Trajectories and the radiation emitted by particles falling into a rotating black hole

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(Submitted February 5, 1974; resubmitted October 8, 1974)
Zh. Eksp. Teor. Fiz. **68**, 369–376 (February 1975)

The trajectories corresponding to small impact parameters for particles incident on a rotating black hole are investigated. With the emission of scalar waves as a model, it is shown that the angular and spectral distributions of the energy emitted by relativistic particles moving along such trajectories depends on the metric of the black hole.

1. INTRODUCTION

The classification of particle trajectories near rotating black holes has been treated in[1-5]. Recently Piragas and Krivenko[5] have indicated among these trajectories a class for which the slope with respect to the axis of rotation of the black hole is limited to a definite angular interval to one side of the equator, and in particular, may remain constant. In that paper it is pointed out that this class of trajectories is realized for sufficiently small impact parameters and corresponds to radial fall in the Schwarzschild metric. Trajectories of relativistic particles with constant slope with respect to the axis of the black hole have been found explicitly.

Information about the metric of a black hole can be extracted, in principle, from angular, spectral, and polarization characteristics of the radiation emitted by surrounding matter. The equation for radiation in coordinates which allow for separation of variables have been written by Teukolsky[6]. In a number of papers[7-11] the radiation emitted by test bodies rotating around the black hole on equatorial orbits have been found. In the present paper we consider the scalar radiation emitted by relativistic particles (with total energy E and mass μ), falling into the black hole along trajectories of constant inclination with respect to the axis of the black hole (of mass M). On the basis of model calculations for the scalar field estimates are given for the characteristic angles, frequencies and also of the total energy of gravitational and electromagnetic radiation. The largest fraction of the energy is emitted on frequencies $\omega \lesssim (c^3/GM)(E/\mu c^2)$, where for $\omega \ll (c^3/GM)(E/\mu c^2)$ the spectrum does not depend on the angular momentum of the black hole. For $\omega \gg (c^3/GM)(E/\mu c^2)$ the spectrum starts falling off exponentially and for isotropic accretion of matter the radiation concentrates near the equator of the black hole. The limit of the fall-off of the spectrum and the opening of the direction diagram depend on the angular momentum of the black hole.

2. THE FALL OF A PARTICLE INTO THE BLACK HOLE

We consider the falling of a particle onto a rotating neutral black hole. We use the Kerr metric in Boyer-Lindquist coordinates t, r, ϑ, φ:

$$ds^2 = \frac{\Delta - a^2\sin^2\vartheta}{\rho^2}dt^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\vartheta^2 - \frac{\sin^2\vartheta}{\rho^2}[(r^2+a^2)^2 - \Delta a^2\sin^2\vartheta]d\varphi^2 + \frac{4ar}{\rho^2}\sin^2\vartheta\, dt\, d\varphi,$$

where $\rho^2 = r^2 + a^2\cos^2\vartheta$ and $\Delta = \Delta(r) = r^2 - 2r + a^2$. The quantities $\hbar$, c, G, the masses of the black hole and of the incident particle are set equal to one, for simplicity. The angular momentum of the black hole is equal to $a\,(0 \le a \le 1)$ and has the orientation $\vartheta = 0$. The radius of the event horizon is $h = 1 + (1 - a^2)^{1/2}$, $\Delta(h) = 0$.

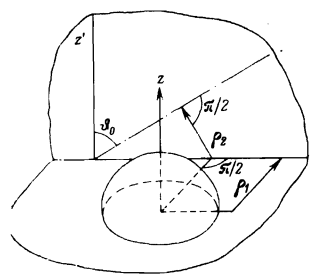

FIG. 1

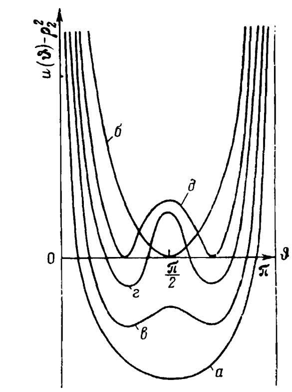

FIG. 2

FIG. 1. The initial parameters of the trajectory of the particle as seen by a remote observer. The horizontal plane coincides with the equatorial plane of the black hole. The vertical plane passes through the rectilinear portion at infinity of the trajectory (dash-dot line) which is perpendicular to the z axis of rotation of the black hole.

FIG. 2. The function $(u(\vartheta) - \rho_2^2)$ for different particle trajectories.

As is well known,[1,2] the equations of the trajectory of a test body contain (in the notation of[1,5]) three conserved quantities, E, Φ and K, where E is the total particle energy, Φ is the projection of the angular momentum of the particle on the axis of rotation of the black hole, and the quantity K is related to the second component of the angular momentum (cf. infra). Let v be the velocity of the particle at infinity, and ϑ_0, φ_0 the initial angles of the trajectory. We introduce two impact parameters: ρ_1 and ρ_2 as illustrated on Fig. 1. The quantities ρ_1 and ρ_2 can be either positive or negative, cf. Fig. 1.

Let $q^2 = \rho_1^2 + \rho_2^2 - a^2\cos^2\vartheta_0$. From the equations of the trajectories[2] for $r \to \infty$ it follows that

$$E = \frac{1}{(1-v^2)^{1/2}}, \quad \Phi = \frac{\rho_1 v \sin\vartheta_0}{(1-v^2)^{1/2}}, \quad K = \frac{a^2 - 2av\rho_1\sin\vartheta_0 + qv^2}{1-v^2}.$$

Then the equations of the trajectories can be represented in the form

$$\left(\frac{d\vartheta}{d\sigma}\right)^2 = \frac{v^2}{1-v^2}[\rho_2^2 - u(\vartheta)], \quad u(\vartheta) = \frac{(\rho_1^2 - a^2\sin^2\vartheta)(\sin^2\vartheta_0 - \sin^2\vartheta)}{\sin^2\vartheta} \tag{1}$$

$$\frac{dr}{d\sigma} = -\frac{1}{(1-v^2)^{1/2}}[v^2r^4 + 2(1-v^2)r^3 + v^2(a^2-q)r^2 + 2(a^2 - 2av\rho_1\sin\vartheta_0 + v^2q)r$$

$$+a^2v^2(\rho_1^2\sin^2\vartheta_0-q)]^{1/2},$$

$$\Delta\sqrt{1-v^2}\frac{dt}{d\sigma}=r^4+r^2a^2(1+\cos^2\vartheta)+2r(a^2\sin^2\vartheta-a\rho_1 v\sin\vartheta_0)+a^4\cos^2\vartheta,$$

$$\Delta\sqrt{1-v^2}\sin^2\vartheta\frac{d\varphi}{d\sigma}=r^2\rho_1 v\sin\vartheta_0+2r(a\sin^2\vartheta-v\rho_1\sin\vartheta_0)+a^2v\rho_1\sin\vartheta_0\cos^2\vartheta.$$

Here the quantity σ is related to the proper time τ of the particle through the relation $d\sigma = \rho^2 d\tau$. The equations (1) simplify considerably for $v \to 0$ and $v \to 1$.

Let us investigate the first equation (1). The character of the trajectory is determined by the region of possible values of the angles ϑ, i.e., by the condition $\rho_2^2 - u(\vartheta) \geq 0$, involving the geometric parameters ρ_1, ρ_2 and ϑ_0, but not the velocity v. For $\rho_1 = a = 0$, $u \equiv 0$; for $\sin\vartheta_0 = 0$, $u = a^2\sin^2\vartheta - \rho_1^2$. In the other cases the function u increases indefinitely near the black hole (Fig. 2), with

$$\rho_2=0, \quad \vartheta_0=\pi/2$$

For $(\rho_1\sin\vartheta_0)^2 \geq a^2$ the function u has a minimum at the point $\vartheta = \pi/2$ (Fig. 2, a, b). If $(\rho_1\sin\vartheta_0)^2 < a^2$ there is a local maximum for $\vartheta = \pi/2$ and symmetrically, for angles $a^2\sin^4\vartheta_{min} = \rho_1^2\sin^2\vartheta_0$ there are two minima $u(\vartheta_{min}) = -(\rho_1 - a\sin\vartheta_0)^2$ (Fig. 2, c, d, e).

If $\rho_2^2 \geq u(\pi/2)$, i.e., if $\rho_2^2 + \rho_1^2\cos^2\vartheta_0 \geq a^2\cos^2\vartheta_0$ (Fig. 2, a, b, c) there is one region of possible values of the angles ϑ which is symmetric with respect to the equator of the black hole[5]. In particular the well studied[3-5] equatorial trajectories $\rho_2 = 0$, $\vartheta_0 = \pi/2$ (Fig. 2, b) belong to this type.

We consider the class of trajectories with the impact parameters

$$\rho_2^2+\rho_1^2\cos^2\vartheta_0<a^2\cos^2\vartheta_0, \tag{2}$$

which are manifestly smaller than the radius of the horizon. In this case two regions of possible values of the angles ϑ are situated on opposite sides of the equator (Fig. 2, d, e) and in the special case $\rho_1 = a\sin\vartheta_0$, $\rho_2 = 0$ the angles ϑ of the trajectory are constant (Fig. 2, e). The particle gets into one of the possible regions, depending on the values of the angle ϑ_0. Without introducing impact parameters this class of trajectories was first pointed out by Piragas and Krivenko[5]. From the analysis of the other equations (1) it follows that the radius r of the trajectory (2) decreases monotonically. The particle reaches the event horizon after a finite proper time τ and asymptotically approaches the horizon as the time t of a remote observer increases. For $\rho_1 \geq 0$ the angle φ of the trajectory increases monotonically. If $\rho_1 < 0$ and far from the black hole the angle φ decreases and for

$$r<1-(a\sin^2\vartheta/v\rho_1\sin\vartheta_0)+\{[1-(a\sin^2\vartheta/v\rho_1\sin\vartheta_0)]^2-a^2\cos^2\vartheta\}^{1/2}$$

it increases. Consequently, near the black hole all trajectories (2) wind around the axis in the direction of rotation of the black hole. Strictly speaking, if the condition (2) is satisfied the angle $\vartheta_0 \neq \pi/2$. As $\vartheta_0 \to \pi/2$ the trajectories under consideration tend to equatorial trajectories with $\rho_1^2 < a^2$. In the limit of the Schwarzschild metric all the trajectories (2) go over into a radial fall. In the Schwarzschild metric the simplest motion is radial fall. In the Kerr metric the simplest trajectories are those with constant angle ϑ ($\rho_1 = a\sin\vartheta_0$, $\rho_2 = 0$), which for relativistic particles $v \to 1$ can be calculated explicitly from the equations (1). It turns out that for any value of a the radius $r = -E\tau v$, and

$$\varphi-\varphi_0=\frac{a}{2\sqrt{1-a^2}}\ln\left(\frac{r-h+2\sqrt{1-a^2}}{r-h}\right),$$

$$-tv=r+\ln\Delta-\frac{1}{\sqrt{1-a^2}}\ln\left(\frac{r-h+2\sqrt{1-a^2}}{r-h}\right), \quad a\neq 1; \tag{3}$$

$$\varphi-\varphi_0=(r-1)^{-1}, \quad -tv=r+2\ln(r-1)-2(r-1)^{-1}, \quad a=1.$$

3. SCALAR RADIATION

On the mathematically simple example of scalar radiation by particles moving along the trajectories (3) we shall show that the radiation carries information on the metric of the black hole. The angular and frequency distributions of the electromagnetic and gravitational fields are in general similar to the distributions obtained for a model scalar field[6].

The potential of a massless scalar field U satisfies the Klein-Gordon equation $\Box U = 4\pi T$, where $\Box$ is the covariant D'Alembertian and T is the density of the scalar charge in the proper volume. In Boyer-Lindquist coordinates[7]

$$\Box U=\rho^{-2}\left\{\frac{\partial}{\partial r}\Delta\frac{\partial}{\partial r}+\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial}{\partial\vartheta}+\left(\sin^{-2}\vartheta-\frac{a^2}{\Delta}\right)\frac{\partial^2}{\partial\varphi^2}\right.$$
$$\left.-\frac{4ar}{\Delta}\frac{\partial^2}{\partial\varphi\,\partial t}-\left[\frac{(r^2+a^2)}{\Delta}-a^2\sin^2\vartheta\right]\frac{\partial^2}{\partial t^2}\right\}U, \tag{4}$$

$$T=\rho^{-2}\sin^{-1}\vartheta\frac{d\tau}{dt}\delta[r-r(t)]\delta[\vartheta-\vartheta(t)]\delta[\varphi-\varphi(t)],$$

where $r(t)$, $\vartheta(t)$ and $\varphi(t)$ are the coordinates of the trajectory of the radiating particle. For the trajectory (3)

$$U=(2\pi)^{-1}\int d\omega\, U_\omega\exp(-i\omega t),$$

$$U_\omega(r,\vartheta,\varphi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{+l}\frac{4\pi f_{lm\omega}(r)}{\sqrt{r^2+a^2}}S_l^m(-a^2\omega^2,\vartheta)S_l^{m*}(-a^2\omega^2,\vartheta_0)\exp(im\varphi). \tag{5}$$

Here $S_l^m(-a^2\omega^2, \vartheta)$ is a spheroidal function of the first kind[12], satisfying the equation (cf.[13])

$$\left(\frac{1}{\sin\vartheta}\frac{d}{d\vartheta}\sin\vartheta\frac{d}{d\vartheta}+\lambda_l^m-a^2\omega^2\sin^2\vartheta-\frac{m^2}{\sin^2\vartheta}\right)S_l^m(-a^2\omega^2,\vartheta)=0 \tag{6}$$

and is normalized as in[7], while $\lambda_l m$ is the eigenvalue corresponding to the function S_l^m.

We substitute (5) in (4) and introduce the new radial variable y[13] $dy/dr = (r^2 + a^2)/\Delta$; $y \to r$ for $r \to \infty$ and $y \to -\infty$ for $r \to h$. Taking into account (3) we obtain for the function $f_{lm\omega}(y)$ the equation

$$\left[\frac{d^2}{dy^2}+k^2(y)\right]f_{lm\omega}=-\frac{\Delta}{vE(r^2+a^2)^{3/2}}\exp[-i\omega yv^{-1}-im\varphi(y)],$$

$$k^2=\left(\omega-\frac{am}{r^2+a^2}\right)^2-\frac{x^2\Delta}{(r^2+a^2)^2}-\frac{\Delta}{(r^2+a^2)^{3/2}}\frac{d}{dr}\left[\frac{r\Delta}{(r^2+a^2)^{3/2}}\right] \tag{7}$$

where $x^2 = \lambda_l^m - 2am\omega$. Since $f_{lm\omega} = f^*_{l-m-\omega}$ it suffices to consider positive frequencies. The characteristic frequencies of the radiation from relativistic particles are $\omega \gg 1$. In this case the numbers l and m are large, and for the solution of Eqs. (5) and (7) we can make use of the quasiclassical method (WKB approximation).

The quasiclassical Green's function $G(y, y')$ of the equation (7) for $y < y'$ and $k^2 > 0$ is of the form

$$G(y,y')=i\cdot 2^{-1}[k(y)k(y')]^{-1/2}\exp\left[i\int_{y}^{y'}dy''\,k(y'')\right].$$

With the help of this expression the function $f_{lm\omega}(y)$ which describes the partial wave excited by the particle, with indices l and m and wave number $k(y)$, can be represented in the form

$$f_{lm\omega}(y)=(i/2\sqrt{k(y)}E)Q\exp\left[-i\omega y+\int_y^\infty dy''\,(k(y'')-\omega)\right]+f_1, \quad (8)$$

where

$$Q=\int_y^\infty \frac{dy'\Delta(r')}{(r'^2+a^2)^{3/2}\sqrt{k(y')}}\exp\left[-i\omega y'(1-v)-im\varphi(y')+i\int_\infty^{y'} dy''\,(k(y'')-\omega)\right],$$

and f_1 denotes the term containing an integral over the region $y' < y$. For the case $x \gtrsim \omega$ of eventual interest, the exponent in Q can be easily computed for $r' \simeq y' \gg 1$ with the help of (3) and turns out to be equal to

$$(i/2)\,(x^2y'^{-1}\omega^{-1}-y'\omega E^{-2})-im\varphi_0,$$

whereas the pre-exponential factor equals $1/y'\sqrt{\omega}$. Since the speed of the relativistic particle is close to the phase velocity ω/k of the wave, the term in the exponential which depends on y' vanishes at a distance $r_1 = Ex/\omega \gg h$ from the black hole. Therefore the main contribution to the integral Q comes from the region of values $y' \sim r_1$ (the region of effective wave generation). For $y' \gg r_1$ the generation stops, owing to the mismatch between the velocities of the wave and particle (the exponential under the integral sign undergoes strong oscillations). Then, for $y \ll r_1$

$$Q\approx 4\omega^{-1/2}K_0(x/E)\exp(-im\varphi_0) \quad (9)$$

(here K_0 is a Macdonald function (Bessel function of second kind of imaginary argument)), the term f_1 in (8) is negligibly small and the function (8) describes a free wave propagating, like the particle, in the direction of the black hold. This wave is either absorbed at the horizon if $k^2 > 0$, or is reflected at some point y_0 on the "radial barrier" $k^2(y_0) = 0$ and can then be recorded by a remote observer. The reflection of the wave occurs (cf. 7)) if $x \gtrsim \omega$, which justifies (9). In the quasiclassical approximation the function which describes the reflected wave (8) has for $r \to \infty$ the form

$$f_{lm\omega}\approx -\frac{2}{\omega E}K_0\left(\frac{x}{E}\right)\exp\left\{i\omega r+i\left[2\int_{y_0}^\infty dy''\,(k(y'')-\omega)-2\omega y_0\right]\right\}. \quad (10)$$

Above-the-barrier penetration is insignificant here[14].

The amount of energy radiated into a unit solid angle in the direction (ϑ_1, φ_1) and unit frequency interval $\omega > 0$ equals

$$d\mathscr{E}/d\omega\, d\Omega_1=\lim_{r\to\infty}|r\omega U_\omega(r,\vartheta_1,\varphi_1)/2\pi|^2. \quad (11)$$

Integrating this expression over the angles (ϑ_1, φ_1) we obtain the spectral distribution of the radiation energy, $d\mathscr{E}(\vartheta_0)/d\omega$ for a particle falling under a fixed angle ϑ_0, and by averaging over the directions (ϑ_0, φ_0) we obtain the angular and spectral distributions of the energy $I(\omega, \vartheta_1)$ for isotropic particle accretion, the particles moving along the trajectories (3). Making use of (5), (8), (9), and of the orthogonality of the functions S_l^m,[7] we obtain

$$\frac{d\mathscr{E}}{d\omega}(\vartheta)=4\pi I(\omega,\vartheta)=\frac{16}{E^2}\sum_{lm}K_0^2\left(\frac{x}{E}\right)|S_l^m(-a^2\omega^2,\vartheta)|^2. \quad (12)$$

The summation over l and m in (5) and (12) will be replaced by an integration with respect to x and m over a region where the condition of radial reflection of the waves for $k^2 = 0$ and $r > h$ is satisfied, and which are allowed classically by Eq. (6).

For $\omega \ll E$ the main contribution to (5) and (12) comes from the quantities $x \simeq E \gg \omega$, far from the limit of vanishing radial barriers. Thus all the radiation is reflected at distances $r_0 \simeq y_0 \simeq x/\omega \gg h$. But

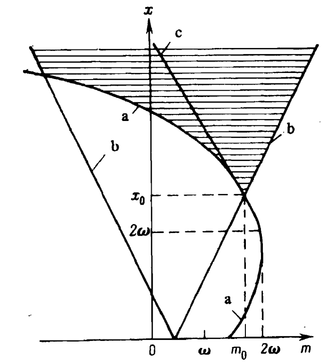

FIG. 3. The integration region (shaded) in Eq. (12) at a = 1.

for $x \gg \omega$ it follows from (6) that $x \to l$ and $S_l^m \exp im\varphi \to Y_{lm}$. Then, taking into account (3) and (7), the term in the square brackets in the exponent of (10) is easily calculated and equals πl. In (5) we have

$$\sum_{m=-l}^{l}(-1)^lY_{lm}(\vartheta_1,\varphi_1)Y_{lm}^*(\vartheta_0,\varphi_0)=(-1)^l(2l+1)P_l(\Theta)/4\pi\approx lJ_0(l(\pi-\Theta))/2\pi.$$

Here Y_{lm} is a spherical function, P_l is a Legendre polynomial, J_0 is a Bessel function, Θ is the angle between the initial direction (ϑ_0, φ_0) of the particle and the direction (ϑ_1, φ_1) under which the radiation is observed. Integrating with respect to l in (5) and making use of (11) we obtain that

$$\frac{d\mathscr{E}}{d\omega\, d\Omega_1}=\frac{4}{\pi^2E^2[(\pi-\Theta)^2+E^{-2}]^2}, \quad \frac{d\mathscr{E}}{d\omega}=\frac{4}{\pi}, \quad (13)$$

i.e., the distribution of the radiation energy for $\omega \ll E$ does not depend on the frequency and the angular momentum of the black hole and the functions (12) do not depend on the angles. The main part of the energy is emitted into a narrow cone $(\pi - \Theta) \sim 1/E$ in a direction opposite to the initial direction of motion of the particle, since the observer records only the radiation which has been reflected back.

For $\omega \gg E$ the main contribution to (12) comes from the quantities x and m near the boundary of vanishing radial barriers. For a = 0 the functions $|S_l^m|^2 = |Y_{lm}|^2$, $x = (l(l+1))^{1/2} \simeq l$, and the radial barriers appear when $x \geq 3\cdot 3^{1/2}\omega$. Then

$$\frac{d\mathscr{E}}{d\omega}=\frac{8}{\pi E^2}\int_{3\cdot 3^{1/2}\omega}^{\infty} dx\, xK_0^2\left(\frac{x}{E}\right)_{\omega\gg E}\approx\frac{2}{3\cdot 3^{1/2}}\exp(-6\cdot 3^{1/2}\omega/E). \quad (14)$$

and the total radiation energy is

$$\mathscr{E}=(8E/3\cdot 3^{1/2}\pi)\int_0^\infty d\xi\,\xi^2K_0^2(\xi). \quad (15)$$

If a = 1, the region of radial reflection of the waves is bounded by the curve (Fig. 3, a) $x_1(m_1)$:

$$x_1=2\omega-\sqrt{16\omega^2-8m_1\omega} \quad \text{for} \quad 3\omega/2\leqslant m_1\leqslant 2\omega,$$
$$x_1=2\omega+\sqrt{8\omega^2-4m_1\omega} \quad \text{for} \quad m_1<2\omega.$$

The classically allowed part of this region,in Eq. (6) is bounded by the curve $x_2 = |m_2\sin^{-1}\vartheta - \omega\sin\vartheta|$ (Fig. 3, b), and the functions $|S_l^m|^2$ computed quasiclassically are

$$|S_l^m(-\omega^2,\vartheta)|^2=\frac{\partial\lambda_l^m/\partial l}{4\pi^2 p\sin\vartheta},$$
$$p^2=\frac{(m^2-\omega^2\sin^2\vartheta_g\sin^2\vartheta)(\sin^2\vartheta-\sin^2\vartheta_g)}{\sin^2\vartheta\sin^2\vartheta_g}, \quad (16)$$

where p is the "wave number for motion along the angle ϑ," ϑ_g is the classical turning point in the Eq. (6), $x = |m\sin^{-1}\vartheta_g - \omega\sin\vartheta_g|$. In the case $\omega \gg E$ the main contribution to (12) comes from a small region of values $\Delta x \sim \Delta m \sim E$ near the point $(x_0, m_0 > 0)$ on the intersection of the curves $x_1(m_1)$ and $x_2(m_2)$ (cf. Fig. 3). The integral (12) over x and m may be calculated between the limits given by the tangents to these curves at the point (x_0, m_0) (Fig. 3, b, c). In the integration region the angles ϑ are close to ϑ_g and

$$p^2 \approx 2(m_0 \sin^{-2}\vartheta - \omega)[(x - x_0)\sin\vartheta - m + m_0].$$

The integral obtained in this manner can be calculated and yields

$$\frac{d\mathscr{E}}{d\omega} = \sqrt{\frac{E(\sin\vartheta - \zeta)}{\pi\omega(\nu - \sin^2\vartheta)}} \exp\left(-\frac{2\varkappa\omega}{E}\right)_{\cos^2\vartheta \ll 1} \approx \frac{1}{2}\sqrt{\frac{E(2+2^{1/2})}{\pi\omega}} \exp\left[-\frac{4\omega(2^{1/2}-1+\cos^2\vartheta)}{E}\right] \tag{17}$$

where

$$\varkappa = x_0/\omega, \quad \nu = m_0/\omega, \quad \zeta = dm_1/dx_1(x_0, m_0);$$
$$\nu = s[2 - s + \sqrt{8(1-s)}], \quad \varkappa = 2[1 - s + \sqrt{2(1-s)}],$$
$$\zeta = -\sqrt{2-\nu} \quad \text{for} \quad 0 < s = \sin\vartheta < \sqrt{3} - 1;$$
$$\nu = s[2 - 3s + 2\sqrt{2s^2 - 4s + 4}], \quad \varkappa = 2[1 - 2s + \sqrt{2s^2 - 4s + 4}],$$
$$\zeta = \sqrt{1 - (\nu/2)} \quad \text{for} \quad \sqrt{3} - 1 \leqslant s \leqslant 1.$$

Equation (17) is not applicable for $\sin^2\vartheta \ll 1$, where the quasiclassical approximation breaks down, and for $\omega \gtrsim E^3$, where the quasiclassical function (16) has to be replaced by an Airy function[14].

It follows from Eqs. (14) and (17) that for $\omega \gg E$ there appears an exponential fall-off of the spectrum which can be explained by absorption of the main part of the radiation (emitted by the particle) from the black hole. The remaining radiation recorded by the observer is reflected at distances $r_0 \sim h$ and therefore contains information on the metric of the black hole: the limit of the fall-off of the spectrum and the angular opening of the direction diagram of the radiation for $\omega \gg E$ depends on the angular momentum of the black hole. For isotropic accretion of matter for a nonrotating black hole the radiation is isotropic, (14), whereas in the case of extremely fast rotation, (17), it is concentrated in a narrow equatorial region $\Delta\vartheta \sim (E/\omega)^{1/2}$. The exponent which determines the rate of exponential fall-off of the spectrum in the equatorial plane turns out for $a = 1$ to be $3 \cdot 3^{1/2}/2(2^{1/2} - 1) \simeq 6.3$ times smaller than for $a = 0$, which can be explained by a less effective absorption of radiation by the rotating black hole.

Recognizing that the density of scalar charge (5) contains a factor E^{-1} we obtain for the total energy of the electromagnetic radiation (e) and gravitational radiation (G) emitted by a particle (of charge e) moving along the trajectory (3) the estimate (we made use of (15); the result is in conventional units):

$$\mathscr{E}_e \sim (e^2c^2/GM)(E/\mu c^2)^3, \quad \mathscr{E}_G \sim (c^2\mu^2/M)(E/\mu c^2)^3. \tag{18}$$

In the same manner as for scalar waves, the particle emits into a narrow cone $\Delta\Theta \sim \mu c^2/E$ in a direction opposite to its initial fall direction (13), the main function of the energy goes into frequencies $\omega \lesssim (c^3/GM) \times (E/\mu c^2)$, and for high frequencies the spectrum falls off exponentially due to absorption of the radiation by the black hole. For comparison we note that the characteristic frequency of radiation from a relativistic particle revolving on an orbit of radius $\sim GM/c^2$ in flat space equals $\omega \sim (c^3/GM)(E/\mu c^2)$ and in the Kerr field[10,11] $\omega \sim (c^3/GM)(E/\mu c^2)^2$.

The author is deeply grateful to É. B. Gliner and V. Unt for proposing this problem, for discussions, and for valuable remarks, and to A. Z. Dolginov and I. G. Dymikova for discussions.

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Translated by M. E. Mayer
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