Anomalous absorption of an electromagnetic wave with a frequency close to twice the plasma frequency

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(Submitted October 28, 1974)
Zh. Eksp. Teor. Fiz. **68**, 1005–1013 (March 1975)

We consider the nonlinear stage of the instability of a monochromatic electromagnetic wave with respect to its decay into two Langmuir oscillations. Its coherence leads to a phase mechanism to limit the amplitude of the excited waves which determines both the induced scattering by ions and the Langmuir turbulence spectra. The distribution of the oscillations in k space is steeply anisotropic and consists of a succession of sharp peaks which extend into the small wavevector region from the maxima of the instability growth rate.

INTRODUCTION

It is well known that the absorption of energy from a powerful electromagnetic wave in a plasma is basically caused by collective effects.[1] It is thus necessary for an understanding of the processes which take place to construct a theory which consistently describes the interaction between the excited plasma oscillations. Recently it has been found possible to understand rather well the problem of the heating of an isothermal plasma by an electromagnetic wave with frequency $\omega_0 \approx \omega_p$.[2-6] The basic features of the parametric excitation of high-frequency potential oscillations in a plasma with a magnetic field have been elucidated.[7] In all these cases the plasma oscillations were excited as a result of the conversion of an electromagnetic wave through scattering by ions:

$$\omega_0=\omega_k+kv_{Ti},$$

and their distribution was described by weak turbulence theory.

In the present paper we consider the non-linear stage of the instability of an electromagnetic wave with respect to its decay into two Langmuir oscillations. This process may, in particular, play a basic role in the absorption of energy by the plasma corona of a deuterium droplet.

In contrast to earlier papers[2-7] we shall show that the induced scattering by ions is not the only important non-linear effect. The coherence of the initial wave leads to the occurrence of a specific phase mechanism to limit the instability. The quantitative results obtained below are not very different from the results from the paper by Pustovalov, Silin, and Tikhonchuk,[8] who did not take phase effects into account, but the distribution of the oscillations in k-space has an essentially different form. The Langmuir turbulence spectra turn out to be steeply anisotropic and have the form of jets extending from the region of the maxima of the instability growth rate into the small wavevector region.

1. BASIC EQUATIONS

We consider a homogeneous isothermal plasma in which a linearly polarized electromagnetic wave $\mathbf{E} = \mathbf{E}_0\cos\{2(\omega_0 t - \boldsymbol{\kappa}\cdot\mathbf{r})\}$ propagates. Its interaction with the Langmuir oscillations which occur is described by the Hamiltonian

$$H_p=\int \rho(\mathbf{v}_0\mathbf{v})\,d^3r. \qquad (1)$$

Here $\mathbf{v}$ is the velocity and ρ the change in the electron density in the Langmuir oscillations while $\mathbf{v}_0$ is the velocity of the motion of the electrons in the field of the electromagnetic wave:

$$\mathbf{v}_0(t)=\mathbf{v}_0\cos[2(\omega_0 t-\boldsymbol{\kappa}\mathbf{r})],\quad \mathbf{v}_0=e\mathbf{E}_0/2m\omega_0,\quad 4\omega_0^2=\omega_p^2+4\kappa^2c^2.$$

Changing to the canonical variables for Langmuir waves[9]

$$\rho_{\mathbf{k}}=k(\rho_0/2\omega_p)^{1/2}(a_{\mathbf{k}}+a_{-\mathbf{k}}^*),\quad \mathbf{v}_{\mathbf{k}}=\frac{\mathbf{k}}{|\mathbf{k}|}\left(\frac{\omega_p}{2\rho_0}\right)(a_{\mathbf{k}}-a_{-\mathbf{k}}^*),$$

we can rewrite H_p in the form

$$H_p=\frac{1}{2}\int [V_{\mathbf{k}}(\boldsymbol{\kappa})a_{\boldsymbol{\kappa}-\mathbf{k}}a_{\boldsymbol{\kappa}+\mathbf{k}}e^{2i\omega_0 t}+\text{c.c.} \qquad (2)$$

$$V_{\mathbf{k}}(\boldsymbol{\kappa})=(\mathbf{k}\mathbf{v}_0)(\boldsymbol{\kappa}\mathbf{k})/|\boldsymbol{\kappa}-\mathbf{k}|\,|\boldsymbol{\kappa}+\mathbf{k}|.$$

The equations of motion are, as usual, obtained by varying the Hamiltonian:

$$\frac{\partial a_{\boldsymbol{\kappa}+\mathbf{k}}}{\partial t}+i\omega_{\boldsymbol{\kappa}+\mathbf{k}}=-i\frac{\delta H_p}{\delta a^*_{\mathbf{k}+\boldsymbol{\kappa}}}=-iV_{\mathbf{k}}(\boldsymbol{\kappa})a^*_{\boldsymbol{\kappa}-\mathbf{k}}e^{2i\omega_0 t}. \qquad (3)$$

One checks easily that the zeroth order solution $a_{\mathbf{k}} = 0$ is unstable with respect to a growth of the oscillations $a_{\boldsymbol{\kappa}+\mathbf{k}}$, $a_{\boldsymbol{\kappa}-\mathbf{k}}$ with maximum growth rate

$$\gamma=V_{\mathbf{k}}(\boldsymbol{\kappa})\sim\omega_p v_0/c.$$

The oscillations which are excited as a result of the instability are arranged near the surface

$$\omega(\boldsymbol{\kappa}-\mathbf{k})+\omega(\boldsymbol{\kappa}+\mathbf{k})=2\omega_0,\quad \omega_k=\omega_p(1+{}^3/_2k^2r_d^2), \qquad (4)$$

which consists of two spheres with radius

$$k=\frac{1}{\sqrt{3}\,r_d}\left[2\frac{(\omega_0-\omega_p)}{\omega_p}-\kappa^2r_d^2\right]^{1/2},$$

and with centers which are at a distance 2κ apart. Owing to the sharp angular dependence the magnitude of the growth rate changes appreciably along the surface (4). We introduce a spherical system of coordinates. We reckon the angle θ from $\boldsymbol{\kappa}$ and the angle φ from $\mathbf{E}_0$. We shall assume in what follows that $\kappa/k \ll 1$. The growth rate then reaches its maximum[10]

$$\gamma_{max}=\frac{\kappa v_0}{2}=\frac{\sqrt{3}}{8}\omega_p\left(\frac{E_0^2}{4\pi n_0mc^2}\right)^{1/2}=\frac{\sqrt{3}}{8}\omega_p\left(\frac{W_{ext}}{n_0mc^2}\right)^{1/2} \qquad (5)$$

in the four points $\varphi = 0, \pi$, $\theta = \pi/4, 3\pi/4$. The sum of the phases of the excited waves is then equal to $\pi/2$.

In order to evaluate the growth rate of the instability of a wave with circular polarization we consider an electric field with two perpendicular components which are shifted in phase by $\pi/2$. Changing the origin of the phases of the waves $a_{\kappa\pm k} \to \exp(i\varphi_k/2)a_{\kappa\pm k}$ (φ_k is the azimuthal angle between **E** and **k**) we are led to Eqs. (2), (3) with the matrix element

$$\tilde{V}_k(\varkappa) = {}^1/_2\varkappa v_0 \sin 2\theta. \quad (6)$$

The maximum growth rate of the instability which has the magnitude (5) is reached on the parallels $\theta = \pi/4$, $3\pi/4$. The amplitude of the excited oscillations is limited by non-linear effects, the main one of which in an isothermal plasma without a magnetic field is the scattering of Langmuir oscillations by ions.

Adding the terms, which describe the interaction between the oscillations,[11] to (3) we get a non-linear dynamic equation

$$\partial a_{\varkappa+k}/\partial t + i\omega_{\varkappa+k}a_{\varkappa+k} + \gamma a_{\varkappa+k} = -iV_k(\varkappa)\dot{a}_{\varkappa-k}e^{2i\omega_0 t}$$
$$-i\int T_{\varkappa+k,k_1,k_2,k_3}a_{k_1}^* a_{k_2}a_{k_3}\delta(\varkappa+k+k_1-k_2-k_3)\,dk_1\,dk_2\,dk_3,$$
$$T_{kk_1k_2k_3} = \frac{\omega_p^2}{2n_0T}\frac{1}{kk_1k_2k_3}\left[(kk_2)(k_1k_3)G\left(\frac{\omega_{k_1}-\omega_{k_3}}{|k_1-k_3|}\right)\right.$$
$$\left.+(kk_3)(k_1k_2)G\left(\frac{\omega_{k_1}-\omega_{k_2}}{|k_1-k_2|}\right)\right], \quad (7)$$
$$G\left(\frac{\omega}{|k|}\right) = \frac{L_{k\omega}}{1-L_{k\omega}},$$
$$L_{k\omega} = \frac{T_e}{Mn_0}\int k\frac{\partial f_{0i}}{\partial v}\frac{dv}{kv-\omega}.$$

Before turning to a discussion of the non-linear effects we draw attention to the expression for the energy flux in the plasma

$$Q = \partial H_p/\partial t = 4\omega_p \operatorname{Im}\int V_k\langle a_{\varkappa+k}a_{\varkappa-k}e^{2i\omega_0 t}\rangle d^3k. \quad (8)$$

(the angle brackets indicate here averaging over the random individual phases of the waves). It is clear that the absorption of the energy of a monochromatic wave by the plasma leads to the appearance of not only the usual, but also the anomalous correlation functions $\sigma_k = \langle a_{\kappa-k}a_{\kappa+k}e^{2i\omega_0 t}\rangle$. Using the fact that κ is small we put it everywhere equal to zero, except in the matrix element $V_k(\kappa)$ of the interaction with the external field. The problem then turns out to be completely analogous to the problem of the excitation by a uniform high-frequency field of two oscillations (see the review[12])[1)] belonging to the same branch of the spectrum.

We neglect to begin with the transfer of energy to the small wavevector region due to scattering by ions. Averaging over the phases we get by analogy with[12,13] from (7) the following set of equations:

$${}^1/_2 dn_k/dt + \gamma n_k = \operatorname{Im} P_k\sigma_k^*,$$
$$d\sigma_k/dt + i\tilde{\omega}\sigma_k + 2\gamma\sigma_k = -2iP_kn_k, \quad (9)$$

where

$$P_k = V_k(\varkappa) + \int S_{kk'}\sigma_{k'}\,dk', \quad \langle a_ka_{k'}^*\rangle = n_k\delta(k-k'),$$
$$\tilde{\omega} = 2\omega_k - 2\omega_0 + 4\int T_{kk'}n_{k'}\,dk', \quad S_{kk'} = T_{k,-k,k',-k'},$$
$$T_{kk'} = T_{kk'kk'}.$$

2. STATIONARY SOLUTIONS

Zakharov, L'vov, and Starobinets[14] have shown that stable stationary solutions of (9) are found on the surface $\tilde{\omega} = 0$, i.e., that oscillations are excited with eigenfrequencies

$$\omega_k + 2\int T_{kk'}n_{k'}\,dk'$$

which satisfy exactly the decay conditions (4). The correlation of the oscillations with opposite wavevectors is complete ($|\sigma_k| = n_k$) even in the transitional regime.[12] As all excited waves have the same frequency we can put G = 1 and we obtain

$$T_{kk'} = \omega_p^2(kk')^2/2n_0Tk^2k'^2, \quad S_{kk'} = 2T_{kk'}.$$

Taking into account the structure of the growth rate we symmetrize the problem with respect to the replacement of k by k′ and φ by $\varphi + \pi$. We get then again Eqs. (9) in which

$$S_{kk_1} = (S_{kk_1} - S_{kk_1'})/2, \quad \sigma_k = 4\sigma_k, \quad n_k = 4n_k,$$
$$k' = k(\varphi+\pi), \quad -\pi/2 < \varphi < \pi/2.$$

We have used here the fact that $V_{-k'} = -V_k$, $\sigma_{-k'} = -\sigma_k$.

If we are just above threshold four monochromatic waves are excited in the points corresponding to the maximum growth rate:[2)]

$$V_{k_0}\sin\Phi = \gamma, \quad S_{k_0k_0}n_k = V_{k_0}\cos\Phi, \quad \sigma_k = n_ke^{-i\Phi}.$$

The threshold of the next group of waves is, as usual,[14] determined by the condition

$$\frac{E^2 - E_{th}^2}{E_{th}^2} = \frac{S^2(V_{k_0}^2 - V_k^2)}{(V_kS - V_{k_0}S_{kk_0})^2}, \quad S = S_{k_0k_0}. \quad (10)$$

Evaluating S_{kk_0} we get

$$S_{k_0k} = \frac{2\omega_p^2}{n_0T}\cos\theta\cos\theta_0\sin\theta\sin\theta_0\cos\varphi = \frac{\omega_p^2}{2n_0T}\sin 2\theta\cos\varphi.$$

We see that $S_{k_0k} \propto V_k$ and that it follows from (10) that the threshold for the production of the next group of waves is infinitely large. When we take into account that the magnitude of κ is finite, the functions V_k and S_{k_0k} are no longer the same and $E/E_{th} \sim c/v_T$. In the case of the decay of a wave with circular polarization the Langmuir oscillations are also described by the set (9) where we have instead of $S_{kk'}$ its average over angles.

We have also exactly $S_{k_0k} \propto V_k$ and for any excess above threshold waves are excited along the two parallels $\theta = \pi/4$, $3\pi/4$.[3)]

We neglected in the preceding section processes which transfer oscillations to other regions of k-space. The most important of these is scattering by ions which decreases the wavevector of the oscillations. We write a_k in Eq. (7) as

$$a_k = A_{k_0} + \tilde{a}_k,$$

where A_k describes waves on the resonance surface (4) while $\tilde{a}_k$ describes waves which are scattered into the region of small k for which no anomalous correlations occur. The presence of waves outside the resonance surface leads to the appearance of a term describing nonlinear scattering in (9):

$$\gamma_{nl} = -\operatorname{Im}\int T_{kk'}N_{k'}\,dk', \quad \langle\tilde{a}_k\tilde{a}_{k'}^*\rangle = N_k\delta(k-k').$$

We showed above that four waves (or two bands) are excited on the resonance surface, if we neglect scattering by ions. In the other points of this surface the energy flux in the medium is less than the damping. It is natural that the presence of additional damping only aggravates this inequality. The oscillations are thus distributed pronouncedly anisotropically on the resonance surface. Breĭzman, Zakharov, and Musher[3] have shown that turbulent spectra excited as a result of induced scattering of parametrically excited waves are also steeply anisotropic and have the form of jets extending into the region of small wavevectors. We must therefore study in what follows the one-dimensional problem of the energy distribution along a jet.

In the case of a wave with linear polarization the distribution of the oscillations in the jet is, when we take the symmetry of the problem into account, described by the equation

$$\partial N_k/\partial t+\gamma N_k=\int_0^\infty T_{kk'}N_{k'}\,dk'N_k+T_{kk_0}n_0N_k. \quad (11)$$

The fact that the problem is one-dimensional appreciably simplifies the form of the matrix element (7):

$$T_{kk'}=\frac{\omega_p^2}{2n_0T}\operatorname{Im}G\left(\frac{3(k-k')}{2k_{diff}}\right),\quad k_{diff}=\frac{1}{r_d}\left(\frac{m}{M}\right)^{1/2}.$$

For n_0 we have from (9)

$$\frac{1}{2}\frac{dn_k}{dt}+\gamma n_k-\int T_{k_0k}N_kdkn_k=V_{k_0}n_k\sin\Phi,$$
$$\frac{1}{2}\frac{d\Phi}{dt}+\tilde{\omega}=n_k(V\cos\Phi+Sn_k). \quad (12)$$

The form of the stationary solutions of (11) was studied in[5] for different forms of the growth rate. It was shown there that if the size of the growth rate is less than the diffusion interval, the solution has the form of a sequence of sharp peaks positioned at distances from one another. The stationary distribution of the oscillations has therefore in our case the form

$$N_k=\sum_m N_m\delta(k_0-mk_{diff}).$$

The energy balance condition in the m-th peak gives

$$\gamma=\tilde{T}(N_{m-1}-N_{m+1}),\quad \tilde{T}=\frac{\omega_p^2}{2n_0T}\max\operatorname{Im}G, \quad (13)$$

and for the first peak excited as the result of the parametric instability we have[4)]

$$S^2N_1^2=V_{k_0}^2-(\gamma+\tilde{T}N_2)^2,\quad N_1=n_k. \quad (14)$$

We proceed to determine the amplitude of the peaks. First of all we note that the amplitude of the penultimate peak is always equal to $N_c=\gamma/\tilde{T}$. If the number of peaks is even, 2m, the amplitude of the first peak is equal to mN_c.

We have from (14) for the amplitude N_2

$$N_2/N_c=[V^2/\gamma^2-(S/\tilde{T})^2m^2]^{1/2}-1.$$

As we get further above threshold, the amplitude of the odd peaks becomes constant, and the amplitude of the even peaks increases until the magnitudes of the last and the last-but-one peaks become comparable. This occurs when the excess above threshold equals

$$V_{2m+1}/\gamma=[(S/\tilde{T})^2m^2+(m+1)^2]^{1/2}. \quad (15)$$

When we get further above threshold there appears the (2m + 1)-st peak, the amplitude of the odd peaks starts to increase, and that of the even peaks to be frozen in. As $N_2=mN_c$ we get for the amplitude of the first peak

$$(S/\tilde{T})^2(N_1/N_c)^2=V^2/\gamma^2-(m+1)^2. \quad (16)$$

This expression remains valid up to the threshold V_{2m+2} for the creation of the next peak:

$$V_{2m+2}/\gamma=[(S/\tilde{T})^2(m+1)^2+(m+1)^2]^{1/2}=\left[(V_{2m+1}/\gamma)^2+(2m+1)\frac{S}{\tilde{T}}\right]^{1/2}.$$

We note that as $S/\tilde{T}=2/(\operatorname{Im}G)_{max}<1$, the region of excesses above threshold for which there is an even number of peaks is larger than the creation for which there is an odd number of peaks.

We now evaluate the energy flux in the plasma. Equation (8) gives

$$Q=4\omega_pN_1V_{k_0}\sin\Phi=4\omega_pN_1(\gamma+\tilde{T}N_2).$$

For an even number of peaks

$$Q_{2m}=4m\omega_p\tilde{T}N_c^2[V^2/\gamma^2-(S/\tilde{T})^2m^2]^{1/2} \quad (17)$$

and for an odd number

$$Q_{2m+1}=4(m+1)\omega_p\tilde{T}N_c^2\frac{\tilde{T}}{S}\left[\frac{V^2}{\gamma^2}-(m+1)^2\right]^{1/2}. \quad (18)$$

When the excess above threshold is large $m\gg 1$, $m\sim V/\gamma(1+s/\tilde{T})^{1/2}$ and we have for the energy flux in the plasma

$$Q\approx 4\omega_pm^2\tilde{T}N_c^2=4\omega_p\frac{V^2}{\gamma^2}\frac{\gamma^2}{\tilde{T}}\approx\frac{8V^2}{\omega_p\max\operatorname{Im}G}n_0T. \quad (19)$$

This result agrees with the estimate obtained by Galeev and Sagdeev.[1] However, the peculiar behavior of the energy flux with a break in the derivative at the moment that a new peak is formed can not be obtained from simple estimates.

So far we have considered stationary solutions of (12) and (11). It was, however, shown in[5] that when we are well above threshold when the oscillations reach the region of Langmuir collapse, the trans-threshold behavior of the system is essentially non-stationary. The released energy is transferred to the region of long wavelengths in the form of periodic pulses—solitons, propagating along the jet. The released energy is then, as before, given by Eq. (19) and the picture of what happens is the same as that described in[5]. We note also that the change to the stationary solution occurs after a long time and only, if thermal noise is present.[4-6]

All results obtained so far referred to the case of a linearly polarized wave. As we showed above, when we neglect transfer along the spectrum, for a circularly polarized wave the oscillations are excited along the parallels $\theta=\pi/4,\ 3\pi/4$. The induced scattering by ions can be taken into account in exactly the same way and the distribution of the oscillations on the cone $\theta=\pi/4,\ 3\pi/4$ has a similar form, while circular bands with different k play the role of the peaks. All results obtained earlier can be applied literally, except that the amplitude of the bands in Eqs. (13) to (19) is written in a spherical normalization and $\tilde{T}$ must be replaced by its angular average.

CONCLUSION

We explain now how an inhomogeneity of the plasma will affect the results obtained by us. It is well known (see, e.g., [1]) that in that case the decay conditions are satisfied in a layer of width

$$\delta\sim\gamma_0/v_{gr}\frac{dk}{dx},$$

so that after the time needed to pass through it the oscillations grow to an amplitude

$$W\infty W_0e^{2\pi\Delta},\quad \Delta=\gamma_0^2/v_{gr}^2\frac{dk}{dx}.$$

If the amplification coefficient $\Delta\gtrsim 1$ the noise density in the plasma is limited by the non-linear mechanisms considered above and the corresponding value of the electrical field can be considered to be the threshold value for the inhomogeneous plasma:

$$\Delta=\gamma_0^2/v_{gr}\frac{d\omega_p}{dx}=\frac{v_T^2}{c^2}\frac{E_{th}^2}{8\pi nT}\omega_p\Big/\frac{d\omega_p}{dx}kr_d^2\sim 1,$$
$$\frac{E_{th}^2}{8\pi nT}\sim\frac{c^2}{v_T^2}(kr_d)\frac{r_d}{L}. \quad (20)$$

One verifies easily that E_{th} is rather small both in laser and in ionospheric experiments.

We now consider the effect of the inhomogeneity on the excited Langmuir turbulence. Oscillations propagating, say, in the direction of increasing density, decrease their wavevector and are removed to other regions of space. However, after a time $\tau \sim (\omega_p W/nT)^{-1}$ the oscillation is scattered backwards and, propagating into the region of lower density, decreases its wavelength. In order that the inhomogeneity does not blur the picture of sharp discrete peaks, it is therefore sufficient that the wavevector changes little over a time τ compared to the diffusion spacing k_{diff}:

$$\frac{W}{nT} \gg \frac{r_d}{L}\left(\frac{m}{M}\right)^{1/2}, \quad L=\left(\frac{d\ln n}{dx}\right)^{-1}.$$

For the first peaks, where

$$\frac{W}{nT} \sim \frac{v_T}{c}\left(\frac{E^2}{nT}\right)^{1/2},$$

this leads to the condition, much weaker than (20):

$$\frac{E^2}{nT} \gg \frac{c^2}{v_T^2}\frac{r_d}{L}\left(\frac{m}{M}\right)^{1/2}.$$

Another effect which was neglected above is the reverse effect of the Langmuir turbulence on the electromagnetic wave. We consider this, for the sake of simplicity, in a uniform plasma. The amplitude of the electromagnetic wave when it penetrates into the medium is diminished and can become less than the threshold amplitude. Moreover, an inhomogeneous profile is, in fact, equivalent to a spectral broadening of the monochromatic wave. If $v_{gr}/L > \gamma_0$ the dynamic regime of the decay instability is shifted stochastically with a growth rate smaller by a factor $\gamma L/v_{gr}$. The relaxation length L can as to order of magnitude be found from the equation of continuity for the energy flux

$$L \sim \frac{cE_0^2}{Q} \sim \frac{cE^2\omega_p}{\gamma_0^2 nT} \sim \frac{c}{\omega_p}\frac{c^2}{v_T^2},$$

and the criterion for the applicability of our results, $\gamma > v_{gr}/L$ is very weak:

$$E^2/nT > (kr_d)^2(v_T/c)^4$$

and is practically always satisfied, even in a thermonuclear plasma. We note also that the relaxation length is rather small ($L \sim 10^{-3}$ cm for the plasma corona of a deuterium droplet) which indicates the essential role of the process considered by us.

The estimates given above show that the Langmuir turbulence developing in a plasma through the action of an electromagnetic wave is practically always uniform and depends on the coordinates as variables through the spatial change of the electromagnetic field. We note that in this form the problem of the dissipation of the energy of an electromagnetic wave near a turning point was solved by Vas'kov and Gurevich.[16]

So far we have considered the case of not too strong electromagnetic fields, $W/nT < kr_d(m/M)^{1/2}$. When we are well above threshold

$$kr_d(m/M)^{1/2} < W/nT < (kr_d)^2$$

the oscillations are transferred to the collapse region, $W/nT \sim (kr_d)^2$ due to the modified decay instability[17]

$$\gamma_{mod} \sim (\omega_p\omega_s^2 W/nT)^{1/3}, \qquad (21)$$

and for yet larger excess above threshold

$$W/nT > (kr_d)^2$$

the oscillations start to collapse into the growth rate region. We note that the nature of such a strong turbulence does no longer depend on the ratio of the electron to the ion temperatures.

The author is grateful to V. S. L'vov for useful discussions.

1) When $\kappa \neq 0$ there is a complete analogy with the parametric excitation of two different kinds of oscillation. [13]

2) A more rigorous consideration shows that we have narrow packets $\Delta\omega \sim Sn$ because of the instability. [13,15]

3) These results are connected in an essential way with the symmetry of the distribution of the oscillations. If this symmetry is violated, e.g., due to the inhomogeneity of the medium, the threshold for the production of the next group of waves is finite, though very large: $E/E_{th} \sim k/(d \ln \omega_0/dx)$.

4) We note that as N_1 is excited on the resonance surface, we do not meet with any complications with the determination of the coordinates of the first peak and the energy flux in it (cf. [4]).

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Translated by D. ter Haar
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