Stimulated Raman scattering in an inhomogeneous plasma

V. P. Silin and A. N. Starodub

P. N. Lebedev Physics Institute, USSR Academy of Sciences
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The effect is studied of spatial inhomogeneity of a plasma on stimulated Raman scattering due to the appearance in the scattered transverse radiation of a satellite produced by a Langmuir wave. It is shown that such an instability in the vicinity of one quarter of the critical density is an absolute instability. The increase in the threshold of the stimulated Raman scattering due to the effect of plasma inhomogeneity is determined.

In the researches of the authors and Baĭkov[1,2] and Perkins and Flick,[3] it was shown that the parametric instability of a spatially inhomogeneous plasma turns out to be absolute under the conditions of transformation of the transverse pump wave into longitudinal plasma waves. In the linear approximation corresponding to this case, the amplitudes of the plasma excitations, which are produced in the plasma under the action of the pump radiation, grow without limit in time, which is direct proof of the importance of the anomalous nonlinear processes in a parametrically unstable plasma. The importance of the assumption of absolute parametric instability can be understood if we compare it with the results of the research of Rosenbluth[4], which was based on the analysis of one contracted field equations, and which reduces to the assumption of a convective (drift) character of the instability.1) It is natural that the drift of the excitations of the plasma generally makes their affect on the anomalous character of the interaction of the radiation with the plasma less significant.

In this connection, the problem of the character of the parametric instability of a plasma takes on importance. This problem is one in which the pump wave is transformed into a potential plasma wave and a non-potential (transverse) wave, which can be emitted from the plasma. An example of such a process is Raman scattering. It will be shown below that in an inhomogeneous plasma, stimulated Raman scattering also represents absolute parametric instability. In contrast with the two-plasmon decay[1] or the decay of the pump wave into Langmuir and ion-acoustic waves,[2,3] stimulated Raman scattering is an example of an instability which leads to the formation of a reflected wave, the amplitude of which is determined by the amplitude of perturbations that grow in a restricted region close to one-quarter of the critical density.

1. BASIC EQUATIONS

The problem of the parametric transformation of a transverse pump wave into two plasma waves, one of which is also transverse, and the other a longitudinal Langmuir electron wave, under conditions of spatial homogeneity of the plasma, was first considered by Andreev.[6] He showed that in the vicinity of one-quarter of the critical density, i.e., for $\omega_0 \approx 2\omega_{Le}(0)$, where ω_0 is the frequency of the pump wave and $\omega_{Le}(0)$ is the Langmuir electron frequency, the instability threshold, which is due to such stimulated Raman scattering, is determined by the following relation:

$$v_E/c = \nu/\sqrt{3}\omega_{Le}(0), \qquad (1)$$

where v_E is the amplitude of the velocity of oscillations of electrons in the electric field of the pump wave, ν is the frequency of electron-ion collisions, and c is the velocity of light.

The purpose of our research is the study of the effect of the inhomogeneity of the plasma on the stimulated Raman scattering. We assume that the plasma is weakly inhomogeneous, and that the profile of its density is a linear function of the x coordinate, i.e., the electron Langmuir frequency can be written down in the form

$$\omega_{Le}^2(x) = \omega_{Le}^2(0)(1 + xL^{-1}),$$

where L^{-1} is the gradient of the plasma density.

Let a plane polarized electromagnetic wave (the pump wave) be incident on such a plasma. The vector potential of this wave is

$$\mathbf{A}(x, t) = \mathbf{A}_0 \exp\{-i\omega_0 t + ik_0 x\}.$$

As a result of the action of such a pump wave in the plasma, a growth of the perturbations is possible. These perturbations are connected with the transverse and Langmuir waves, and appear as products of the decay of the pump wave. If $\delta\mathbf{v}(\mathbf{r}, t)$ is the velocity of the oscillations of the electrons in the electric field of the transverse wave and $\delta n_e(\mathbf{r}, t)$ is the perturbation of the density of electrons in the Langmuir wave, then the following set of equations holds for $\delta\mathbf{v}(\mathbf{r}, t)$ and $\delta n_e(\mathbf{r}, t)$:[7,8]

$$\frac{\partial^2 \delta n_e}{\partial t^2} + \nu \frac{\partial \delta n_e}{\partial t} + \omega_{Le}^2(x)\delta n_e - v_{Te}^2 \Delta \delta n_e = n(x)\Delta(\mathbf{v}_0 \delta\mathbf{v}),$$

$$\operatorname{rot}\operatorname{rot}\delta\mathbf{v} + \frac{1}{c^2}\frac{\partial^2 \delta\mathbf{v}}{\partial t^2} + \frac{\nu}{c^2}\frac{\partial \delta\mathbf{v}}{\partial t} + \frac{\omega_{Le}^2(x)}{c^2}\delta\mathbf{v} = -\frac{4\pi e^2}{mc^2}\mathbf{v}_0^* \delta n_e. \qquad (2)$$

Here $v_{Te} = (3T_e/m)^{1/2}$ is the thermal velocity of the electrons, $\mathbf{v}_0 = e\mathbf{A}(x, t)/mc$ is the velocity of the oscillations of the electrons in the electric field of the pump wave, and n(x) is the density of the plasma.

In obtaining this set of equations, we have limited ourselves to the situation in which the Landau damping of the Langmuir wave is small in comparison with the damping due to collisions of electrons with ions. We have also neglected the motion of the ions and assumed their distribution to be fixed, inasmuch as their role is unimportant for the considered instability.

We further stipulate that we shall refer everywhere to stimulated Raman back scattering, if the wave vector of the transverse wave is antiparallel to the wave vector of the pump wave, and to stimulated Raman lateral scattering if these vectors are directed at some angle with respect to one another. Equations (2) describe the perturbations of the velocity of oscillations of electrons and density, coupled parametrically through the pump wave. A similar set of equations has been analyzed[5,9] for the case of stimulated Raman back scattering. The authors of these researches obtained an expression for the threshold of this instability under conditions in which the effect of the inhomogeneity appears only as a small correction to the result of the theory of a homogeneous plasma (1) (see below, Eq. (5)). The interpolation formula proposed by Drake and Lee[9], which takes into account the role of the inhomogeneity under conditions in which the threshold of the stimulated Raman scattering is determined by such an inhomogeneity, has not been proved up to the present time. It should also be noted that the equation written down by them[9] for a transverse wave corresponds to that obtained by us below only in the limit as $L \to \infty$ and $T_e \to 0$.

In the second section of this paper, we study the consequences of a set of equations (2) for stimulated Raman back scattering, and a comparison is made between the developed theory and experimental researches.[10,11] In the third section, expressions are obtained for the threshold of the studied instability under conditions in which the transverse wave is propagated at some angle to the wave vector of the pump wave.

2. STIMULATED RAMAN BACK SCATTERING

We write down the set of Eqs. (2) in the form of a single equation for the velocity of oscillations of electrons in the field of a transverse wave. Here we take it into account that, as has been pointed out by Andreev,[6] in the vicinity of one-quarter of the critical density, i.e., at $\omega_0 \approx 2\omega_{Le}(\theta)$, the wave number of the transverse wave is less than the wave number of the pump wave. Therefore, we can obtain the following second-order differential equation for the Laplace transform of the velocity of the oscillations $\delta v(x, \omega)$ from the set (2) (for simplicity, we assume that the polarizations of the pump wave and the transverse wave are identical):

$$\delta v''(x, \omega)+q^2(x)\delta v(x, \omega)=0,$$

$$q^2(x)=\frac{\omega_{Le}^2(0)}{c^2}\frac{(x_+-x)(x-x_-)}{L(x-x_0)},$$

$$x_\pm=\left\{-\frac{3}{2}\frac{v_{Te}^2}{c^2}\pm\left[\left(\frac{3}{2}\frac{v_{Te}^2}{c^2}-2\delta-i\frac{\nu+2\gamma}{\omega_{Le}(0)}\right)^2+3\frac{v_E^2}{c^2}\right]^{1/2}\right\}L,$$

$$x_0=-3\frac{v_{Te}^2}{c^2}L+\left(2\delta+i\frac{\nu+2\gamma}{\omega_{Le}(0)}\right)L, \tag{3}$$

where the frequency of the Langmuir wave ω^l is represented in the form $\omega^l = \omega_{Le}(0)(1+\delta)+i\gamma$, γ being the increment of the studied instability.

We first consider the situation which corresponds to the limit of an almost homogeneous plasma. In this case, the distance between the turning points x_+ and x_- is small in comparison with the distance x_0 from each turning point to the pole. Therefore, we can neglect the dependence of the denominator of the function $q^2(x)$ on the coordinate x. As a result of this neglect, we obtain the equation of the harmonic oscillator with a complex potential. It is obvious that in such an approximation, both the longitudinal Langmuir wave and the transverse wave turn out to be locked between the turning points x_+ and x_-, while the amplitudes of these waves increase in time without limit if $v_E > v_{E\,thr}$, where $v_{E\,thr}$ is determined by Eq. (5). Therefore, such a solution corresponds to an absolute parametric instability, and the wave reflected from the plasma should penetrate through the potential barrier.

For the determination of the correction δ and the increment γ, we can write down the following quantization condition

$$\int_{x_-}^{x_+} q(x)\,dx=\pi\left(n+\frac{1}{2}\right).$$

From this it follows that

$$\delta=\frac{3}{4}\frac{v_{Te}^2}{c^2}+\frac{1}{2}\sqrt{\frac{3}{2}}(2n+1)\frac{1}{k_0L}\left[\frac{\omega_{Le}(0)}{\nu+2\gamma}\right]^{1/2},$$

$$\gamma=-\frac{1}{2}\nu-\frac{2n+1}{4\sqrt{6}}\frac{1}{k_0L}\left(\frac{v_E}{c}\right)^{-1}\left(\frac{\nu}{\omega_{Le}(0)}\right)^{1/2}\omega_{Le}(0)+\frac{\sqrt{3}}{2}\frac{v_E}{c}\omega_{Le}(0). \tag{4}$$

The expression (4) for γ coincides with that obtained in[5] but differs from that given by Drake and Lee.[9]

The condition of vanishing of the increment determines the threshold of stimulated Raman scattering in an inhomogeneous plasma:

$$\frac{v_{E\,thr}}{c}=\frac{1}{\sqrt{3}}\frac{\nu}{\omega_{Le}(0)}+\frac{2n+1}{2\sqrt{2}}\frac{1}{k_0L}\left(\frac{\omega_{Le}(0)}{\nu}\right)^{1/2}. \tag{5}$$

This result is valid if $(k_0L)^{-1} \ll (\nu/\omega_{Le}(0))^{3/2}$. Therefore the next component in formula (5) is a small correction.

It is seen that the threshold of instability increases under the action of the inhomogeneity of the plasma. This increase in the threshold is due to the finiteness of the region in which the parametric growth of the waves takes place (cf.[3]).

If the effect of the inhomogeneity of the plasma is so great that the threshold of the stimulated Raman scattering is determined by such an inhomogeneity, then the consideration of the limit of a collision-free plasma as $\nu \to 0$ is of interest. It is obvious that the turning points x_+ and x_-, and also the pole x_0, the values of which are determined by just such expressions as

$$x_\pm=-\frac{3}{2}\frac{v_{Te}^2}{c^2}L\pm\left[\left(\frac{3}{2}\frac{v_{Te}^2}{c^2}-2\delta\right)^2+3\frac{v_E^2}{c^2}\right]^{1/2}L,$$

$$x_0=-3v_{Te}^2L/c^2+2\delta L, \tag{6}$$

lie in this case on the real axis, while the pole x_0 is located half-way between the points x_+ and x_-.

In the region of large values of the coordinates, i.e., as $|x| \to \infty$, the "potential $q^2(x)$ takes the following form:

$$q^2(x)=-\omega_{Le}^2(0)x/c^2L.$$

It then follows that as $x \to -\infty$, the potential $q^2(x) > 0$, while, as $x \to +\infty$, we have $q^2(x) < 0$. This behavior of the potential $q^2(x)$ indicates that it is necessary to seek such solutions of Eq. (3) which correspond to an exponentially decaying wave at $x > x_+$ and a wave which travels to the left in the direction of decreasing density at $x < x_-$.

We introduce the dimensionless coordinate $\eta = x/L$ and rewrite Eq. (3) in the following form:

$$\delta v''(\eta, \omega)+q^2(\eta)\delta v(\eta, \omega)=0,$$

$$q^2(\eta)=\frac{\omega_{Le}^2(0)}{c^2}L^2\frac{(\eta_+-\eta)(\eta-\eta_-)}{\eta-\eta_0} \tag{7}$$

where $\eta_\pm = x_\pm/L$ and $\eta_0 = x_0/L$. Since, for $\eta > \eta_+$,

$$\int_{\eta_+}^{\eta} q(\eta)\,d\eta=+i\frac{2}{3}(\eta-\eta_+)^{3/2}\mu_1^{1/2},\quad \mu_1=\left[\frac{\omega_{Le}(0)}{c}L\right]^2\frac{\eta_+-\eta_-}{\eta_+-\eta_0},$$

it is evident that the asymptotic solution of Eq. (7)

$$\delta v_0(\eta,\omega)=\frac{\delta v_0}{|q(\eta)|^{1/2}}\exp\left\{i\int_{\eta_+}^{\eta}q(\eta)d\eta\right\},\quad \eta>\eta_+, \tag{8}$$

represents an exponentially damped wave. In order to continue this solution in the region $\eta_0 < \eta < \eta_+$, where the asymptotic solution of Eq. (7) is of the form

$$\delta v_1(\eta,\omega)=\frac{\delta v_1^{(1)}}{q^{1/2}(\eta)}\exp\left\{i\int_{\eta_+}^{\eta}q(\eta)d\eta\right\}+\frac{\delta v_1^{(2)}}{q^{1/2}(\eta)}\exp\left\{-i\int_{\eta_+}^{\eta}q(\eta)d\eta\right\}, \tag{9}$$

it is necessary to match Eqs. (8) and (9) with the aid of the Airy function.

Actually in a small region about the linear turning point η_+, Eq. (7) has an asymptotic solution in the form of an Airy function of the first kind:

$$\delta v_2(\eta,\omega)=\delta v_2\,\mathrm{Ai}[\mu_1^{1/3}(\eta-\eta_+)]$$

(since the solution should fall off at $\eta > \eta_+$, the second, increasing solution has been omitted). Since $k_0L \gg 1$, where $k_0 = \sqrt{3}\,\omega_{Le}(0)/c$ is the wave number of the pump wave, there is a neighborhood of the point η_+[12] such that both the solution $\delta v_0(\eta,\omega)$ and the solution $\delta v_2(\eta,\omega)$ are valid in this region. Equating them, we find that $\delta v_2 = 2\sqrt{\pi}\,\mu_1^{-1/6}\delta v_0$. It is understood that to the left of the turning point η_- there is also a region in which the solutions $\delta v_1(\eta,\omega)$ and $\delta v_2(\eta,\omega)$ are valid simultaneously. By requiring them to be identical, we determine the coefficients $\delta v_1^{(1,2)}$. They turn out to be equal to $\delta v_1^{(1)} = \delta v_0 e^{-i\pi/4}$, $\delta v_1^{(2)} = \delta v_0 e^{i\pi/4}$. It then follows that the components in Eq. (9) are shifted in phase by $\pi/2$ relative to one another. Therefore, the solution (9) represents a standing wave in the region $\eta_0 < \eta < \eta_+$.

In the vicinity of the pole η_0 the asymptotic solution (9) is written in the form

$$\delta v_1(\eta,\omega)=\frac{\delta v_1^{(1)}}{q^{1/2}(\eta)}e^{-i\Psi}\exp\left\{i\int_{\eta_0}^{\eta}q(\eta)d\eta\right\}+\frac{\delta v_1^{(2)}}{q^{1/2}(\eta)}e^{i\Psi}\exp\left\{-i\int_{\eta_0}^{\eta}q(\eta)d\eta\right\},$$

$$\Psi=\int_{\eta_0}^{\eta_+}q(\eta)d\eta. \tag{10}$$

We continue this solution in the region $\eta < \eta_-$. For this purpose, it is convenient to consider the plane of the complex variable η. The necessary continuation of the solution (10) in the region $\eta < \eta_-$ is obtained as a result of going around the pole η_0 and around the turning point η_- in this plane. It must be taken into account that in so doing we cross the Stokes line on which the coefficients in the expression (10) change jumpwise. The corresponding Stokes distance β was found by Denisov.[13] It turns out to be equal to

$$\beta=e^{i\pi/2}[1-e^{-2i\Phi}],\quad \Phi=\int_{\eta_-}^{\eta_0}q(\eta)d\eta.$$

Finally, on going from η_0 to η_-, the coefficients $\delta v_1^{(1)}$ and $\delta v_1^{(2)}$ must be multiplied by $e^{-i\Phi}$ and $e^{i\Phi}$, respectively. Taking these remarks into account, we can write down the asymptotic solution of Eq. (7) in the region $\eta < \eta_-$ in the form

$$\delta v_3(\eta,\omega)=\frac{1}{q^{1/2}(\eta)}[\delta v_1^{(1)}e^{-i(\Psi+\Phi)}+\delta v_1^{(2)}\beta e^{i(\Psi+\Phi)}]\exp\left\{i\int_{\eta_-}^{\eta}q(\eta)d\eta\right\}$$

$$+\frac{1}{q^{1/2}(\eta)}\delta v_1^{(2)}e^{i(\Psi+\Phi)}\exp\left\{-i\int_{\eta_-}^{\eta}q(\eta)d\eta\right\}. \tag{11}$$

Inasmuch as there is no wave that travels to the right at $\eta < \eta_-$, the condition of the vanishing of the coefficient in the first component in Eq. (11), along with the relation $\delta v_1^{(1)} = \delta v_1^{(2)}e^{-i\pi/2}$ found above, gives the following dispersion equation:

$$e^{-2i(\Psi+\Phi)}=1-e^{-2i\Phi}. \tag{12}$$

Separating the real and imaginary parts in this equation, we find the desired quantization rule:

$$\int_{x_0}^{x_+}q(x)dx=\pi m,\quad i\int_{x_-}^{x_0}q(x)dx=\ln\sqrt{2}. \tag{13}$$

Performing the indicated integration, we write down the condition (13) in the form of the following set of two equations for the determination of the corrections to the frequency of the Langmuir wave and the threshold of stimulated Raman scattering:

$$(x_+-x_-)^{1/2}\{(2r^2-1)\mathrm{E}(r)+(1-r^2)\mathrm{K}(r)\}=\frac{3}{2}\pi m\frac{c}{\omega_{Le}(0)}L^{1/2},$$

$$(x_+-x_-)^{1/2}\{(2p^2-1)\mathrm{E}(p)+(1-p^2)\mathrm{K}(p)\}=\frac{3}{2}\ln\sqrt{2}\frac{c}{\omega_{Le}(0)}L^{1/2}, \tag{14}$$

where E(r) and K(r) are the complete elliptic integrals of the first and second kinds, respectively,

$$r^2=(x_+-x_0)(x_+-x_-)^{-1},\quad p^2=(x_0-x_-)(x_+-x_-)^{-1},$$

and the values of x_+, x_-, x_0 are determined by Eqs. (6).

Since the arguments p and r are connected by the condition $p^2 + r^2 = 1$, we obtain, dividing one of the Eqs. (14) by the other, an equation for the determination of r:

$$\frac{(2p^2-1)\mathrm{E}(p)+(1-p^2)\mathrm{K}(p)}{(2r^2-1)\mathrm{E}(r)+(1-r^2)\mathrm{K}(r)}=\frac{\ln\sqrt{2}}{\pi m}, \tag{15}$$

which can be solved, say, graphically.

Let r_m be a solution of Eq. (15) corresponding to the mode number m. Then the expressions for the correction δ and the threshold of the instability considered above can be written down in the form

$$\delta=\frac{3}{4}\frac{v_{Te}^2}{c^2}+\frac{\sqrt{3}}{4}\alpha_m\frac{r_m^2-p_m^2}{r_mp_m}\left[\frac{c}{\omega_{Le}(0)L}\right]^{2/3},\quad \frac{v_{E,\mathrm{thr}}}{c}=\alpha_m\left[\frac{c}{\omega_{Le}(0)L}\right]^{2/3}$$

$$\alpha_m=3^{1/4}2^{-2/3}(\pi m)^{2/3}[(2r^2-1)\mathrm{E}(r)+(1-r^2)\mathrm{K}(r)]^{-2/3}. \tag{16}$$

Equation (16) for the threshold was obtained under conditions in which the threshold is determined by the inhomogeneity of the plasma, i.e., the effect of dissipative effects connected with electron-ion collisions can be neglected. Therefore, the condition for the validity of Eqs. (16) is

$$(k_0L)^{-2/3}\gg(\nu/\omega_{Le}(0)),$$

i.e., the limit that is the opposite to that for which Eq. (5) was obtained.

The solution of Eq. (15) for m = 1 yields $r_1 = 0.95$. Therefore, the minimal threshold which corresponds to the excitation of this mode is equal to

$$v_{E,\mathrm{thr}}/c=0{,}43[c/\omega_{Le}(0)L]^{2/3}. \tag{17}$$

We compare this result with the threshold (1) of stimulated Raman scattering in a homogeneous plasma. The ratio of the threshold energy fluxes of the pump wave in a strongly inhomogeneous and a homogeneous plasma is equal to

$$\mu\approx1.6\frac{\omega_{Le}^2(0)}{\nu^2}\left[\frac{c}{\omega_{Le}(0)L}\right]^{4/3}.$$

It then follows that under such conditions of interest for obtaining controlled thermonuclear reactions with the help of a laser the threshold of stimulated Raman scattering is determined by the inhomogeneity of the plasma. Actually, if the plasma density is $n = 10^{21}$ cm^{-3}, the temperature is $T = 1$ keV, and the size of the inhomogeneity of the plasma is $L = 5\times 10^{-3}$ cm, then $\mu \approx 10^3$.

We now show that even in this case we are dealing with an absolute instability. For simplicity, we limit ourselves to situations in which the increment of the instability is small, i.e., the field of the pump is only slightly greater than the threshold field (17). It is evident that the pole x_0 and the turning points x_+ and x_-, which are now determined by formulas ([illegible]), have a small imaginary part and are located close to the real axis. Therefore, it follows from the dispersion equation that the first of Eqs. (13) does not change, while a small component that is linear in γ appears in the second. It then follows that the correction δ to the frequency of the Langmuir wave remains as before, i.e., it is equal to the expression (16), and we have for the increment

$$\gamma=-\frac{1}{2}\nu+0.07\frac{v_E-v_{E,\,\mathrm{thr}}}{v_{E,\,\mathrm{thr}}}\omega_{Le}(0). \tag{18}$$

The first term in Eq. (18) is small, by virtue of the assumptions that have been made. It is seen that actually, if the amplitude of the pump exceeds the threshold value (16), then $\gamma > 0$. Thus, we have established that the stimulated Raman scattering near one-quarter the critical density is an absolute instability.

If $E^{tr}(x, \omega)$ and $E^{l}(x, \omega)$ are the amplitudes of the electric fields of the transverse and longitudinal waves, then, with the help of Eqs. (2), we can show that these amplitudes are expressed in terms of the velocity of the oscillations $\delta v(x, \omega)$ in the following fashion:

$$E^{tr}(x,\omega)=\frac{m\omega_{Le}(0)}{e}\delta v(x,\omega), \tag{19}$$

$$E^{l}(x,\omega)=\frac{m}{e}\frac{c^2}{v_E}\int dx\, e^{ik_0x}\left\{\frac{\partial^2}{\partial x^2}-\frac{\omega_{Le}^2(0)}{c^2}\left[\frac{x}{L}+2\delta+i\frac{\nu+2\gamma}{\omega_{Le}(0)}\right]\right\}\delta v(x,\omega).$$

With the help of these relations and the complete equation for $\delta v(x, \omega)$, which takes the form

$$\begin{aligned}&\delta v^{\mathrm{IV}}-2ik_0\delta v'''-\left\{k_0^2+\frac{\omega_{Le}^2(0)}{v_{Te}^2}\left(\frac{x}{L}-2\delta-i\frac{\nu+2\gamma}{\omega_{Le}(0)}\right)\right\}\delta v''\\&+2ik_0\left\{\frac{\omega_{Le}^2}{c^2}\left(\frac{x}{L}+2\delta+i\frac{\nu+2\gamma}{\omega_{Le}(0)}\right)+\frac{\omega_{Le}^2(x)}{c^2}\frac{v_E^2}{v_{Te}^2}\right\}\delta v'\\&+\left\{\frac{\omega_{Le}^2(0)}{c^2}\left(\frac{x}{L}+2\delta+i\frac{\nu+2\gamma}{\omega_{Le}(0)}\right)\left[k_0^2+\frac{\omega_{Li}^2(0)}{v_{Te}^2}\left(\frac{x}{L}-2\delta-i\frac{\nu+2\gamma}{\omega_{Le}(0)}\right)\right]\right.\\&\left.-k_0^2\frac{\omega_{Le}^2(x)}{c^2}\frac{v_E^2}{v_{Te}^2}\right\}\delta v=0,\end{aligned} \tag{20}$$

we can observe how the amplitude of the transverse wave changes in the region $x < x_-$.

If the value of the coordinate x is sufficiently large, i.e.,

$$\frac{x}{L}\gg\frac{v_{Te}^2}{c^2},\ \frac{v_E}{c},\ \frac{\nu+2\gamma}{\omega_{Le}(0)},$$

then Eq. (20) can be rewritten in the form

$$\delta v''-\frac{\omega_{Le}^2(0)}{c^2}\frac{x}{L}\delta v=\frac{v_{Te}^2}{\omega_{Le}^2(0)}\frac{L}{x}F(x,\delta v), \tag{21}$$

$$F(x,\delta v)=\delta v^{\mathrm{IV}}-2ik_0\delta v'''-k_0^2\delta v''+2ik_0\frac{\omega_{Le}^2(0)}{c^2}\frac{x}{L}\delta v'+k_0^2\frac{\omega_{Le}^2(0)}{c^2}\frac{x}{L}\delta v.$$

Inasmuch as the right side of Eq. (21) is small, its solution can be sought by the perturbation theory method, i.e.,

$$\delta v=\delta v_0+\delta v_1+\dots$$

Substituting this expansion in Eq. (21), we obtain

$$\delta v_0(\xi,\omega)=\frac{\delta\bar v}{q^{1/2}(\xi)}\exp\left\{i\int_{\xi_-}^{\xi}q(\xi)\,d\xi\right\},$$

$$\delta v_1''(\xi,\omega)+q^2(\xi)\delta v_1(\xi,\omega)=-v_{Te}^2\omega_{Le}^{-2}(0)L^2\xi^{-1}F(\xi,\delta v_0), \tag{22}$$

where we have for convenience introduced the dimensionless coordinate $\xi = -x/L$, and $q^2(\xi) = \omega_{Le}^2(0)c^{-2}L^2\xi$.

It follows from Eqs. (22) and from expression (19) for $E^{l}(x, \omega)$ that δv_0 does not make a contribution to the longitudinal field and therefore, the amplitude of the electric field of a plasma wave is determined by the correction δv_1:

$$E^{l}(\xi,\omega)\approx-\frac{m}{e}\frac{c^2}{v_E}\frac{v_{Te}^2}{\omega_{Le}^2(0)}L\int_{\xi_-}^{\xi}\frac{d\xi}{\xi}\exp\{-ik_0L\xi\}F(\xi,\delta v_0).$$

It is easy to show that

$$F(\xi,\delta v_0)=\frac{\omega_{Le}^4(0)}{c^4}\{\xi^4+2\sqrt{3}\,\xi^3+(3-2\sqrt{3})\xi^2+3\xi\}\delta v_0(\xi,\omega).$$

Therefore, we can estimate the ratio of the amplitude of the transverse and Langmuir waves. In the region $\xi \sim 1$, for example, it turns out to be equal to

$$\frac{E^{l}(\xi,\omega)}{E^{tr}(\xi,\omega)}=-\frac{v_{Te}^2}{v_Ec}\left[\frac{\omega_{Le}(0)}{c}L\right]^{1/2}\cdot\int_{\xi_-}^{1}d\xi\,\frac{\xi^3+2\sqrt{3}\,\xi^2+(3-2\sqrt{3})\xi+3}{\xi^{1/4}}\exp\left\{-ik_0L\left[\xi-\frac{2}{3\sqrt{3}}\xi^{3/2}\right]\right\}.$$

Inasmuch as the derivative $\varphi'(\xi)$ of the phase $\varphi(\xi) = [\xi - 2\xi^{3/2}/3\sqrt{3}]k_0L$ does not vanish on the cut $[\xi_-, 1]$ it follows that, integrating by parts, we can estimate this integral. With accuracy $\sim(k_0L)^{-1}$, this estimate takes the form

$$\frac{E^{l}(\xi,\omega)}{E^{tr}(\xi,\omega)}\sim\begin{cases}-i\dfrac{c}{v_E}\left(\dfrac{v_{Te}}{c}\right)^{3/2}(k_0L)^{-1/2}, & \dfrac{v_{Te}^2}{c^2}(k_0L)^{2/3}>1,\\[2ex] -i\dfrac{v_{Te}^2}{c^2}\left(\dfrac{c}{v}\right)^{5/4}(k_0L)^{-1/2}, & \dfrac{v_{Te}^2}{c^2}(k_0L)^{2/3}<1.\end{cases}$$

Replacing v_E here with $V_{e\,\mathrm{thr}}$, it is easy to show, by means of Eq. (17), that in the region $\xi \sim 1$ we have $E^{l}(\xi, \omega) \ll E^{tr}(\xi, \omega)$. We also note that E^{l} and E^{tr} are shifted by $\pi/2$.

Inasmuch as $E^{l} \sim E^{tr}$ at $x_+ > x > x_0$, as follows from Eqs. (2) and the asymptotic solution (9), we can represent the physical picture of the interaction of the pump wave with the inhomogeneous plasma in the following way: In the vicinity of one-quarter of the critical density $x_+ > x > x_0$ an increase occurs in the transverse and Langmuir waves, which are formed as a result of the decay of the pump wave. The growing perturbations penetrate through the barrier $x_- < x < x_0$ into the region $x < x_-$ and propagate in the direction of decreasing density of the plasma. Here the amplitude of the Langmuir wave falls off, while the amplitude of the transverse wave does not change appreciably, and tends to some limiting value. Using Eq. (19), it is easy to prove that the wave number of the transverse wave at the point $x = -L$, i.e., on entering the vacuum from the plasma, is equal to $k = \omega_{Le}(0)/c$.

For an estimate of the threshold flux of the pump wave for stimulated Raman scattering, we can point out the following interpolation formula (cf. with[9]):

$$\frac{v_{E\,\mathrm{thr}}}{c}=\frac{1}{\sqrt{3}}\frac{\nu}{\omega_{Le}(0)}+0.43\left[\frac{c}{\omega_{Le}(0)L}\right]^{2/3}. \tag{23}$$

It is interesting to compare Eq. (23) with the results of experimental researches.[10,11] In both experiments, the plasma was formed as a result of the irradiation of a target of lithium deuteride with powerful radiation from a neodymium laser. By comparing the data of these researches, we can note that right up to fluxes of $q \sim 10^{15}\,\mathrm{W/cm^2}$ the reflection coefficient is found at the level 0.1–0.5%,[10] while in the region of fluxes

$q = (0.25-0.50) \times 10^{16}\ \mathrm{W/cm^2}$, the reflection coefficient increases appreciably.[11] The authors of[11] attempted to connect such a growth of the reflection coefficient with the development of instability in the plasma, due to stimulated Mandelshtam-Brillouin scattering. However, inasmuch as the threshold of this instability amounts to $3 \times 10^{11}\ \mathrm{W/cm^2}$, the growth in the reflection coefficient with growth in the radiation flux should also have been observed in[10].

It is possible to explain the growth in the reflection coefficient for $q > 2 \times 10^{15}\ \mathrm{W/cm^2}$ with the help of the instability caused by the stimulated Raman scattering. Inasmuch as the temperature of the electrons of the plasma amounted to 1 keV, the threshold of such an instability is due to the inhomogeneity of the plasma. If $L = 3 \times 10^{-3}$ cm, then the growth in the reflection coefficient takes place in that region of fluxes whose value exceeds q_{thr}. It follows from Eq. (18) that the instability connected with stimulated Raman scattering manages to develop in the course of the laser pulse. Actually, if the pump field surpasses the threshold value by 10%, then the product of the increment γ and the duration of the pulse τ, which in[11] amounted to 120×10^{-12} sec, is equal to $\gamma\tau \approx 10^2$.

Thus we can assume that the observed growth in the reflection coefficient for fluxes $q > 2.5 \times 10^{15}\ \mathrm{W/cm^2}$ is connected with the development in the plasma of an instability due to stimulated Raman scattering. The spectrum of the reflected signal should then include a line that corresponds to the harmonic $\frac{1}{2}\omega_0$, where ω_0 is the frequency of the pump wave.

3. STIMULATED RAMAN SCATTERING AT AN ANGLE TO THE DIRECTION OF PROPAGATION OF THE PUMP WAVE

The theory developed above can be applied to the case of stimulated Raman lateral scattering. Actually, let the wave vectors of the pump wave and the transverse wave lie in the xy plane, and let the vectors of the electric fields of these waves be directed along the z axis. As in the second section, we restrict ourselves to the consideration of the neighborhood of one-quarter the critical density, i.e., $k_0 \gg k$, where k_0 is the wave number of the transverse wave.

Under these assumptions, the equation for the velocity of the oscillations of the electrons $\delta v(x, \omega)$ can be written in the form

$$\delta v''(x,\omega) + Q(x)\,\delta v(x,\omega) = 0,$$
$$Q(x) = \frac{\omega_{Le}^2(0)}{c^2}\,\frac{(x_+ - x)(x - x_-)}{L(x - x_0)},$$
$$\frac{x_\pm}{L} = -\frac{1}{2}\,\frac{k_0^2 v_{Te}^2 + k_y^2 c^2}{\omega_{Le}^2(0)} \pm \left\{ \frac{(k_0^2 v_{Te}^2 + k_y^2 c^2)^2}{4\omega_{Le}^4(0)} + \frac{k_0^2 v_E^2}{\omega_{Le}^2(0)} \right.$$
$$\left. + \left[\frac{k_y^2 c^2}{\omega_{Le}^2(0)} + 2\delta + i\frac{\nu + 2\gamma}{\omega_{Le}(0)} \right] \left[-\frac{k_0^2 v_{Te}^2}{\omega_{Le}^2(0)} + 2\delta + i\frac{\nu + 2\gamma}{\omega_{Le}(0)} \right] \right\}^{1/2},$$
$$\frac{x_0}{L} = -\frac{k_0^2 v_{Te}^2}{\omega_{Le}^2(0)} + 2\delta + i\frac{\nu + 2\gamma}{\omega_{Le}(0)}. \qquad (24)$$

It is seen that this equation is similar to that investigated in Sec. 2. Therefore, the quantization conditions obtained above are applicable also to the study of Eq. (24).

If the inhomogeneity of the plasma depends weakly on the stimulated Raman scattering, then we can write the following expressions for the correction δ to the frequency of the Langmuir wave and the instability threshold:

$$\delta = \frac{1}{4}\,\frac{k_0^2 v_{Te}^2 - k_y^2 c^2}{\omega_{Le}^2(0)} + \frac{1}{2}\sqrt{\frac{3}{2}}(2n+1)\frac{1}{k_0 L}\left(\frac{\omega_{Le}(0)}{\nu}\right)^{1/2},$$
$$\frac{v_{E,\,thr}}{c} = \frac{1}{\sqrt{3}}\,\frac{\nu}{\omega_{Le}(0)} + \frac{2n+1}{2\sqrt{2}}\,\frac{1}{k_0 L}\left(\frac{\omega_{Le}(0)}{\nu}\right)^{1/2}. \qquad (25)$$

It is seen that the stimulated Raman lateral scattering threshold coincides with the back scattering threshold (5). Only the correction to the frequency of the Langmuir wave changes. The expressions (25) are valid if $(k_0 L)^{-2/3} \ll (\nu/\omega_{Le}(0))$.

In the opposite limiting case, the instability threshold is determined by the inhomogeneity of the plasma:

$$v_{E,\,thr}/c = \alpha_m [c/\omega_{Le}(0)L]^{2/3}. \qquad (26)$$

Comparison of this result with Eq. (16) shows that even at $(k_0 L)^{-2/3} \gg (\nu/\omega_{Le}(0))$ the thresholds of stimulated Raman back scattering and lateral scattering are identical. The correction to the frequency of the Langmuir wave is determined in this limit as

$$\delta = \frac{1}{4}\,\frac{k_0^2 v_{Te}^2 - k_y^2 c^2}{\omega_{Le}^2(0)} + \frac{\sqrt{3}}{4}\alpha_m \frac{r_m^2 - p_m^2}{r_m p_m}\left[\frac{c}{\omega_{Le}(0)L}\right]^{2/3},$$

where α_m is determined by Eq. (14), $p_m^2 = 1 - r_m^2$, r_m is the solution of Eq. (15).

By repeating the discussions given in Sec. 2, we can show that even in lateral scattering we are dealing with an absolute instability.

CONCLUSION

In this research, the instability due to stimulated Raman scattering has been analyzed in the vicinity of one-quarter of the critical density, i.e., at $\omega_0 \approx 2\omega_{Le}(0)$, where ω_0 is the frequency of the pump wave. The obtained results (5), (16), (25), (26) for the thresholds of back and lateral stimulated Raman scattering provide the basic content of the paper. These results allow us to show the following interpolation formula for the threshold flux of the pump wave (in $\mathrm{W/cm^2}$):

$$q_{thr} = 1.79\cdot 10^{-31} z^2 n_e^2 T^{-3} \lg^2[2.7\cdot 10^{13} n_e^{-1/2}] + 1.9\cdot 10^5 n_e^{1/3} L^{-4/3}, \qquad (27)$$

where the critical density of the plasma n_e is measured in $\mathrm{cm^{-3}}$, the temperature of the electrons T_e in keV, the size of the inhomogeneity of the plasma L in cm, and z is the multiplicity of the charge of the ions.

The inhomogeneity of the plasma has an important effect on the threshold of the stimulated Raman scattering; it grows significantly under such an influence. It suffices to note that for conditions that are of interest for obtaining controlled thermonuclear reactions by means of a laser, the threshold of this instability is determined by the inhomogeneity. The instability itself, which is known as stimulated Raman scattering, is an absolute parametric instability.

A comparison of the threshold of scattering (27) with the results of experimental researches[10,11] allows us to assert that this instability can explain the increase in the reflection coefficient of radiation incident on the plasma for fluxes $q > 2.5 \times 10^{15}\ \mathrm{W/cm^2}$. The experimental confirmation of such an assertion can also explain the spectral composition of the reflected signal of the line corresponding to the harmonic $\frac{1}{2}\omega_0$, where ω_0 is the frequency of the pump wave.

1)In [5], the possibility of absolute parametric instability is allowed only close to the vanishing points of the space derivative of the sum of the wave vectors of the waves which take part in the decay of the instability.

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