Gravitational radiation emitted by an electron in the field of a circularly polarized electromagnetic wave

D. V. Gal'tsov and Yu. V. Grats

Moscow State University

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The gravitational radiation emitted by a charged particle in the field of a plane circularly polarized electromagnetic wave is considered in the linear approximation of general relativity. It is shown that, in distinction from the case of the gravitational radiation emitted by a relativistic particle in a slowly varying field, the intensity of the radiation increases proportional to the sixth power of the ratio E/m, rather than the fourth power. A formula is also obtained for the annihilation of a pair into one graviton in the presence of an electromagnetic wave. The limit of a static field is discussed.

INTRODUCTION

The photoproduction of a graviton on a charged particle is one of the most essential microscopic processes that lead to the generation of high-frequency gravitational radiation in stellar interiors. For instance, for the sun (nonrelativistic electron energies) the effectiveness of this process is by several orders of magnitude larger than the Coulomb bremsstrahlung mechanism, and the corresponding magnitude of the flux of gravitational radiation near the earth turns out to be of the same order of magnitude as the flux of low-frequency gravitational radiation from the nearest double stars [1]. Other astrophysical applications require consideration of the region of relativistic temperatures. In this case, in addition to the one-photon photoproduction, the emission of a graviton accompanied by the absorption of one or more quanta becomes possible. All these processes are described uniformly as the gravitational radiation emitted by a charge moving in an electromagnetic wave.

In the present paper the problem of gravitational radiation emitted by an electron in the field of a plane circularly polarized electromagnetic wave is discussed both within the framework of classical and quantum theory; in the latter case the exact solutions of the Dirac equation in the field of the wave are used. The units used are such that $\hbar = c = 1$, $e^2 = 1/137$. The metric of free space is $\eta_{00} = 1$, $\eta_{ii} = -1$ ($i = 1, 2, 3$).

1. CLASSICAL THEORY

In distinction from electromagnetic radiation, the gravitational radiation emitted by a charged particle is a nonlocal effect, since the contribution to the radiation come from the stresses of the total electromagnetic field of the particle, stresses which are in general distributed throughout the whole space. For this reason the gravitational radiation emitted by a charge moving on a circular orbit in a homogeneous magnetic field [2,3] and in the field of a plane, circularly polarized, electromagnetic wave turn out to be different, although the trajectories of the motion are identical.

Using the equations for a weak gravitational field [4] one can obtain in the linear approximation a formula that expresses the power of the gravitational radiation, P_G, emitted by a charged particle, in terms of an integral of the product of the energy-momentum tensor $T_{\mu\nu}$ and the derivatives of the potentials $\psi_{\mu\nu}$ of the gravitational field (an analog of the Umov-Poynting theorem in electrodynamics) [3]:

$$P_G + \frac{dW}{dt} = -\frac{1}{2}\int_V d^3x\left(T_{\mu\nu}\frac{\partial\psi^{\mu\nu}}{\partial x^0} - \frac{1}{2}T_\mu{}^\mu\frac{\partial\psi_\nu{}^\nu}{\partial x^0} - 2T_\nu{}^\lambda\frac{\partial\psi^{0\nu}}{\partial x^\lambda}\right). \quad (1)$$

Here dW/dt is the rate of change of the energy of the gravitational field in a volume V, $\psi_{\mu\nu}$ is the solution of the equation for the weak gravitational field

$$\Box\psi_{\mu\nu} = 16\pi G T_{\mu\nu} \quad (2)$$

with the subsidiary condition $\partial\psi^{\mu\nu}/\partial x^\nu = 0$, where in the right-hand side of (1) one must substitute the semi-difference between the retarded and the advanced solutions of Eq. (2), in the same manner as in electrodynamics [5].

In the case of periodic motion of the particle, the second term in the left-hand side of (1) vanishes, and we obtain for the average power of the gravitational radiation

$$dP_G = \frac{G}{\pi}\sum_{l=1}^{\infty}(l\omega)^2 T_{\mu\nu}{}^l(\omega\mathbf{n})T^{*l}_{\lambda\rho}(\omega\mathbf{n})\Lambda^{\mu\nu\lambda\rho}\,d\Omega,$$

$$T_{\mu\nu}{}^l(\omega\mathbf{n}) = \int_0^{2\pi}\frac{d(\omega t)}{2\pi}\int d\mathbf{r}\,T_{\mu\nu}(\mathbf{r},t)e^{il\omega(t-\mathbf{n}\mathbf{r})}, \quad (3)$$

$$\Lambda^{\mu\nu\lambda\rho} = {}^1/_2(\eta^{\mu\lambda}\eta^{\nu\rho} + \eta^{\mu\rho}\eta^{\nu\lambda} - \eta^{\mu\nu}\eta^{\lambda\rho}),$$

where ω is a characteristic frequency, and $\mathbf{n}$ is a unit vector in the direction of the gravitational wave. For the case of motion of a charge in a given external field $F^{\text{ext}}_{\mu\nu}$ satisfying the free Maxwell equations, the conserved energy-momentum tensor $T_{\mu\nu}$ consists of a mass term and the energy-momentum tensor of the electromagnetic field, in which one has to retain only the mixed contribution from the proper retarded field $F^{\text{ret}}_{\mu\nu}$ and the external field $F^{\text{ext}}_{\mu\nu}$:

$$T_{\mu\nu} = m\int ds\frac{dx_\mu}{ds}\frac{dx_\nu}{ds}\delta(x - x(s)) + \frac{1}{4\pi}\Big(F^{ext}_{\mu\lambda}F_{ret\,\nu}^{\lambda} + F^{ret}_{\mu\lambda}F_{ext\,\nu}^{\lambda} + \frac{1}{2}\eta_{\mu\nu}F^{ext}_{\lambda\rho}F^{\lambda\rho}_{ret}\Big). \quad (4)$$

We note that taking into account the conservation law $\partial T^{\mu\nu}/\partial x^\nu = 0$ (since all calculations are done in the linear approximation for the gravitational field the gravitational interaction is not taken into account here) one can write the tensor $\Lambda_{\mu\nu\lambda\rho}$ in a three-dimensionally transverse form

$$\Lambda_{ijkl} = {}^1/_2(\Delta_{ik}\Delta_{jl} + \Delta_{il}\Delta_{jk} - \Delta_{ij}\Delta_{kl}),$$

$$\Delta_{ij} = \delta_{ij} - n_i n_j \quad (i, j, k, l = 1, 2, 3) \quad (5)$$

(all other components vanish), and also represent this tensor as an expansion with respect to two independent polarization states:

$$\Lambda_{\mu\nu\lambda\rho}=\sum_{s=1,2} e_{\mu\nu}^{(s)} e_{\lambda\rho}^{(s)*}, \quad e^{(s)\mu\nu}e_{\mu\nu}^{(s')*}=\delta_{ss'}. \tag{6}$$

In this section we shall use the three-dimensionally-transverse representation of the polarization tensors

$$e_{ij}^{(1)}=2^{-1/2}(e_i^\theta e_j^\theta-e_i^\varphi e_j^\varphi), \quad e_{ij}^{(2)}=2^{-1/2}(e_i^\theta e_j^\varphi+e_i^\varphi e_j^\theta), \tag{7}$$

where $\mathbf{e}^\theta$ and $\mathbf{e}^\varphi$ are the unit vectors of spherical coordinates with the radius—vector along $\mathbf{n}$.

Let us consider a monochromatic plane electromagnetic wave with circular polarization. We represent its 4-potential in the form (cf. [4])

$$A_{ext}^\mu=a_1^\mu\cos\varphi+a_2^\mu\sin\varphi, \quad \varphi=(kx);$$
$$a_1^2=a_2^2=-m^2\xi^2/e^2, \quad (a_1a_2)=0, \quad (a_ik)=0 \quad (i=1,2). \tag{8}$$

If one neglects radiation damping, the motion of a charge in the field of a potential (8) in a special reference frame will be a circular motion. Therefore the contribution of the energy-momentum tensor of the particle coincides with the analogous quantity for the motion in a uniform magnetic field [3].

In order to calculate the contribution of the electromagnetic stresses we write the retarded potential A^μ_{ret} in the form

$$A_{ret}^\mu=\frac{1}{2\pi^2}\sum_{m=-\infty}^{\infty}\int d\mathbf{q}\, j_{(m)}^\mu(\mathbf{q})\,e^{-im\omega t+i\mathbf{q}\mathbf{r}}\left\{\frac{P}{\mathbf{q}^2-(m\omega)^2}+i\pi\frac{m}{|m|}\delta(\mathbf{q}^2-(m\omega)^2)\right\},$$
$$j_{(m)}^\mu(\mathbf{q})=\int_0^{2\pi}\frac{d(\omega t)}{2\pi}e^{im\omega t-i\mathbf{q}\mathbf{r}(t)}V^\mu(t), \quad V^\mu=(1,\mathbf{v}). \tag{9}$$

The term proportional to the delta function describes the electromagnetic radiation which has detached itself from the particle, and the corresponding term in (4) is a secondary effect of the transformation of electromagnetic radiation into gravitational radiation [2,6]. It is easy to show that in the case under consideration this effect vanishes. Indeed, since the external field is a plane wave, we are talking in fact about the conversion of two photons into a graviton. Since all three particles have zero mass, the law of conservation of energy and momentum requires that their momenta be collinear. Then the amplitude of the process, constructed out of the antisymmetric tensors $F_{\mu\nu}$ of the photons and the symmetric transverse tensor $e_{\mu\nu}$ of the graviton vanishes on account of the transversality condition.

Calculating the contribution of the principal value in (9) to the Fourier transform of the energy-momentum tensor of the field and combining it with the contribution of the mass, we obtain

$$T_{\mu\nu}^l(\omega\mathbf{n})=\sum_{s=1,2}e_{\mu\nu}^{(s)}T_{(s)}^l(\omega\mathbf{n}),$$
$$T_{(s)}^l=\frac{m\xi}{\sqrt{2}}e^{il\lambda\varphi}\int_0^{2\pi}\frac{d\alpha}{2\pi}e^{i(l\alpha-v\sin\theta\cos\alpha)}\tau_{(s)}^l, \tag{10}$$
$$\tau_{(1)}^l=\cos\theta(-v\sin^2\alpha(1+\cos\theta)+i\sin\theta\cos\alpha/l(1-\cos\theta)),$$
$$\tau_{(2)}^l=\lambda(v^{\cdot 1}/_2\sin 2\alpha(1+\cos\theta)+i\sin\theta\sin\alpha/l(1-\cos\theta)).$$

Here θ and φ are the spherical angles of the vector $\mathbf{n}$, $\lambda = \pm 1$ corresponds to right (left) circular polarization of the plane electromagnetic wave and the absolute value of the velocity v is related to the intensity parameter ξ by $v = \xi/(1+\xi^2)^{1/2}$.

For small values of the parameter ξ (nonrelativistic limit) the main contribution comes from radiation with the frequency ω ($l = 1$). In this case one may neglect in the exponent the term proportional to v, which yields

$$T_{(1)}^1=i\frac{m\xi\sin 2\theta e^{i\lambda\varphi}}{4\sqrt{2}(1-\cos\theta)}, \quad T_{(2)}^1=-\frac{\lambda m\sin\theta e^{i\lambda\varphi}}{2\sqrt{2}(1-\cos\theta)}. \tag{11}$$

These quantities correspond to the production of a graviton by a photon in a Coulomb field [1], and has a singularity for $\cos\theta = 1$. This singularity is related to the long range of the Coulomb field and is removed by taking screening into account, which results in the difference $(1-\cos\theta)$ in the denominator being replaced by

$$1-\cos\theta+{}^1/_2(\omega R)^{-2}, \tag{12}$$

where R is the screening radius.

Radiation on the harmonic $l = 2$ is described by amplitudes without singularities. The angular distribution for the two independent polarizations has the form

$$dP_{(1)}^2=\cos^2\theta\, dP_{(2)}^2=2\pi^{-1}G(m\omega)^2\xi^4\cos^2\theta\cos^4(\theta/2)\,d\Omega, \tag{13}$$

and the total power radiated is

$$P_{(1)}^2={}^2/_7P^2, \quad P_{(2)}^2={}^5/_7P^2, \quad P^2={}^{56}/_{15}G(m\omega)^2\xi^4. \tag{14}$$

We note that this quantity does not coincide with the intensity of radiation emitted by nonrelativistic gravitationally coupled particles, calculated according to the Landau-Lifshitz formula [6]. The reason for this is the different contribution of the stresses (in essence this effect is relativistic, even for $v \to 0$, since a photon participates in it).

In the general case of arbitrary values of the parameter ξ the angular distribution of the intensity of gravitational radiation has the following form:

$$dP_G=dP_{(1)}+dP_{(2)};$$
$$dP_{(1)}=Gm^2(1+\xi^2)\,\mathrm{ctg}^2\frac{\theta}{2}\sum_{l=1}^{\infty}(l\omega)^2\,\mathrm{ctg}^2\theta J_l^2(z)\sin\theta\,d\theta, \tag{15}$$
$$dP_{(2)}=Gm^2(1+\xi^2)\,\mathrm{ctg}^2\frac{\theta}{2}\sum_{l=1}^{\infty}(l\omega)^2v^2J_l'^2(z)\sin\theta\,d\theta,$$

where $J_l(z)$ is the Bessel function of $z = v\sin\theta$.

It is easy to see that $dP_{(1)}$ and $dP_{(2)}$ are proportional to the known expressions of the intensities of radiation for the so-called π and σ components of linear polarization in electromagnetic synchrotron radiation [7]. This circumstance is not fortuitous and, as will be shown in the next section, it survives also in the quantum theory.

Let us compute the total power of the gravitational radiation for the case of relativistic electron energies ($\xi \gg 1$). Since the maximum of the spectral intensity in (15) falls into the high harmonics ($l \sim \xi^3$) the sum over l and the integration with respect to θ can be carried out with the help of the quasiclassical asymptotic expressions for the Bessel functions [7], neglecting the term with $l = 1$, which has a nonintegrable singularity in θ. The result is

$$P_G\approx{}^2/_3Gm^2\omega^2(1+\xi^2)^3={}^2/_3Gm^2\omega^2(E/m)^6, \tag{16}$$

where $E = m(1+\xi^2)^{1/2}$ is the energy of the electron in the frame in which it is on the average at rest.

We note that the fact that the power is proportional to the sixth power of the energy is, generally speaking, characteristic for the conversion of electromagnetic radiation (of synchrotron type) into gravitational radiation, for a charge which moves in a slowly varying electromagnetic field [6]; at the same time the proper gravitational radiation of the particle increases as E^4. In the case under consideration one cannot, however, consider the external field as slowly varying[1)]; the effect of direct conversion of electromagnetic radiation into gravitational radiation is totally absent, as we already remarked. However, in our case the energy E is in fact determined by the value of the invariant ξ^2 which characterizes the intensity of the electromagnetic wave and is thus not an independent parameter.

2. QUANTUM THEORY

The usual description of the photoproduction of a graviton on an electron in terms of perturbation theory [8-11] is applicable if $\xi \ll 1$. For large intensities of the incident wave ($\xi \gtrsim 1$) one must take into consideration the contribution of multiphoton diagrams. The corresponding calculations can be carried out in the Furry representation. Making use of the solutions of the Dirac equation in the field of a plane electromagnetic wave [12,13]; this is done under the assumption that the interaction with the electromagnetic and gravitational fields can still be treated as a small perturbation.

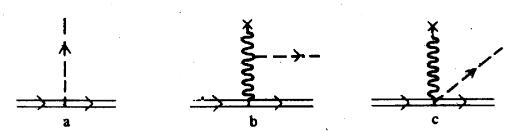
In an arbitrary graviton gauge the matrix element of the process under consideration has a rather complicated form:

$$S_{fi}=\frac{\varkappa}{4\sqrt{\omega'}}e_{\mu\nu}^{*}(k')\int dx\, e^{ik'x}(\bar\psi_{q'}\gamma^\mu\psi_q^{,\nu}-\bar\psi_{q'}^{,\nu}\gamma^\mu\psi_q) \tag{17}$$

$$-i\frac{e\varkappa}{2\sqrt{\omega'}}e_{\mu\nu}^{*}(k')\int dx\, e^{ik'x}\bar\psi_{q'}\gamma^\mu\psi_q A^{\nu}_{ext}+i\frac{e\varkappa}{4\pi\sqrt{\omega'}}e_{\mu\nu}^{*}(k')\int dx\,dx'\, e^{ik'x'}\bar\psi_{q'}(x)$$

$$\times(\gamma^\nu\partial_{x'}^{\lambda}D_0^c(x-x')-\gamma^\lambda\partial_{x'}^{\nu}D_0^c(x-x'))F^{\mu}_{ext\,\lambda}(x')\psi_q(x),$$

where $\psi_q(x)$ is an exact solution of the Dirac equation in the field of a circularly polarized electromagnetic wave [12,13], $\psi_q'^{\nu} \equiv \partial^\nu\psi_q$, $\kappa = (16\pi G)^{1/2}$. The first, second and third terms in (17) are represented by the diagrams a, b, c in the Figure, where the double line represents



the exact electron wave function, the cross denotes the external field of the wave, considered as classical and the dotted line corresponds to the emitted graviton.

The polarization tensor of a graviton with four-momentum k′ satisfies the relations

$$e_{\mu\nu}(k')=e_{\nu\mu}(k'),\quad e_\mu^{\ \mu}(k')=0,\quad e_{\mu\nu}(k')k'^\nu=0,$$
$$e_{\mu\nu}^{*}(k')e^{\mu\nu}(k')=1. \tag{18}$$

All calculations become significantly simpler if one makes use of the technique of helicity amplitudes [10,11]. For this purpose we construct two spacelike 4-vectors:

$$\varepsilon_\lambda^{(1)}=\frac{1}{(kk')}e_{\lambda\mu\nu\rho}\varepsilon^{(2)\mu}k'^\nu k^\rho,\quad \varepsilon_\lambda^{(2)}=\frac{e_{\lambda\mu\nu\rho}k'^\mu k^\nu q^\rho}{(kk')\,(2(kq)\,(k'q)/(kk')-m_*^2)^{1/2}} \tag{19}$$

having the following properties:

$$(\varepsilon^{(1)}\varepsilon^{(1)})=(\varepsilon^{(2)}\varepsilon^{(2)})=-1,\quad(\varepsilon^{(1)}\varepsilon^{(2)})=0,$$
$$(\varepsilon^{(i)}k)=(\varepsilon^{(i)}k')=0\quad(i=1,2),\quad(\varepsilon^{(2)}q)=(\varepsilon^{(2)}q')=0, \tag{20}$$
$$(\varepsilon^{(1)}q)=(\varepsilon^{(1)}q')=(2(kq)\,(k'q)/(kk')-m_*^2)^{1/2};$$

q and q′ are the quasi-momenta of the electron in the initial and final states, respectively ($q^2 = m_*^2 = m^2(1+\xi^2)$). In (20) we have taken into account the fact that the matrix element of the process under consideration is represented as a sum of amplitudes of partial processes with the conservation laws $sk + q = q' + k'$. The relations (20) allow us to fix the gauge of the electromagnetic wave in the following manner:

$$a_1=m\xi\varepsilon^{(1)}/e,\quad a_2=m\xi\varepsilon^{(2)}/e, \tag{21}$$

and to characterize the two independent polarization states of the graviton by the chiral tensors (helicity tensors)

$$e_{\mu\nu}^{(\pm)}=\varepsilon_\mu^{(\pm)}\varepsilon_\nu^{(\pm)},\quad \varepsilon_\mu^{(\pm)}=2^{-1/2}(\varepsilon_\mu^{(1)}\pm i\varepsilon_\mu^{(2)}). \tag{22}$$

Indeed, the validity of the relations (8) and (18) follows obviously from (20); moreover tensors of opposite helicity are orthogonal:

$$e_{\mu\nu}^{(+)*}e^{(-)\mu\nu}=0.$$

In the gauge (21), (22) the total contribution of the second and third terms of (17) (the diagrams b and c of the figure) vanishes and the matrix elements of the S-matrix corresponding to the emission of a graviton with positive or negative helicity turns out to be proportional to the helicity matrix elements for the emission of a photon:

$$S_{fi}^{(\pm)}=-i\frac{\varkappa}{\sqrt{2}}(\varepsilon^{(\pm)*}q)\int dx\frac{e^{ik'x}}{\sqrt{2\omega'}}\bar\psi_{q'}\hat\varepsilon^{(\pm)*}\psi_q. \tag{23}$$

Making use of the results of [14], this allows us to write directly an expression for the partial wave probability of emission of a graviton (from unit volume, per unit time) with the absorption of s ($s \ge 1$) quanta from the wave:

$$dW_G^{(s)}=\frac{n}{4q_0}Gm^4(1+\xi^2)\frac{du}{(1+u)^2}\left(\frac{u_s}{u}-1\right)$$
$$\times\left\{-4J_s^2(z)+\xi^2\left(2+\frac{u^2}{1+u}\right)(J_{s+1}+J_{s-1}-2J_s^2)\right\}, \tag{24}$$

here n is the electron density

$$u=\frac{(kk')}{(kq')},\quad 0\leqslant u\leqslant u_s=\frac{2s(kq)}{m_*^2},$$

and the variable in the Bessel functions is

$$z=2s\frac{\xi}{(1+\xi^2)^{1/2}}\left(\frac{u}{u_s}\left(1-\frac{u}{u_s}\right)\right)^{1/2}.$$

For $\xi \ll 1$ the right-hand side of the expression (24) can be expanded in powers of ξ. The first term of the expansion in $dW_G^{(1)}$, with appropriate normalization of the 4-potential of the wave [13] coincides with the probability for the photoproduction of a graviton, calculated according to perturbation theory [11]. Passing to the classical limit in (24) ($\omega'/q_0 \ll 1$), then in the frame where the electron is at rest on the average, the formula obtained here coincides with the classical intensity of the gravitational radiation (15) divided by $s\omega$.

The probabilities of the processes of one-graviton annihilation and pair-creation by a graviton can also be calculated making use of the gauge (21), (22). Just as in the case of emission, these probabilities turn out to be proportional to the corresponding quantities for processes with the participation of a photon. In particular, the probability for the annihilation of a pair into one graviton in the presence of a circularly polarized wave has for $\xi \gg 1$ the form

$$W_G=\frac{Gm_*^2}{e^2}\left(\zeta\frac{\chi\chi'}{(\chi+\chi')^2}-1\right)W_\gamma,$$

$$\chi=\frac{e}{m^3}[-(F_{\mu\nu}^{ext}q^\nu)^2]^{1/2},\quad \chi'=\frac{e}{m^3}[-(F_{\mu\nu}^{ext}q'^\nu)^2]^{1/2},\quad \zeta=\frac{(q+q')^2}{m_*^2},\qquad (25)$$

and W_γ is the total probability for one-photon annihilation[15]. We note that for $\xi\to\infty$ (for fixed other invariants on which the probability depends) (25) tends to zero.

3. THE EMISSION OF A GRAVITON IN A STATIC FIELD

The limit $\xi\to\infty$, which allowed us to go over from the probability of emission of a photon by an electron in the field of a circularly polarized wave to the corresponding probability in static crossed fields[15], makes the probability of emission of a graviton infinite. Indeed, the 4-momentum conservation law implies that in a static field the momentum of the intermediate photon (diagram b) is on the mass shell. Therefore the gravitational radiation of an electron in a static field will be determined principally by the succession of two events: the emission of a photon by the electron with successive transformation of the electromagnetic radiation into gravitational radiation. The formal becoming infinite of the probability means in fact that the amplitude of the gravitational wave increases with the distance.

Within the framework of classical theory the intensity of the gravitational radiation in an arbitrary static field can be determined by means of solving the equations for the gravitational potentials by means of the method of slowly varying amplitudes[6]. This intensity turns out to be proportional to the square of the distance to the observation point, in agreement with the fact that the integral over a spherical volume of the stress tensor of the total field of the electromagnetic radiation and the external field, which in this case is the main source of gravitational radiation, grows in the same proportion:

$$\frac{dP_G^{res}}{d\Omega}=-Gr^2(F_{\mu\nu}^{ext}n^\nu)^2\frac{dP_\gamma}{d\Omega}.\qquad (26)$$

One should however keep in mind that we neglect the variation of the amplitude of the electromagnetic radiation on account of the effect of conversion into gravitational radiation, as well as the inverse transition. As was shown by Zel'dovich[16], a joint consideration of the direct and inverse processes leads to oscillations of the amplitude for large r.

In the quantum theory of the problem we restrict our attention to the diagram b, and replace the propagator in it by its absorptive part. For the 4-vectors $\epsilon^{(1)}$ and $\epsilon^{(2)}$ from which the graviton polarization tensors (22) are constructed it is convenient to choose

$$\varepsilon^{(1)}=\frac{1}{(k'p)}e_{\mu\nu\lambda\sigma}\varepsilon^{(2)\nu}k'^\lambda p^\sigma,\quad \varepsilon_\mu^{(2)}=\frac{e_{\mu\nu\lambda\rho}k'^\nu p^\lambda F_{ext}k'^\rho}{(k'p)[-(F_{\mu\nu}^{ext}k'^\nu)^2]^{1/2}}.\qquad (27)$$

Then the contribution to the amplitude of one-graviton emission, related to the conversion of the electromagnetic radiation of the particle into gravitational radiation in a static field $F^{\mu\nu}_{ext}$ takes the form

$$M_G^{(\pm)}=-(4G)^{1/2}\varepsilon_\mu^{(1)}n_\nu F_{ext}^{\mu\nu}IM_\gamma^{(\pm)},\qquad (28)$$

where $M_\gamma^{(\pm)}$ is the helicity amplitude for the emission of a photon in the field $F^{\mu\nu}_{ext}$ and I is a divergent integral:

$$I=\int d\mathbf{q}\,\delta(\mathbf{q}+\mathbf{k}')\,\delta(\omega'^2-\mathbf{q}^2).\qquad (29)$$

We make use of the following formal trick. We represent $\delta(\mathbf{k}+\mathbf{k}')$ in the form

$$\delta(\mathbf{q}+\mathbf{k}')=\lim_{r\to\infty}\frac{1}{\pi^2 r}((\mathbf{q}+\mathbf{k}')^2+r^{-2})^{-2},\qquad (30)$$

which has the result that (29) takes the form $r/2\pi\omega'$. The corresponding contribution to the intensity of the gravitational radiation due to the direct conversion of the electromagnetic radiation of the electron into gravitational radiation turns out to be proportional to the intensity of the electromagnetic radiation and we obtain a formula analogous to (26). In the special case of the crossed field

$$\frac{dP_G^{res}}{d\Omega}=4GB^2r^2\sin^4\frac{\theta}{2}\frac{dP_\gamma}{d\Omega},$$

where θ is the angle between **k** and **k**′, **k** is the "wave vector" of the crossed field, and B is the absolute value of the field strength. The electromagnetic radiation of an electron in a crossed field has been discussed in detail in[15].

[1]Essentially the slowly varying field approximation of Sushkov and Khriplovich [6] corresponds to neglecting the momentum carried away by the external field, which is not admissible in the case under consideration.

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86