Transverse focusing of electrons in a metal by a magnetic field

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The recently observed [7] transverse focusing of electrons in a metal by a magnetic field is theoretically investigated. The nonmonotonic part of the potential difference between the contact onto which the electrons are focused and a peripheral point of the sample is analyzed in detail. It is shown that the nature of the singularities of the potential difference is determined by the shape of the Fermi surface in the vicinity of the extremal diameters, the possible types of singularities are classified, and focusing in a thin plate, when collisions of the electrons with both surfaces of the plate are important, is studied. It is shown how the effect of transverse focusing can be used to investigate the Fermi surface and to study the nature of the interaction of the elctrons with the sample surface.

As is well known, in a strong nonquantizing magnetic field (when the Larmor radius $r = cp_F/eH$ is less than the mean free path l of the carriers) a characteristic scale of the spatial inhomogeneity of the current and the potential is the Larmor radius. In samples of small dimensions, when the mean free path is comparable to some geometric characteristics of the sample, this inhomogeneity leads to the appearance of specific galvanomagnetic size effects. The most thoroughly studied—both theoretically [1,2] and experimentally—are the galvanomagnetic properties of thin plates and wires; in this case in constructing the theory it is sufficient to take into account the inhomogeneity of the current and potential across the sample. Advances in experimental techniques now allow a detailed investigation of the inhomogeneity along a thin sample as well; in particular, we can, with the aid of a felicitously chosen configuration of the electrical contacts, create a situation in which the distance between the contacts is comparable to the magnetic-field dependent trajectory length. Since the contact configuration can be varied within sufficiently wide limits, such experiments may become important sources of information about the properties of metals.

At present the most important manifestation of effects of this sort is electron focusing in thin plates. The focusing effect was first predicted [3] and observed [4,5] by Sharvin and his co-workers. They studied longitudinal electron focusing, when the contacts located on the opposite sides of a thin (~0.4 mm) tin plate turned out to be on the same line of force as the magnetic field was rotated. After this, in [6], the magnetoresistance of such a plate was computed, and those characteristics of the Fermi surface (FS) that could be determined in this way were pointed out. The recently published paper [7] by Tsoĭ demonstrates the solution of a more complex experimental problem: the location of the contacts on the same surface of the sample at a distance from each other of less than the mean free path. It is shown in his paper that the potential difference between such contacts in a magnetic field parallel to the surface is extremely sensitive to the parameters of the electron trajectory: when the distance between the contacts was an integral multiple of an extremal FS diameter multiplied by c/eH, sharp peaks appeared in the potential difference.

In the present paper we carry out a theoretical investigation of transverse electron focusing in parallel and oblique magnetic fields, classify the types of singularities that the potential difference can have, and indicate what information about the Fermi surface and the nature of the electron-sample surface interaction can be obtained in the experimental investigation of the focusing effect.

1. FORMULATION OF THE PROBLEM AND THE COMPLETE SET OF EQUATIONS

Let us consider a metallic plate with two current contacts A, B and one potential contact C (Fig. 1). The current in the circuit AB is given; the quantity that is measured is the potential difference between the contact C and a peripheral point of the sample where the potential is equal to zero. This is precisely how Tsoĭ set up his experiment [7]; we shall indicate possible modifications below. The magnetic field, which is perpendicular to BC, curls the electron trajectories, as shown in the figure; clearly, the magnitude of the potential at C should nonmonotonically depend on the magnetic field. Indeed, when L, the distance between B and C, becomes equal to an extremal FS diameter multiplied by c/eH, the number of electrons capable of reaching C from B in one stage is significantly greater than for other values of the magnetic field, and therefore φ_C should have a spike. As H is increased, there come moments when electrons from the vicinity of the extremal diameter reach C from B by a chain of several trajectories geometrically similar to the trajectory shown above, undergoing specular reflections from the x = 0 plate surface as they move along the chain; in this case φ_C should also have spikes.

The complete system of equations of the problem consists of a linearized—in the weak electric field—kinetic equation (in the τ-approximation)

$$\frac{\partial\psi}{\partial t}+v_i\frac{\partial\psi}{\partial x_i}+\frac{\psi}{t_0}=-v_i\frac{\partial\varphi}{\partial x_i}, \quad (1)$$

the electroneutrality condition

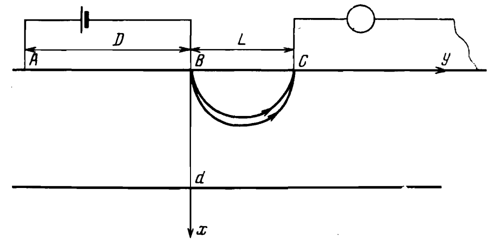

FIG. 1. D is the distance between the current contacts and d is the thickness of the plate.

$$\rho'=0, \quad \text{or} \quad \langle\psi\rangle=\frac{e^2H}{c}\iint_{\epsilon=\epsilon_F}\psi dp_H dt=0, \tag{2}$$

the reflection conditions for the charge carriers

$$\psi_r\ (\mathbf{r},\mathbf{p})=q\psi_i\ (\mathbf{r},\mathbf{p}^*)+\chi(\mathbf{R}) \tag{3}$$

and the expressions for the current flowing through the contacts

$$\langle v_x\psi\rangle|_{x=0,\,d}=i_{0,\,d}(\mathbf{R}). \tag{4}$$

Here the specularity parameter q is equal to the probability of specular reflection from the surface, t_0 is the relaxation time, φ is the potential, and ψ is a correction to the distribution function: $f(\mathbf{r},\mathbf{p}) = f_0(\epsilon) - e\psi\partial f_0/\partial\epsilon$, $f_0(\epsilon)$ being the Fermi distribution function. The current $i(\mathbf{R})$ is the current flowing through all the contacts ($\mathbf{R} = (y, z)$ is a two-dimensional radius vector), but if the resistance of the measuring circuit is sufficiently high, then the current through the contact C is small and, as was pointed out in [6], the distortions introduced by it can be neglected. Therefore, we shall seek the potential $\varphi_C(H)$ at the contact C under the assumption that $i_C(R) = 0$.

Let us seek the solution to Eq. (1) in the following form:

$$\psi(\mathbf{r},\mathbf{p})=\exp\left(\frac{\lambda-t}{l_0}\right)f[\lambda(x-x(t));\ y-y(t),\ z-z(t)]$$

$$-\varphi(\mathbf{r})+\frac{1}{t_0}\int_\lambda^t\exp\left(\frac{t'-t}{t_0}\right)\varphi[\mathbf{r}+\mathbf{r}(t')-\mathbf{r}(t)]dt'. \tag{5}$$

Here f is an arbitrary function, $\lambda = \lambda(x - x(t))$ denotes the moment an electron undergoes its last reflection from any one of the plate surfaces, and $\mathbf{r} - \mathbf{r}(t)$ denotes the characteristic curve of Eq. (1). Let us expand $i(\mathbf{R})$, $\varphi(x, \mathbf{R})$, and $f(\lambda, \mathbf{R} - \mathbf{R}(t))$ in Fourier integrals and substitute them into Eqs. (2)–(4). We obtain as a result a system of three equations—the two reflection conditions (3) (for reflections at x = 0 and at x = d) and the electroneutrality condition (2). Let us write out the reflection condition (3) for x = 0, the remaining equations having similar structures:

$$F_+(t,\mathbf{k})\exp[i\mathbf{k}\mathbf{R}(t)]=q_0\Big\{\sum_i S_i(\mathbf{p}^*,0)\exp\left[\frac{\lambda-t^*}{t_0}+i\mathbf{k}\mathbf{R}(t^*)\right]$$

$$\times F_-(\lambda_i;\mathbf{k})+\sum_i S_i(\mathbf{p}^*,0)\frac{1}{t_0}\int_{\lambda_i}^{t}\exp\left[\frac{t'-t^*}{t_0}+i\mathbf{k}(\mathbf{R}(t^*)-\mathbf{R}(t'))\right]$$

$$\times\Phi[0+x(t')-x(t),\mathbf{k}]dt'\Big\}+\frac{1-q_0}{\langle v_x\rangle_-}\langle v_x\{\ldots\}\rangle_-+\frac{I_0(\mathbf{k})}{\langle v_x\rangle_+}. \tag{6}$$

Here $I(\mathbf{k})$, $\Phi(x, \mathbf{k})$, and $F(\lambda, \mathbf{k})$ are respectively the Fourier transforms of $i(\mathbf{R})$, $\varphi(x, \mathbf{R})$, and $f(\lambda, \mathbf{R})$; the index i, which assumes the values 0 and d, denotes the surface with which the electron collides for the last time; the indices ± denote: for the functions $F_\pm$, the sign of v_x and, for the angle brackets, averaging over that part of the FS where $v_x \gtrless 0$; $\mathbf{p}^*$ is the incident-electron momentum, which is connected with the reflected-electron momentum $\mathbf{p}$ by the specularity conditions $\mathbf{p}_{\parallel} = \mathbf{p}^*_{\parallel}$ [1]; S_i is the Heaviside unit function: $S_0(\mathbf{p}^*, 0) = 1$ if the next collision of an electron of momentum $\mathbf{p}^*$ that has just been reflected from the x = 0 surface is with the same surface, otherwise $S_0(\mathbf{p}^*, 0) = 0$.

For arbitrary relations between r, l, and d and for an arbitrary orientation of the magnetic field, it is not possible to solve the obtained system analytically, and the method of successive approximations must be used (as was done in [2] for the homogeneous distribution of current along the sample). As we shall see, analytic solutions can be obtained in the investigation of the focusing effects.

2. FOCUSING IN A THICK PLATE IN A MAGNETIC FIELD PARALLEL TO THE SURFACE

Let us consider the case when the plate thickness d is significantly larger than the Larmor radius r, the distance L between the contacts, and the mean free path l. Then in a magnetic field that is parallel to the surface, the electrons that collide specularly with the surface can be divided into two groups. The electrons of the first group come to the surface from the interior of the sample and, after one or several collisions, return to the interior. In order for such electrons to exist, there must exist at the Fermi surface open trajectories properly oriented with respect to the surface of the plate. For $d \gg l$ these electrons certainly collide on their way from one surface to the other with other particles inside the sample, and, consequently, they lose the momentum information they acquire in a collision with a surface before they reach and collide with the other surface. Thus, these electrons do not participate in the focusing effect, and we assume in this section that they are altogether absent, i.e., that $S_0(\mathbf{p}^*, 0) = 1$ and $S_d(\mathbf{p}^*, 0) = 0$. The second group consists of electrons that continually collide with the x = 0 surface. It is not difficult to show that their motion is periodic with a period $T_\lambda \le T$, where $T = 2\pi/\Omega$ and $\Omega = eH/m^*c$. Using this circumstance and taking account of the fact that the functions F_+ and F_- are constant along the characteristic curves, we can easily show that $F_+(t, \mathbf{k})$ and $F_-(\lambda_0, \mathbf{k})$ in Eq. (6) coincide, and, consequently, Eq. (6) goes over into an integral equation with a degenerate kernel, which is easy to solve. For $r \ll l$ and, consequently, $T_\lambda \ll t_0$, it is sufficient to limit ourselves to the zeroth approximation in T_λ/t_0. Substituting the obtained $F_+(t, \mathbf{k})$ into the electroneutrality condition, and inverting the Fourier integral, we obtain

$$\varphi_C(H)=\frac{1}{(2\pi)^2}\iint d^2k\exp(-i\mathbf{k}\mathbf{L})I_0(\mathbf{k})\left\langle\frac{1+\exp(i\mathbf{k}\Delta\mathbf{R})}{1-q_0\exp(i\mathbf{k}\Delta\mathbf{R})}\right\rangle_-[1-a(\mathbf{k})]^{-1}. \tag{7}$$

Here $\Delta\mathbf{R} = \mathbf{R}(t^*) - \mathbf{R}(\lambda(t^*))$, $\mathbf{L} = (0, L, 0)$, and

$$a(\mathbf{k})=\frac{1-q_0}{\langle v_x\rangle_-}\langle(v_x\exp(i\mathbf{k}\Delta\mathbf{R}))/(1-q_0\exp(i\mathbf{k}\Delta\mathbf{R}))\rangle_-.$$

Notice that we cannot set $q_0 = 1$ in (7), since the $\mathbf{k}$ integral then diverges. This circumstance has a simple physical meaning: in purely specular scattering the condition that there should be no current flowing through the surface is, as has been shown in [2], automatically fulfilled, the nonzero current $i(\mathbf{R})$ being incompatible with the reflection conditions in this case.

Let us proceed to analyze the formula (7). Using the asymptotic forms of the averages for $|\mathbf{k}\cdot\Delta\mathbf{R}| \le 1$ and for $|\mathbf{k}\cdot\Delta\mathbf{R}| > 1$, we can easily derive the smooth part of the magnetic-field dependence of φ_C. These dependences are as follows (the unimportant factors are omitted):

$$\varphi_C\sim\begin{cases}r^{-2}\ln(L/r), & L<D, \quad (8a)\\ \exp(-L/r), & L>D, \quad (8b)\end{cases}$$

if $\mathrm{Im}\,a(\mathbf{k}) = 0$ (when the magnetic field is parallel to the line BC), or

$$\varphi_C\sim\begin{cases}\ln(L/r), & L<D, \quad (9a)\\ \exp(-L/r), & L>D, \quad (9b)\end{cases}$$

if $\mathrm{Im}\,a(\mathbf{k}) \ne 0$ (for any other orientation of the magnetic field). The plot given in [7] of φ_C as a function of the magnetic field is in good agreement with (9a).

To analyze the nonmonotonic part of $\varphi_C(H)$, let us expand $[1-a(\mathbf{k})]^{-1}$ in a power series in a and limit ourselves to the first term. This is permissible, since the multiplication of the integrand of the $\mathbf{k}$ integral by the factor $[a(\mathbf{k})]^m$ (with $m > 0$), which decreases as $k \to \infty$, can lead only to the weakening of the singularity[2]. Expanding the function $[1-q_0 \exp(i\mathbf{k}\Delta\mathbf{R})]^{-1}$ in a series, we obtain

$$\tilde{\varphi}_C(H) = \frac{1}{(2\pi)^2}\iint d^2k \exp(-i\mathbf{k}\mathbf{L})\,[I_B(\mathbf{k}) + \exp(ik_yD)\,I_A(\mathbf{k})] \times \Big[1 + (1+q_0)\sum_{n=1}^{\infty} q_0^{n-1}\langle\exp(in\mathbf{k}\Delta\mathbf{R})\rangle_-\Big]. \quad (10)$$

Here $I_A(\mathbf{k})$ and $I_B(\mathbf{k})$ are the Fourier transforms of the currents through the contacts A and B, computed under the assumption that the contact is located at the coordinate origin (of course, the resultant current through them is zero: $I_A(0) = -I_B(0)$). In the case when $D \gg L$, it is sufficient to analyze the expression

$$B_n(\mathbf{L}) = \iint d^2k I_B(\mathbf{k}) \exp(-i\mathbf{k}\mathbf{L})\langle\exp(in\mathbf{k}\Delta\mathbf{R})\rangle_-. \quad (11)$$

Let us represent it in the form of a convolution:

$$B_n(\mathbf{L}) = \iint d^2X\, i_B(\mathbf{L}-\mathbf{X})\, b_n(\mathbf{X}),$$
$$b_n(\mathbf{X}) = (2\pi)^{-2}\iint d^2k \exp(-i\mathbf{k}\mathbf{X})\langle\exp(in\mathbf{k}\Delta\mathbf{R})\rangle_- = \langle\delta[\mathbf{X}-n\Delta\mathbf{R}]\rangle_- = \sum_i \Big(\Big|\frac{\partial(n\Delta y, n\Delta z)}{\partial(p_H, t)}\Big|\Big)^{-1}_{\mathbf{X}_i = n\Delta\mathbf{R}} \quad (12)$$

Since $i_B(\mathbf{R})$ is different from zero only at the location of the contact, such a convolution carves out from $b_n(\mathbf{X})$ a certain part, which is such that the smaller the maximum diameter a_{max} of the contact is, the smaller it is. It can be seen from the formula (10) that all the terms of the series have similar analytical structures, and therefore the singularities in $\tilde{\varphi}_C(H)$ should recur periodically with the amplitudes of the neighboring peaks differing by the factor q_0, which has a simple physical meaning.

Let us consider the first peak of $\tilde{\varphi}_C(H)$ ($n = 1$). Clearly, $\tilde{\varphi}$ will have its greatest value when the convolution encompasses the points where the Jacobian vanishes or where it is small. Analysis shows that the following cases are possible.

1. The Jacobian goes to zero as $|\xi d_{extr} - X|^{1/2}$, where d_{extr} is the extremal diameter of the Fermi surface and $\xi = c/eH$). This corresponds to the trajectories shown in Fig. 2, a)—c) (below we shall, when discussing the various types of singularities, indicate the number of the figure where the singularities are shown). Assuming that the current $i_B(\mathbf{R})$ does not have singularities, we find after evaluating the convolution integral for $a_{max} \ll L$ that $\tilde{\varphi} \sim |L-2r|^{1/2}$ for $H \to H_{extr} - 0$, and is equal to zero when $H > H_{extr}$ in the cases 2a and 2b; $\tilde{\varphi} \sim |L-2r|^{1/2}$ for $H \to H_{extr} + 0$, and is equal to zero

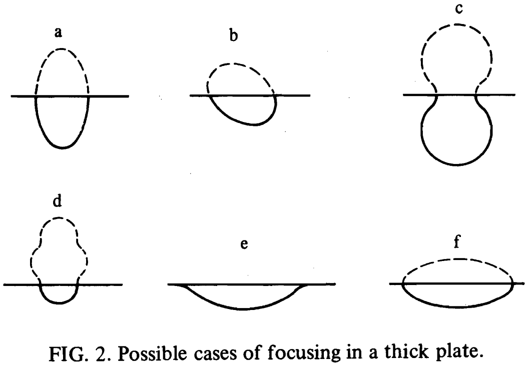

FIG. 2. Possible cases of focusing in a thick plate.

when $H < H_{extr}$ in the case 2c. In the experiment[7], singularities of the type 2a with a period of 6 Oe were observed.

2. The Jacobian goes to zero as $|\xi d_{extr} - X|^{2/3}$. This corresponds to the trajectory shown in Fig. 2d. In this case $\tilde{\varphi} \sim |L-2r|^{1/3}$ for $H \to H_{extr} \pm 0$.

3. The Jacobian does not vanish at the end point (i.e., on the boundary of the region defined by the delta function in (12)). Such a case is possible on open trajectories (Fig. 2e). In this case the potential has a jump with finite values of the derivatives $\partial\tilde{\varphi}/\partial H$ from the left and from the right. Such will be the behavior of the potential in the cases when the Jacobian goes to zero so slowly that the dimension of the region where $b_n(\mathbf{X}) \gg 1$ does not exceed a_{max} (Fig. 2f).

The simplest case is when the magnetic field is perpendicular to the line BC and $v_z = 0$ on the effective trajectory. Then in the vicinity of an extremal diameter Δy behaves in the following manner: $\Delta y = \Delta y_0 - \alpha(\Delta t)^2$ in the cases 2a and 2b; $\Delta y = \Delta y_0 + \beta(\Delta t)^3$ in the case 2d; $\Delta y = \Delta y_0 + \alpha(\Delta t)^2$ in the case 2c. To the trajectories 2a and 2d may correspond the dependences $\Delta y = \Delta y_0 - \alpha(\Delta t)^{2n}$ and $\Delta y = \Delta y_0 + \beta(\Delta t)^{2n+1}$ respectively, and then $\tilde{\varphi}$ behaves like $|L-2r|^{1/2n}$ and $|L-2r|^{1/(2n+1)}$; in these cases the FS in the vicinity of the extremal diameters is flattened. Furthermore, in the case of multiply-connected FS it is possible to orient the magnetic field relative to the crystal axes in such a way as to make the extremal diameters of the various cavities equal, thereby making the appearance of other types of singularities that are combinations of the singularities described in Subsecs. 1—3 possible.

We see that for a real FS the pattern of peaks can be fairly complicated; the situations shown in Fig. 2, a)—d) can occur in succession as the magnitude of the magnetic field is varied, and to them will correspond different periods in the magnetic field. Naturally, such a pattern will also be observable on sufficiently complex open trajectories, such as those shown in Fig. 63 in the book[8] by I. Lifshitz et al.

The variation of the orientation of the surface relative to the crystallographic directions can lead to very abrupt changes in the periods. Such a possibility is illustrated in Fig. 3a, where we show the effective trajectories on a corrugated-cylinder type FS; the solid lines represent the projections of the trajectories onto the plane of the drawing and the magnetic field lies in this plane; the mutually corresponding directions of $\mathbf{H}$ and the projection of the trajectories are marked by the same numbers. In the field $\mathbf{H}_1$ there are two type-2a periods corresponding to the trajectories T_{11} and T_{12}. As $\mathbf{H} \to \mathbf{H}_2$, the period T_2 increases rapidly, while the height of the peaks decreases until it vanishes at $\mathbf{H} = \mathbf{H}_2$. The reason for this is that the end points of the effective trajectory coincide with the stop points, with the result that the period of the electron orbital motion is logarithmically large, so that

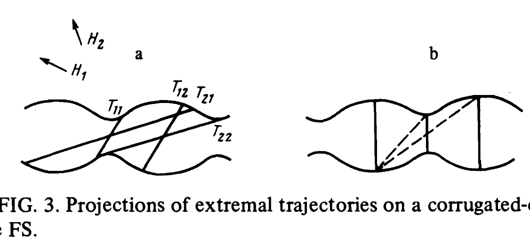

FIG. 3. Projections of extremal trajectories on a corrugated-cylinder type FS.

the electron gets scattered inside the plate and does not participate in the focusing. Upon further rotation of **H** in the plane of the drawing, the period T_2 reappears.

It follows from the foregoing that the investigation of the transverse-focusing effect can become a very convenient method of investigating the FS, since, by varying the orientation of the surface relative to the crystallographic axes, we can, in principle, measure all the extremal diameters of the FS. In particular, it would be desirable to obtain the curve $\varphi_C(H)$ for bismuth with a higher resolution than was achieved in the experiment [7]. The point is that Fig. 2 in [7] shows at a field H = 3.5–4 Oe a peak of amplitude less than that of the principal peaks at fields H = 6 Oe, H = 12 Oe, etc. If more thorough measurements confirm the existence of this peak and clarify its shape, then this will give additional information about the FS of bismuth.

If the magnetic field is not perpendicular to the line BC, then the connection between the peak-recurrence periods and the FS parameters gets, as a rule, significantly complicated. An exception is the case when the shape of the FS in the vicinity of an extremal diameter is close to being cylindrical; for then at not too large ω (ω is the angle between the direction of the magnetic field and the normal to BC) $H_{extr}(\omega) = H_{extr}(0)/\cos\omega$. It is precisely this case that was realized in bismuth [7].

3. FOCUSING IN A PLATE OF FINITE THICKNESS

Let us consider focusing in a plate whose thickness does not exceed the distance L between the contacts. In this case it is necessary to take into account the collisions of the electrons with both surfaces of the plate. Now the motion of each of the electrons is periodic, and therefore an analysis similar to the one carried out in Sec. 2 leads to a system of two integral equations with degenerate kernels for $F_-(x = 0)$ and $F_+(x = d)$. After substituting the solution of the system into the electroneutrality equation we find that the quantities responsible for the $\tilde{\varphi}_C(H)$ peaks have the following forms

$$b^{(1)}(\mathbf{k}) = \left\langle S_0 \frac{1+\exp(i\mathbf{k}\Delta\mathbf{R}_1)}{1-q_0\exp(i\mathbf{k}\Delta\mathbf{R}_1)} \right\rangle_-, \tag{13a}$$

$$b^{(2)}(\mathbf{k}) = \left\langle S_d \frac{1+q_d\exp(i\mathbf{k}\Delta\mathbf{R}_2)}{1-q_0 q_d\exp(i\mathbf{k}\Delta\mathbf{R}_2)} \right\rangle_-. \tag{13b}$$

The trajectories corresponding to $\Delta\mathbf{R}_1$ and $\Delta\mathbf{R}_2$ are shown in Fig. 4, a) and b). Analysis of the Jacobians arising from (13) can be performed in the same manner as was done before, and leads to the following results.

1. If the effective trajectories are parts of the trajectories 2a–2c, then the Jacobian does not vanish at $\mathbf{X} = \Delta\mathbf{R}_{max}$, and $\tilde{\varphi}_C$ has the same jump as in Subsec. 3 of the preceding section. The effective trajectory is shown in Fig. 4c (for the isotropic dispersion law). For such a trajectory the peaks will not be equidistant until Δx_{max} becomes less than the thickness of the plate. The same peaks will be observed if H is replaced by −H; the

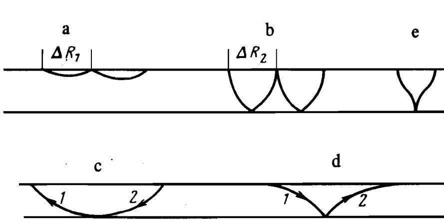

FIG. 4. Possible cases of focusing in a thin plate.

trajectory is shown in Fig. 4d, where the numbers 1 and 2 denote the centrally symmetric parts of the trajectories. In a thick plate, the peaks vanished when the sign of H was changed [7], since the electrons were then twisted in the opposite direction.

2. If the trajectories are sections of the trajectory 2e (such sections may also be on open trajectories, which, for $d \gg l$, did not participate in the focusing—see the beginning of Sec. 2), then the Jacobian goes to zero as in Subsecs. 1 and 2 of Sec. 2, and $\tilde{\varphi}_C(H)$ will then have singularities of the type 2a–2d. The effective trajectory is shown in Fig. 4e. In order for such a singularity to be observable, a definite relation between the plate thickness and the distance between the contacts should be fulfilled; otherwise $\tilde{\varphi}_C(H)$ will have a jump, as in Subsec. 3 of Sec. 2.

As can be seen, the distances between the singularities of $\tilde{\varphi}_C$ for a thin plate are not directly related to the extremal diameters of the FS. However, besides its obvious use for investigating open trajectories (the case 4e), the investigation of the focusing effect in such a formulation allows us to find the curvature of the FS. In fact, for $r \gg L \gg d$, effective trajectories of the type 4c encompass only a small part $\delta\varphi \sim L/r$ of the full period of the trajectory. In this case the quantity $(eH_{extr}/c)(L^2/2d)$ coincides to within small terms of second order in $\delta\varphi$ with the radius of curvature at the location of the extremal trajectory. Let us stress that in this case the magnetic field may be so weak that the Larmor radius may exceed the mean free path; this circumstance is not reflected in the course of the computations, since in a thin plate the small parameter for the solution of Eqs. (2)–(4) is L/l and not r/l.

If in the case 4c the magnetic field is rotated in the plane x = 0, then Δx_{max} on the effective trajectory decreases, and at some critical value of the angle between H and BC the trajectories cease to intersect with the surface x = d. Analysis shows that the connection between the critical angle and the FS parameters is quite unwieldy and cannot, apparently, be conveniently used to investigate FS. In the case of bismuth, where the shape of the electron FS is close to that of a highly prolate ellipsoid, Δx_{max} weakly depends on the orientation of the magnetic field in a considerable interval of angles, and the observation of the indicated effect is difficult.

4. FOCUSING IN AN OBLIQUE FIELD

If the magnetic field is inclined to the sample surface, then the electrons can periodically collide with the sample surface only in special particular cases. In Fig. 3b, we illustrate such a possibility on the example of a corrugated-cylinder type FS; here the magnetic field is parallel to the axis of the cylinder and the dashed lines join points related by the specularity condition. In the overwhelming majority of cases the motion of the electrons is not periodic: they either go away into the interior of the plate after one or several collisions, or, after an infinite number of collisions, reach the line $v_x = 0$, traversing in the process a finite (of the order of r) distance along the sample surface. This means that only the magnitude of the magnetic field at the first peak of $\tilde{\varphi}_C$ can be comparatively easily related to the FS parameters. Mathematically, the connection between $F_+(t)$ and $F_-(\lambda)$ in Eq. (6) gets significantly complicated, and, even for a thick plate, a closed solution is possible to obtain only for purely diffusive reflection. Analysis shows that singularities of the type 2a—2f are possible here too.

In a thin plate ($d \ll l$), when it is necessary to take the collisions of the electrons with both surfaces into account, the foregoing likewise remains valid. In this case the observation of the first peak in $\tilde{\varphi}$ is possible not only in the case when the current and potential contacts are approximately located on the same line of force, but also in the case when there exist open trajectories on which the mean value of **v** over a period is directed along the line joining the contacts (see the beginning of Sec. 2). It is precisely such an effect that was observed in the experiment[5] (see Fig. 3 there) at $\alpha = 4.5°$.

Furthermore, another type of nonmonotonic magnetic-field dependence of $\tilde{\varphi}_C$ arises in an oblique field in a thin plate. Let us analyze it for the simplest case when the magnetic field is perpendicular to the plate and under the assumption that the FS is sufficiently symmetric, so that the specularity conditions reduce to the conditions $p_x^* = -p_x$, $t^* = t$, and that no electron collides twice in succession with one and the same surface of the plate, i.e., that $S_0(\mathbf{p}^*, 0) = 0$; as was pointed out in[1,6], these restrictions are not, in the present case, fundamental restrictions. Then $\lambda_0 = t - d/v_x$, and the equations for F_+ and F_- are integro-difference equations that admit of an exact solution. As a result, it turns out that the nonmonotonic part of the potential contains terms of the type

$$\tilde{\varphi}_n^{\perp}(C) = (2\pi)^{-2}\iint d^2k \exp(-i\mathbf{k}\mathbf{L}) I_B(\mathbf{k}) q_d (q_0 q_d)^n \times \left\langle \exp\left(-\frac{2nd}{t_0 v_x} + i\mathbf{k}\Delta\mathbf{R}_n^{(1)}\right)\right\rangle_-,$$

$$\Delta\mathbf{R}_n^{(1)} = \mathbf{R}(t) - \mathbf{R}\left(t - \frac{2nd}{v_x}\right); \quad n = 1, 2, \ldots \tag{14a}$$

if the current and potential contacts are located on the same surface of the plate, and

$$\tilde{\varphi}_n^{\perp}(C) = (2\pi)^{-2}\iint d^2k \exp(-i\mathbf{k}\mathbf{L}) (q_0 q_d)^n I_B(\mathbf{k}) \times \left\langle \exp\left[-\frac{(2n+1)d}{t_0 v_x} + i\mathbf{k}\Delta\mathbf{R}_n^{(2)}\right]\right\rangle_-,$$

$$\Delta\mathbf{R}_n^{(2)} = \mathbf{R}(t) - \mathbf{R}\left(t - \frac{(2n+1)d}{v_x}\right); \quad n = 0, 1, \ldots, \tag{14b}$$

if the contacts are located on different surfaces. Notice that here we must retain the exponential functions $\exp(-nd/t_0 v_x)$, in contrast to the parallel-field case, where the resultant path traversed by an electron along the chain of extremal trajectories did not depend on the number of the peak and allowance for the finiteness of the mean free path weakened all the peaks to the same extent.

The analysis of the expressions (14a) and (14b) can be performed in the same way as was done in Sec. 2. It turns out that for any $|X| < 2r$ the equation $\mathbf{X} = \Delta\mathbf{R}$ has a denumerable number of roots, while the sum analogous to (12) has a denumerable number of terms; the Jacobians do not vanish at these points and, as averaging over the momenta shows, $\tilde{\varphi}$ has no singularities. For $|X| = 2r$ the solution of the equation $\mathbf{X} = \Delta\mathbf{R}$ is unique, and, although the Jacobian vanishes, no singularity arises in this case too, since v_x and, hence, the function $\exp(-d/t_0 v_x)$ tend to zero on these trajectories; physically, it is clear that the focusing effect is inhibited in this case by the collisions that occur in the interior of the sample. A more thorough investigation shows that $\tilde{\varphi}$ nevertheless has a nonmonotonic part, the dominant contribution to it being made by the set of trajectories on which the factor $\exp(ideH/m^*cv_x)$ is stationary, which, as is well known, is the condition for the occurrence of Sondheimer oscillations. Precisely these oscillations have been observed in experiments[4,5] on tin.

As is well known[9], the case (mentioned in the footnote 1) when the specularity conditions do not effect a one-to-one correspondence between the points of the FS is possible. Two different situations are then possible. If the equations $\mathbf{p}_\parallel = \mathbf{p}_\parallel^*$ have a finite number, and then not too large a number, of solutions (the crystallographic indices of the surface are not too large), then we can, by dividing the set of solutions into pairs with inverse values of v_x, assign to each pair a value q_{ik} that is the analog of the specularity coefficient, thereby transforming the reflection conditions into a system of integral equations with degenerate kernels that admit, as before, of an exact solution. Then on the dependence $\tilde{\varphi}_C(H)$ will arise a finite number of periods. If, on the other hand, the equations $\mathbf{p}_\parallel = \mathbf{p}_\parallel^*$ have a large (infinite, if the indices of the surface are irrational numbers) number of solutions, then an electron can, after a specular reflection, reach practically any point of the FS, and such a situation is equivalent to diffusive reflection.

CONCLUSION

Thus, transverse-focusing observation can become a very convenient tool for investigating FS. From the investigations of focusing in a thick plate we can, by varying the orientation of the surface relative to the crystal axes, measure all the extreme diameters of the FS and, from the nature of the singularities, determine the type of diameter. From the investigations of the focusing in a thin plate we can, by varying the arrangement of the contacts and the orientation of the plate relative to the axes, measure the curvature over the entire FS. From measurements on the dynamics of the peak-recurrence periods of $\tilde{\varphi}(H)$ and the measurement of the peak amplitude we can extract very valuable information on the shape of the cross sections of the FS, on the location of the saddle points, and on the details of the structure of the open trajectories.

The experimental investigations of the transverse focusing effect can also yield valuable information about the nature of the interaction of the electrons with the boundaries of the sample. In the computations we assumed q to be a constant, but the generalization of the formulas (7), (12), and (13) to the case when q is momentum dependent presents no difficulties; in this case, since the contribution to the singularities of $\tilde{\varphi}$ is made by small regions of the FS, it is natural to assume that q is a constant in these regions. Thus, the quantity q figuring in (12) and (13) is $q(\mathbf{p})$ at the ends of the extremal diameter, and, from the ratio of the amplitudes of neighboring peaks, we can determine the specularity coefficient. Focusing in the cases 2a—2d in a magnetic field perpendicular to BC allows the measurement of q in the case of nearly normal incidence. Combining the rotation of the magnetic field in the $x = 0$ plane with the variation of the orientation of the surface relative to the crystal axes, we can, in principle, measure q at all values of the momentum.

1)The cases when the specularity conditions do not guarantee a one-to-one correspondence between the FS points will be discussed below.

2)To verify this, it is necessary to consistently convolute $B_n(X)$ from (11) with $A(X) = (2\pi)^{-2}\iint d^2ka(\mathbf{k})\exp(-i\mathbf{k}X)$, as is done in the text.

Note Added in Proof (November 25, 1974). After the present paper had been sent for publication, there appeared articles in which similar galvanomagnetic effects are investigated. In one paper (J. Low Temp. Phys. **16**, 317 (1974)), J. Clarke and L. A. Schwarzkopf investigate the resistance of bismuth located in a magnetic field parallel to its surface in the case when the contacts are periodically located on the surface of the sample; as can be easily demonstrated with the aid of the apparatus used in the present paper, the singularities in the resistance should be much less pronounced, which was experimentally confirmed; another factor that weakened the influence of the focusing effect on the resistance was a number of inapt characteristics of the configuration of the contacts. In a theoretical paper by C. E. Confalves da Silva (same journal, p. 337), where the resistance is considered in such a formulation, Chambers's method is used, which did not allow the author to consider the case of arbitrary specularity coefficient; furthermore, because of several inaccuracies, a number of the results do not agree with the experiment mentioned above.

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