Pinning of a vortex lattice on the interface of two superconductors and the critical current

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The problem of calculating the interaction energy between a vortex lattice and the interface between two superconductors is solved. The vortex lattice is assumed to be parallel to the plane of the interface. The superconductors differ with respect to the penetration depth of the magnetic field and their coherence length. The interaction between the vortices and the interface makes possible the flow of a transport current perpendicular to the vortices along the interface. The current flows in a strip of width $\sim\lambda_1+\lambda_2$ near the boundary (λ is the penetration depth). Estimates show that the critical current is a monotonically decreasing function of the magnetic field. The critical current density may reach $\sim 10^5$ A/cm^2 when the difference in λ amounts to only 1%.

1. INTRODUCTION AND STATEMENT OF THE PROBLEM

The flow of a nondissipative volume transport current through a type II superconductor in the mixed state is possible only if the superconducting vortices are pinned by some sort of inhomogeneities. These inhomogeneities can be macroscopic inclusions of another superconducting phase. The inhomogeneous cooling of an ingot, for example, can lead to the appearance of such inclusions. After transformation of this material into a wire, these superconducting inclusions are stretched into long superconducting filaments or plates located inside the basic superconducting matrix. The simplest case of such an inhomogeneity of which we shall speak will be simply a plane boundary between two superconducting half-spaces, when the vortices are located parallel to the boundary.[1)]

The problem now arises: how the pinning of the vortices takes place and what will be the critical current in such a material? We first discuss the physics of the problem. There are two completely different approaches here. Let the boundary between the two superconductors coincide with the plane $x = 0$, and let the external magnetic field H_0 be parallel to the z axis.

Dew-Hughes and Witcomb [1] explained the pinning of the vortices on the boundary between two superconductors as follows: Let the penetration depth of the magnetic field in the right half-space ($x > 0$) be equal to λ_1, and let the coherence length be ξ_1. For the left half-space ($x < 0$), we have, correspondingly, λ_2 and ξ_2. Then, for a given external field H_0, the equilibrium induction will be B_1 for $x > 0$ and B_2 for $x < 0$. This means that a superconducting current flows along the boundary between the two superconductors in the direction of the y axis:

$$I_M=\frac{c}{4\pi}(B_1-B_2).$$

This current flows in a layer $\sim\lambda_1 + \lambda_2$ near the boundary. The interaction of vortices located in this layer with the current I_M leads to pinning of the entire vortex structure, and an estimate of the force of this pinning, carried out in the spirit of the work of Campbell and Evetts [2], (where pinning of the vortex system on the boundary of a superconductor with a vacuum, due to interaction of the vortices with their own images and with the Meissner current, is considered) gives the following final expression for the critical current density for a material with a system of dislocation cells:

$$j_c=\frac{\Delta\varkappa}{\varkappa}\frac{S_v c(H_{c2}-2B)}{8\pi\beta\varkappa^2\lambda}\sqrt{\frac{\Phi_0}{B}}. \quad (1)$$

Here $\Delta\kappa$ is the difference between the constants κ of the Ginzburg-Landau theory of two contacting superconductors (it is assumed that $\Delta\kappa/\kappa \ll 1$), S_V is the area of the interface per cm^3 of material, $\beta = 1.16$. For a certain field H^*, the curves of the reciprocal of the magnetic moment intersect, i.e., $\Delta M(H^*) = 0$. This means that, in accord with (1), $j_c(H^*) = 0$. Therefore, the authors of [1] confirm that the critical current will be a nonmonotonic function of the field H_0 in the considered case. In the case in which the superconducting properties of the right and left superconducting half-spaces are close together, $H^* \approx H_{c2}/2$.

A completely different mechanism of interaction of the vortex with the boundary of two superconducting half-spaces was considered in [3]. Even if there is no external field and, consequently, no other vortices, a single vortex placed near the boundary between two superconductors can turn out to be entrapped by this boundary. This can easily be understood. The lines of the superconducting current are refracted on going from one superconductor to the other. Actually, the tangential component of the vector potential **A** should be continuous at the boundary. In the opposite case, as can easily be seen, there would be an infinite magnetic field at the boundary. This means that (by virtue of the Ginzburg-Landau equations in the London approximation) the tangential components of the vector $\lambda^2\mathbf{j}$ should be continuous. On the other hand, it follows from the law of charge conservation that the normal components of the vector $\mathbf{j}$ should be continuous at the boundary. Thus we arrive at the following boundary conditions:

$$j_{n1}=j_{n2},\quad \lambda_1^2 j_{t1}=\lambda_2^2 j_{t2}. \quad (2)$$

This also means that the lines of the superconducting current refract at the boundary. That is, the shape of the current lines for a vortex located close to the boundary differs from circular, as is shown schematically in Fig. 1. And this indicates the appearance of a force acting on the normal center—the force of interaction of the vortex with the boundary. Actually, in the case shown in Fig. 1, the superfluid velocity to the left of the normal center is greater than to the right. This means that the pressure difference, which appears because of Bernoulli's law, acts on the center in the negative direction of the x axis, i.e., the vortex is drawn to the boundary.[3)] Further, it is necessary to recognize that in the transition from one material to another, a change takes

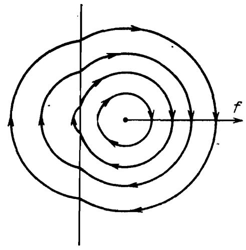

FIG. 1. Shape of a vortex located near the interface of two superconductors. The center of the vortex is in the medium with the smaller penetration depth (see note[3]).

place, the self-energy of the vortex changes, i.e., in the energy of the vortex at a great distance from the boundary.

Such is the state of affairs with regard to the interaction of a single vortex with the interface between two superconductors. It is now natural to take the next step and consider the complete picture of interaction of the entire vortex lattice with the interface, with account taken of interaction of the vortices with one another and with this boundary.

Thus, let the boundary between two superconductors be the plane $x = 0$. The right half-space ($x > 0$) is filled with a superconductor with penetration depth λ_1 and coherence length ξ_1. For the left half-space ($x < 0$) we have, correspondingly, λ_2 and ξ_2. The difference between the superconducting characteristics of the two superconducting half-spaces is assumed to be small in what follows. The external magnetic field H_0 is directed parallel to the z axis, $H_{c1} \ll H_0 \ll H_{c2}$. In both halves, an intermediate state is established, and the inductions B_1 and B_2 are in equilibrium with the external field H_0. Calculation of the equilibrium vortex structure near the superconductor-vacuum interface [4] has shown that the parameter of the vortex lattice is not changed even in the immediate vicinity of the boundary of the superconductor. It is natural to extend this result to our problem and to assume that the periods a_1 and a_2 do not depend on the coordinate x.

Qualitatively, the picture of pinning of the vortex lattice on the interface can be represented as follows. According to [3], for a single vortex, there exists a potential well which extends along the entire interace. One naturally expects that if there exists a vortex lattice, then an entire vortex row falls in this well. This row (owing to the interaction of vortices of this series with all the remaining vortices) now presents an obstacle to the motion of the entire vortex lattice in the direction of the x axis. This means that a superconducting transport current can flow along the interface in the direction of the y axis (see [5]). To determine its critical value, it is necessary to find the maximum force gradient that acts on the entire vortex system if it is displaced as a whole by a small distance along the x axis. We speak of the displacement of the lattice as a whole since we neglect its deformation under the action of the transport current. Actually, in the geometry considered, this deformation is determined by the elastic modulus of the vortex lattice C_{11}, which is equal to [6]

$$C_{11}=\frac{B^2}{4\pi}\frac{\partial H_0(B)}{\partial B}+C_{66}. \tag{3}$$

Inasmuch as the shear modulus $C_{66} \ll C_{11}$, we neglect it in this formula. The relative deformation of the lattice $\delta a/a$ at the critical current I_c along the interface is determined by the formula

$$\frac{\delta a}{a}=C_{11}^{-1}(I_cB/c),$$

here $I_c \sim j_c\lambda$, where j_c is the critical current density. Using (3) and taking $\partial H_0(B)/\partial B \approx 1$, we have

$$\frac{\delta a}{a}=\frac{1}{c}\frac{4\pi}{B}j_c\lambda.$$

If we take the reasonable estimates $j_c \sim 10^5$ A/cm² and $\lambda \sim 10^{-5}$ cm at $B \sim 10^3$ G, then we get $\delta a/a \sim 10^{-3}$. We shall not take such a deformation into account.

The calculation will be carried out for a square lattice, since it is easier than that of the triangular lattice, and the difference in the results of the calculations is quite unimportant for the final result, where we are dealing with orders of magnitude.

We now proceed to formulation of the problem. A vortex lattice is considered, which fills the right and left half-spaces with periods a_1 and a_2, respectively. The problem consists in the calculation of the dependence of the free energy of the system F on the displacement Δ of the entire vortex system as a whole.[4)] In other words, the free energy of the system will be calculated when the vortices are located at points with coordinates $(na_1 + \Delta, ma_1)$ for $n = 0, 1, 2, \ldots$, $m = 0, \pm1, \pm2, \ldots$ and $(na_2 + \Delta, ma_2)$ for $n = -1, -2, \ldots$, $m = 0, \pm1, \pm2, \ldots$. Moreover, we calculate the value of the restoring force $-\partial F/\partial\Delta$ and the critical current density j_c.

2. CALCULATION OF THE FREE ENERGY

According to [3], the free energy F of our system, per unit length along the z axis, can be written in the form

$$F=\frac{\Phi_0}{8\pi}\sum_L H(\mathbf{r}_L),$$

where Φ_0 is the magnetic flux quantum ($\Phi_0 = \pi\hbar c/e$), $\mathbf{r}_L$ is the radius vector of the center of the L-th vortex, $H(\mathbf{r}_L)$ is the field created by all the vortices of the system at the center of the L-th vortex, summation being carried out over all the vortices of the system.

For the calculation of F, we first find the field created by a single vortex located at the point (x_0, y_0). This field is easily found by solving the Ginzburg-Landau equation in the London approximation, with use of the boundary conditions (2) (see [3]). We give the final results:

$$H(x,y;x_0,y_0)=\int_{-\infty}^{\infty}\frac{dk}{2\pi}e^{ik(y-y_0)}H_{\alpha k}(x,x_0),\quad \alpha=1,2, \tag{4}$$

where the following notation is used:

$$H_{1k}(x,x_0)=\frac{\Phi_0}{2u_1\lambda_1^2}\left(e^{-u_1|x-x_0|}+\frac{\lambda_1^2u_1-\lambda_2^2u_2}{\lambda_1^2u_1+\lambda_2^2u_2}e^{-u_1(x+x_0)}\right),\quad x_0>0,\quad x>0;$$
$$H_{2k}(x,x_0)=\frac{\Phi_0}{\lambda_1^2u_1+\lambda_2^2u_2}e^{u_2x-u_1x_0}\quad x_0>0,\quad x<0; \tag{5}$$

$$u_\alpha=\sqrt{k^2+\lambda_\alpha^{-2}},\quad \alpha=1,2. \tag{6}$$

We now write out the expression for the energy of the vortex system, referred to a single vortex row located on the x axis and to a unit height along the z axis:

$$F=\frac{\Phi_0}{8\pi}\sum_{x_0,x_0',y_0'}H(x_0',y_0';x_0,0). \tag{7}$$

Here x_0 and x_0' run through all the values of the abscissa of the vortex lattice sites, independently, y_0' runs through all the values of the ordinates of these sites. Substituting (4) in (7), we carry out calculations similar to those which were performed in [4,5]. Without changing notation, we shall now refer the energy F to a horizontal band of unit width along the y axis and unit height along the z axis. We assume that both the contacting superconductors differ little from one another in their superconducting parameters:

$$|\lambda_1-\lambda_2|\ll\lambda_1, \lambda_2;\quad |a_1-a_2|\ll a_1, a_2;\quad |\xi_1-\xi_2|\xi\ll_1, \xi_2.$$

We introduce the notation: $\delta\lambda = \lambda_1 - \lambda_2$, $\delta a = a_1 - a_2$, $\delta\xi = \xi_1 - \xi_2$ and carry out the following operations:

1) We expand all the formulas in the small parameters $\delta\lambda/\lambda$ and $\delta a/a$.

2) Taking into account the smallness of Δ/a, we omit terms $\sim\Delta\delta\lambda$, $\Delta\delta a$, and $\delta a\delta\lambda$.

3) We omit terms which transform into themselves (i.e., remain unchanged) under the substitution $1 \rightleftarrows 2$ (which corresponds to a change in the sign of Δ). Here we simply change the point assumed to be zero energy.

4) We establish such a starting point for measuring the energy $F(\Delta)$ that $F(+0) = -F(-0)$.

As a result, we get

$$F(\Delta)=-\frac{\Phi_0^2}{16\pi}\operatorname{sign}(\Delta)\left[\frac{\delta\lambda}{\pi a\lambda^3}\ln\left(\frac{2|\Delta|\sqrt{e}}{\xi}+2\frac{\delta a}{a^4}+\frac{\delta a}{2\pi a^2\lambda^2}\ln\frac{a}{2\pi\xi}-\frac{\delta\xi}{2\pi a\lambda^2\xi}\right)\right]. \qquad (8)$$

Using the formula for the induction B in the London approximation: [7,8]

$$H_0\approx B+\frac{\Phi_0}{4\pi\lambda^2}\ln\frac{a}{2\pi\xi},$$

we can easily find the connection between δa and $\delta\lambda$:

$$\frac{\delta a}{a}=-\frac{\delta\lambda}{4\pi\lambda}\frac{a^2}{\lambda^2}\ln\left(\frac{ae^p}{2\pi\xi}\right), \qquad (9)$$

where the parameter p is determined by the characteristics of the contacting materials:

$$p=\frac{1}{2}\varkappa\frac{\delta\xi}{\delta\lambda}. \qquad (10)$$

It is useful to write down one more expression for p, which follows from (10):

$$p=-\frac{1}{2}\left(1-2\frac{\delta H_{cm}}{\delta H_{c2}}\frac{H_{c2}}{H_{cm}}\right)^{-1}. \qquad (11)$$

We can now carry out some simplifications in Eq. (8). We first note that the third term of this formula is small in comparison with the second. Actually, their ratio is equal to $(a^2/4\pi\lambda^2)\ln(a/2\pi\xi)$. If we also assume that $a \sim \lambda$ and $\kappa \sim 100$, then this ratio will be of the order of 1/4. Therefore, we shall neglect the third component in Eq. (8) in what follows. We join the last component in this formula with the first and, using (9) and (10), we obtain

$$F(\Delta)=-\frac{\Phi_0}{8\pi a}\frac{H_{c2}}{\varkappa^2}\frac{\delta\lambda}{\lambda}\operatorname{sign}(\Delta)\ln\left(\frac{2\sqrt{2\pi}|\Delta|}{\sqrt{a\xi}}e^{(p+1)/2}\right). \qquad (12)$$

We now analyze the result. We can rewrite Eq. (12) in a more convenient form if we introduce the field H_1, which is the characteristic field for the pair of superconductors considered:

$$H_1=\frac{e^{-2(p+1)}}{32\pi}H_{c2}.$$

Then Eq. (12) takes the form

$$F(\Delta)=-\frac{\Phi_0}{32\pi a}\frac{H_{c2}}{\varkappa^2}\frac{\delta\lambda}{\lambda}\operatorname{sign}(\Delta)\ln\left[\left(\frac{|\Delta|}{\xi}\right)^4\frac{H_0}{H_1}\right]. \qquad (13)$$

We first consider the case $H_0 \ll H_1$. Then $F(\Delta)$ takes the form shown in Fig. 2a. Actually, all our calculations are correct if the vortex row that is closest to the boundary is still sufficiently far from it, i.e., at $\Delta \gg \xi$. If $\Delta \to 0$, a logarithmic divergence appears which we shall, as always, cut off at distances of the order of ξ. We emphasize that $F(\Delta)$ is the change in the energy of the entire vortex lattice when it is shifted as a whole (rigid shift)

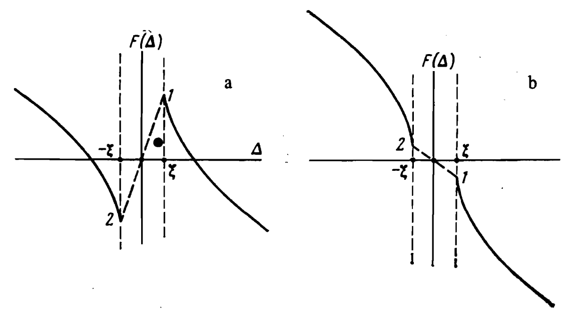

FIG. 2. Free energy of the vortex lattice $F(\Delta)$ as a function of its one-dimensional displacement by an amout Δ: a) $H_0 \ll H_1$, b) $H_0 \gg H_1$.

by a distance Δ along the x axis. The energy $F(\Delta)$ is referred to a unit surface of the interface between the two superconductors. It is clear that in this case the vortex lattice will be in a stable state at the point 2 (see Fig. 2a) and will experience finite and different restoring forces (f_+ and f_-) when the lattice is shifted to the right or left of the equilibrium position.

If $H_0 \gg H_1$, the energy $F(\Delta)$ takes the form shown in Fig. 2b. This means that there exists a restoring force f_- for the displacement of the lattice to the left, but no restoring force f_+. This, of course, does not mean that there will be instability in the vortex lattice as a whole. The edge of the sample is simply not taken into account in the given analysis.

3. CRITICAL CURRENT

Our next task will be the estimate of the critical current, which is determined by the pinning of the vortex lattices on the potential barrier, given by Eq. (13). Real superconducting materials, to which we want to apply our discussions, are not of course two contacting half-spaces. Actually it is as though there were alternating bands of different superconductors. We shall therefore consider such a model further: inside the infinite second superconductor there is a plane parallel plate of the first superconductor, and $\lambda_1 > \lambda_2$. The thickness of the plate is arbitrary, but greater than the penetration depth. An external magnetic field H_0 is imposed parallel to the surface of this plate. The vortex filaments fill the entire space both in the plate and in the second superconductor.

In the case in which $H_0 \ll H_1$, the restoring force f, which arises upon the displacement of the vortex lattice, will now be symmetric and equal to $|f_+| + |f_-|$, where

$$f_+=-\xi^{-1}F(\xi),\quad f_-=-\left.\frac{\partial F}{\partial\Delta}\right|_{\Delta=-\xi}=\frac{\Phi_0 H_{c2}}{8\pi a\varkappa^2}\frac{\delta\lambda}{\lambda\xi}.$$

Thus,

$$f=\frac{\Phi_0}{8\pi a}\frac{H_{c2}}{\varkappa^2}\frac{\delta\lambda}{\lambda\xi}\left(1+\frac{1}{4}\left|\ln\left(\frac{H_0}{H_1}\right)\right|\right).$$

We determine the critical current, as always, by equating the Lorentz force with the restoring force:

$$\frac{1}{c}I_cB=f.$$

Using the connection between H_{c2} and ξ ($2\pi\xi^2H_{c2} = \Phi_0$), we get

$$I_c\approx\frac{cH_{c2}^{3/2}}{4\sqrt{2\pi}\varkappa^2}\frac{\delta\lambda}{\lambda}\left(1+\frac{1}{4}\left|\ln\frac{H_0}{H_1}\right|\right)\Big/\sqrt{H_0}. \qquad (14)$$

The current determined by this formula is given per unit height of the plate (along the z axis) if the current flows along the y axis. If the plate is ideally homogeneous and the only inhomogeneity is this boundary between the two superconductors, then the current flows along this boundary in a layer of thickness of the order of λ. Here we use the result of [5], where it was shown that in a similar situation, the current flows along the wall of the pinned vortices.

If $H_0 > H_1$, then, obviously, the restoring force is $f = f_-$. According to Fig. 2b,

$$f_- = \max \begin{cases} |F(-\xi)/\xi| \\ \dfrac{H_{c2}^{3/2}}{4\sqrt{2\pi}\varkappa^2}\dfrac{\delta\lambda}{\lambda} \end{cases}.$$

Actually, if H_0 is not much larger than H_1, then the slope of the dashed line in Fig. 2b is less than the derivative of $F(\Delta)$ at the point 2 and, in the critical regime, the lattice is shifted to the left to the point 2 and will be maintained there by the restoring force f_-. If the points 2 and 1 are sufficiently far apart, then the slope of the dashed line determines the sufficiently large restoring force.

Thus, in the case $H_0 > H_1$, we have

$$I_c = \max \begin{cases} \dfrac{cH_{c2}^{3/2}}{4\sqrt{2\pi}\varkappa^2}\dfrac{\delta\lambda}{\lambda}\dfrac{1}{\sqrt{H_0}}, \\ \dfrac{cH_{c2}^{3/2}}{16\sqrt{2\pi}\varkappa^2}\dfrac{\delta\lambda}{\lambda}\dfrac{1}{\sqrt{H_0}}\ln\dfrac{H_0}{H_1} \end{cases} \tag{15}$$

4. DISCUSSION OF THE RESULT

Thus, the dependence of the critical current of the plate on the external field is determined by Eqs. (14) and (15). We first consider the physical picture that has been developed (see [4,5]).

A sufficiently thin superconducting plate of the first superconductor is placed in the second superconductor, and $\lambda_1 > \lambda_2$. The external magnetic field H_0 creates a mixed state in both the superconductors and the vortices are parallel to the surfaces of the plate. In the previous section, it was shown that if the entire system of vortices is displaced as a whole transverse to the plate, then the force gradient begins to act on the vortices. This means that one can create such a state in which a finite transport current proceeds along the plate perpendicular to the vortices. Actually, let the surfaces of the plate coincide with the plates $x = \pm d/2$. A stable state of the system of vortices is possible if the mean magnetic field B at $x > d/2$ for $-d/2 \le x \le d/2$ and at $x < -d/2$ will be $B_2 + H_1$, B_1 and $B_2 - H_I$, respectively. Here B_1 and B_2 are the equilibrium inductions for a given field H_0 in the first and second superconductors. It has been assumed, of course, that the value of H_I does not exceed some critical value (which is determined by the critical current). In each of the three enumerated parts of the space (to the right, in the plate, and to the left of it) the density of the vortices will be different, but homogeneous. The jump in the vortex density, which is produced on the boundaries of the plate by H_I, indicates the existence of a transport current which is localized near the surface of the plate. The region of localization is of the order of λ from each of the sides of the surface of the plate. Actually, the vortex density changes at the boundary in discontinuous fashion and the change in the magnetic field takes place at a distance of the order of λ. Thus, the surfaces of the plate serve as conductors of sort for the nondissipative current in the mixed state, which flows along a "corridor" of width about 2λ along the first and second superconductors near each surface of the plate. In the flow of the transport current, a Lorentz force is generated which acts on the vortices and the entire vortex system as a whole is displaced in the direction of this force. Then a force gradient arises immediately, which counterbalances the Lorentz force. The equilibrium is preserved until the current becomes less than critical.

We now sum up the results obtained in the previous section. It is convenient to do this by introducing the normalized current I_{c0} and referring the external field to the quantity H_1:

$$i_c = I_c/I_{c0}, \quad h = H_0/H_1,$$
$$I_{c0} = \frac{cH_{c2}^{3/2}}{4\sqrt{2\pi}\varkappa^2}\frac{\delta\lambda}{\lambda}\frac{1}{\sqrt{H_1}}. \tag{16}$$

Equation (15) now takes the form $(h > 1)$

$$i_c = \max \begin{cases} 1/\sqrt{h} \\ (1/4\sqrt{h})\ln h \end{cases}.$$

It follows unequivocally from this formula that at $1 < h < 54.6$ we have $i_c = \frac{1}{4}h^{-1/2}\ln h$. The quantity 54.6 is the root of the equation $1 = \frac{1}{4}\ln h$. We now write out the final result for the critical current:

$$i_c = h^{-1/2}(1 - \tfrac{1}{4}\ln h), \quad h < 1, \tag{17}$$

$$i_c = h^{-1/2}, \quad 1 < h < 54.6, \tag{18}$$

$$i_c = \tfrac{1}{4}h^{-1/2}\ln h, \quad h > 54.6. \tag{19}$$

Thus, over the entire range of magnetic fields, the dependence of the critical current of the plate on the magnetic field is a monotonically decreasing function. The value of the field H_1 is determined by the difference between the characteristics of the two contacting superconductors.

We now consider a specific case. Let both metals differ only by the concentrations of the impurities, i.e., by the mean free paths of the electrons. This means that $\delta H_{cm} = 0$ and, in accord with (11), $p = -\frac{1}{2}$. Then $H_1 = H_{c2}/(32\pi e) \approx 3.7 \times 10^{-3} H_{c2}$. In this case, all three formulas (17)—(19) will be realized.

We now estimate the order of the critical current density $j_{c0} = I_{c0}/2\lambda$, where I_{c0} is determined by Eq. (16), taking it into account here that for the considered case $H_1 = H_{c2}/32\pi e$ we have

$$j_{c0} = \frac{cH_{cm}}{\lambda\varkappa}\frac{\delta\lambda}{\lambda}.$$

Let $\delta\lambda/\lambda = 0.01$, $\kappa \approx 100$, $H_{cm} \sim 10^3$ Oe, $\lambda \sim 10^{-5}$ cm. Then $j_{c0} \sim 10^5$ A/cm^2, i.e., a difference in the penetration depth of only 1% guarantees a sufficiently large density of the critical current along the interface.

A similar situation arises in a strongly plastically deformed superconductors. For a sufficiently high degree of plastic deformation, a cellular dislocation structure arises (see, for example, [9]). Here the mean free paths of the electron inside the cell and outside will be different. This should lead to somewhat different values of the penetration depth. Narlikar and Dew-Hughes [9] were the first to point out such a difference as the reason for pinning of the vortices on the dislocation cells. It is clear that the results of our calculation can be used to explain the pinning of the vortex lattice by the dislocation cells.

Further, our calculation with $p = -\frac{1}{2}(\delta H_{cm} = 0)$ can explain the strong interaction of the vortices with the grain boundaries. It is shown in some papers that the second critical field is anisotropic (see, for example, [10,11]), and the anisotropies can reach values of several percent. It then follows from our calculation that the boundary between the grains can effectively pin the vortex lattice and serve as the guide for the superconducting transport current.

Finally, we estimate how the dependence of the mean volume force of pinning of the vortices (f_p) should depend on the temperature. For this it is convenient to express the pinning force in terms of H_{c2}. Defining $f_p = c^{-1}j_cB$, neglecting the differences between B and H_0, and assuming that $\delta\lambda/\lambda$ does not depend on the temperature (such would be the case if $\delta\lambda$ were to arise from a difference in the mean free paths of the electrons), we get from Eq. (16)

$$f_p \sim H_{c2}^{5/2}.$$

Fietz and Webb observed just such a dependence of f_p on H_{c2} [12] in strongly deformed Nb-Ti alloys.

We note in conclusion that the difference between our final result (Eqs. (17)—(19)) and the result of [1] (Eq. (1)) arose because the effect of refraction of the lines of flow of the vortices on the interface of two superconductors was not taken into account in [1].

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[2]Of course, if the plane boundary has infinite length in the direction of the vortex filaments, then the pinning of the vortices by the boundary can occur only if the magnetic field will be strictly parallel to the boundary. There cannot be pinning at the smallest inclination of the vortex relative to the boundary—this is understood from simple geometric considerations. Actually, however, the surface of separation has a very complicated and irregular form. In this case, one can always separate the portions of the separation surface which are parallel to the vortex filaments. The vortex filaments will be pinned in these sections.

[3]The arrow in Fig. 1 is mistakenly directed in the opposite direction.

[4]Inasmuch as the final result will be obtained in what follows for the case in which the superconducting characteristics of the contacting materials are close to one another, we shall not take into account the possibility that the right and left systems of vortices can be displaced by different distances (λ_1 and λ_2) under the action of the transport current.

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