)].

3V. G. Peschanskiĭ and I. A. Privorotskiĭ, Fiz. Metal. Metall. **12**, 327 (1961).

4Yu. M. Gal'perin, V. D. Kagan and V. I. Kozub, Zh. Eksp. Teor. Fiz. **62**, 1521 (1972) [Sov. Phys.-JETP **35**, 798 (1972)].

5Yu. M. Gal'perin and V. I. Kozub, Zh. Eksp. Teor. Fiz. **63**, 1083 (1972) [Sov. Phys.-JETP **36**, 570 (1973)].

6I. M. Lifzhitz, M. Ya. Azbel' and M. I. Kaganov, Elektronnaya teoriya metallov (Electron Theory of Metals) Nauka, 1971, p. 227.

7Yu. M. Gal'perin, V. L. Gurevich and V. I. Kozub, Zh. Eksp. Teor. Fiz. **65**, 1045 (1973) [Sov. Phys.-JETP **38**, 517 (1974)].

8A. A. Abrikosov, Vvedenie v teoriyu normal'nykh metallov (Introduction to the Theory of Normal Metals) Nauka, 1972, p. 231.

Translated by R. T. Beyer
110