

Experimental study of the loss and capture of electrons by fast unexcited and metastable heliumlike ions in ion-atomic collisions

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We have determined the average cross sections for loss and capture of one and two electrons for beams of heliumlike particles containing a metastable component, formed in the process of loss of one electron by lithiumlike ions and obtained in passage of a beam of fast particles through a thin celluloid film. From the results of these measurements we have found the cross sections for loss and capture of electrons by unexcited heliumlike ions and the cross sections for loss of an electron by metastable heliumlike ions with nuclear charge Z from 3 to 8 in passage through helium and nitrogen with velocity v from 4×10^8 to 12×10^8 cm/sec. The electron capture and loss cross sections obtained are compared with the cross sections for other very simple atomic systems. The relative number of metastable particles in beams of heliumlike ions formed in this way has been determined.

1. INTRODUCTION

In beams of fast helium-like ions which are used in experiments on determination of the cross sections for loss and capture of electrons by fast particles, as a rule, a significant fraction of the ions^[1] are in $(1s2s)^1S$ metastable states. Therefore the cross sections found in these experiments, generally speaking, cannot be assigned to ions in the $(1s^2)^1S$ ground state. The cross sections for loss of an electron by unexcited and metastable particles have been determined separately only for helium atoms by the method of atomic-beam attenuation in a collision chamber.^[2-4] The cross sections for electron loss by metastable helium ions have been determined, in addition, from the corresponding cross sections for unexcited atoms and the average cross sections for atomic beams with a known metastable component.^[5] In the present work we have proposed and used an experimental method for determination of the cross sections for electron loss by unexcited helium-like ions from measurements of the average cross section for electron loss by helium-like ions formed on ionization of fast lithium-like particles in single collisions with gas atoms. In contrast to the beam-attenuation method, this method can be used also in cases where the electron-loss cross sections being determined are not only comparable with but even much smaller than the cross sections for electron capture.

In the experiments described here we have measured the average cross sections for loss and capture of one and two electrons by helium-like ions formed in passage of a beam of fast particles through a celluloid film and of a beam of fast lithium-like ions through a thin gas target. The cross sections were measured for ions with nuclear charge Z from 3 to 8 in passage through helium and nitrogen with velocities $v = 4 \times 10^8$, 8×10^8 , and 12×10^8 cm/sec. From the results of these measurements we have determined the cross sections for loss and capture of one and two electrons by unexcited helium-like ions and the cross sections for loss of one electron by metastable particles, and we have estimated the relative number of metastable ions in beams of helium-like particles for the two indicated means of formation. A comparison has been made of the cross sections for unexcited and metastable helium-like particles with the corresponding cross sections for other

very simple ionic systems. A brief report of some of the results obtained has been given previously.

2. THE EXPERIMENT AND ITS DIRECT RESULTS

The cross sections for loss and capture of electrons were determined by a mass-spectrometric method in the experimental apparatus described previously.^[7] In these experiments we utilized the ions Li^{i+} , Be^{i+} , B^{i+} , C^{i+} , N^{i+} , and O^{i+} accelerated in a 72-cm cyclotron. The lithium-like ions Be^{i+} , B^{i+} , C^{i+} , and N^{i+} extracted from the accelerator with charges $i = Z - 3$ were passed either through a flowing gas target of thickness $\sim 10^{15}$ atoms/cm² or through a celluloid film of thickness 2–3 $\mu\text{g}/\text{cm}^2$ (which corresponds to $\sim 10^{17}$ atoms/cm²). The fast helium-like particles formed, part of which were in metastable states, were separated by a magnetic mass monochromator and were directed into a collision chamber for determination of the electron loss and capture cross sections. In those cases where B^{i+} , N^{i+} , and O^{i+} ions with charges $i < Z - 3$ were accelerated in the cyclotron, the lithium-like particles B^{i+} , N^{i+} , and O^{i+} were separated by a focusing magnet from the beam of ions, part of which changed charge as the result of interaction with atoms of the residual gas in the ion pipe. These ions were then used in the same way as the lithium-like particles obtained directly from the accelerator. When the helium-like ions Li^{i+} , Be^{i+} , and B^{i+} with charges $i = Z - 2$ were accelerated in the cyclotron, the cross sections were measured for the helium-like ions both occurring in the composition of the ion beam which passed through a solid target, and obtained directly from the accelerator. On acceleration of the hydrogen-like ions Li^{i+} , measurements were made of the cross section for loss and capture of electrons by the helium-like ions Li^{i+} produced in passage of the ion beam through a celluloid target. In each experiment the collision chamber was filled alternately with nitrogen and helium; the gas target was filled with nitrogen, and in a number of experiments also with helium.

The intensity of the beam of helium-like ions obtained by ionization of lithium-like ions in a thin gas target was one to two orders of magnitude lower than the intensity of the initial beam, as a result of which the statistical error in the corresponding cross sections

was somewhat greater than usual. The total error in the values obtained in the experiment for the cross sections for loss and capture of one electron are 10–15%, and for the cross sections for loss and capture of two electrons—20–30%.

The cross sections obtained in these experiments for loss and capture of electrons by helium-like ions produced in the celluloid film and also the similar cross sections for the hydrogen-like and helium-like ions B^{+2} , C^{+3} , N^{+4} , and O^{+5} measured as a control agree within experimental error with the cross sections measured previously.^[7–10] These experiments also permitted some refinements of the experimental cross sections for loss and capture of an electron by hydrogen-like and lithium-like ions in helium and nitrogen. In all of the figures of the present article, the improved cross-section values are presented.

The cross-section measurements were carried out for relatively small thicknesses of the gas layer in the collision chamber, where the loss of intensity of the ion beam entering the collision chamber as a result of loss and capture of electrons did not exceed 10–20%. In satisfying this condition, the average cross sections obtained in the experiment for change of charge of helium-like ions σ^j were related to the corresponding cross sections σ^0 and σ^m for unexcited and metastable particles as follows:

$$\sigma^j = \sigma^0 + r\alpha_j(\sigma^m - \sigma^0), \quad (1)$$

where α_j is the relative number of metastable particles in the ion beam immediately after it leaves the target. The superscript j refers to the indices s , l , and a which indicate the means of formation of the ions respectively in solid and gas targets and in the accelerator ion

source. The coefficient r takes into account the decrease in the number of excited particles as a result of radiative capture in the 1–3-m path from the target to the collision chamber. Since the lifetimes of metastable helium-like particles are rather large,^[11] we have in all cases: $0.8 \leq r \leq 1$.

The results of the measurements of cross sections for loss and capture of electrons by helium-like ions in helium and in nitrogen are shown in Figs. 1 and 2. In these figures we have also indicated the values of the corresponding cross sections for helium atoms He^0 and helium-like ions of oxygen O^{+6} and fluorine F^{+7} , which were taken from refs. 3 and 12–14. As can be seen from the figures, the cross sections σ_{78} and σ_{79} for electron loss by F^{+7} ions in nitrogen at $v = 12 \times 10^8$ cm/sec are apparently exaggerated, perhaps because of incomplete separation of particles with neighboring charge values after passage of the beam through the solid target.

In all cases the cross sections for loss of one electron $\sigma_{Z-2, Z-1}^s$ by helium-like ions formed in a solid target are greater than the similar cross sections $\sigma_{Z-2, Z-1}^a$ and $\sigma_{Z-2, Z-1}^l$ for ions extracted directly from the accelerator or obtained as the result of stripping of lithium-like ions in a thin gas target, the values of the ratios $\sigma_{Z-2, Z-1}^s / \sigma_{Z-2, Z-1}^a$ and $\sigma_{Z-2, Z-1}^s / \sigma_{Z-2, Z-1}^l$ varying from 2–6 for $v = 4 \times 10^8$ cm/sec. This means that in a beam of helium-like ions formed in solid film the relative number of metastable particles is greater than in ion beams obtained by the two other means, i.e., $\alpha_s > \alpha_a$ and $\alpha_s > \alpha_l$.

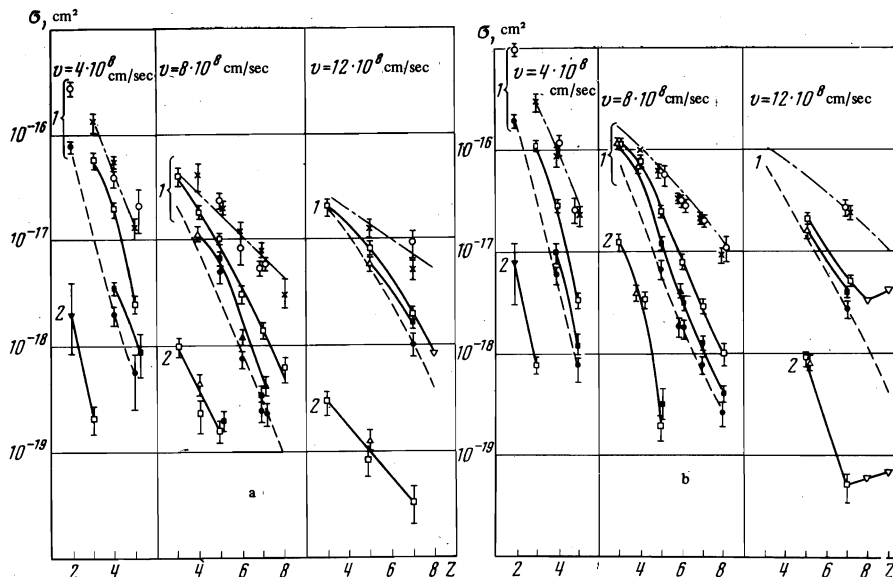


FIG. 1. Cross sections for electron loss by helium-like ions in helium (a) and in nitrogen (b), for v equal to 4×10^8 , 8×10^8 , and 12×10^8 cm/sec, as a function of the nuclear charge of the ion Z . The numbers 1 and 2 indicate the cross sections for loss respectively of 1 and 2 electrons $\sigma_{Z-2, Z-1}$ and $\sigma_{Z-2, Z}$. The average cross sections directly obtained in the experiment are as follows: for ions formed by stripping of lithium-like particles in nitrogen— \blacksquare , in helium— \blacktriangle ($j = l$); for ions which have passed through a solid film— \square ($j = s$); \triangle —cross sections for ions obtained from a cyclotron ($j = a$); ∇ —cross sections for the ions O^{+6} and F^{+7} which have passed through a solid film, from refs. 13 and 14. The solid circles show the cross sections $\sigma_{Z-2, Z-1}^0$ for unexcited particles, the hollow circles and crosses show the cross sections $\sigma_{Z-2, Z-1}^m$ for metastable particles, obtained from Eqs. (5) and (6), respectively. The dashed and dot-dash lines show the most probable values of $\sigma_{Z-2, Z-1}^0$ and $\sigma_{Z-2, Z-1}^m$. For $Z = 2$ the symbols \circ and \bullet show the cross sections σ_{10}^m and σ_{10}^0 from ref. 3, and \blacktriangledown shows the cross sections σ_{02} from ref. 12.

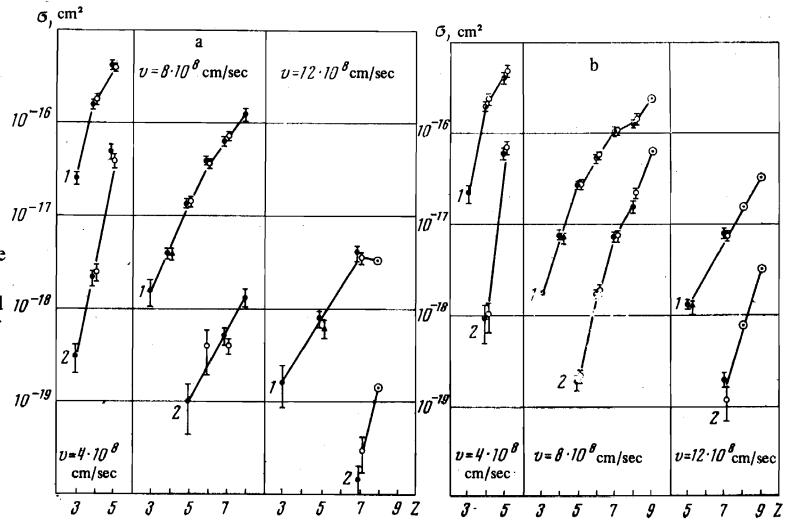


FIG. 2. Cross sections for electron capture by helium-like ions in helium (a) and nitrogen (b), for v equal to 4×10^8 , 8×10^8 , 12×10^8 cm/sec, as a function of the nuclear charge of the ion Z : ●—for ions which have passed through a solid target ($J = s$); ○—for ions obtained by ionization of lithium-like ions ($j = l$); ▲—for ions obtained directly from a cyclotron ($j = a$); ◊—for ions produced in a solid target, from refs. 13 and 14. The numbers 1 and 2 indicate cross sections for capture of respectively one and two electrons, $\sigma_{Z-2, Z-3}$ and $\sigma_{Z-2, Z-4}$.

The values of the cross sections for capture of one electron $\sigma_{Z-2, Z-3}^j$ and also those of the cross sections for loss and capture of two electrons $\sigma_{Z-2, Z}^j$ and $\sigma_{Z-2, Z-4}^j$ for helium-like ions produced by the different methods agree within the experimental errors (see Figs. 1 and 2).

3. DETERMINATION OF CROSS SECTIONS FOR ELECTRON LOSS BY UNEXCITED AND METASTABLE HELIUM-LIKE IONS

In beams of fast helium-like particles formed by ionization of unexcited lithium-like ions in single collisions, the numbers of unexcited and metastable particles in the respective states ($1s^2$) and ($1s2s$) are proportional to the cross sections for loss of $2s$ and $1s$ electrons by lithium-like ions $\sigma_{2s}^{(1)}(1s^2 2s)$ and $\sigma_{1s}^{(1)}(1s^2 2s)$

(here and in what follows we have shown the initial state of the ions in parentheses, and the upper indices 1 and 2 indicate that the cross sections refer respectively to the gas in the target in which the helium-like ions are formed or to the gas in the collision chamber). Therefore, for the relative number of metastable particles among these helium-like ions we will have

$$\alpha_l = \frac{\sigma_{1s}^{(1)}(1s^2 2s)}{\sigma_{2s}^{(1)}(1s^2 2s) + \sigma_{1s}^{(1)}(1s^2 2s)}. \quad (2)$$

From Eqs. (1) and (2), for the ratio K^0 = $\sigma_{Z-2, Z-1}^0 / \sigma_{Z-2, Z-1}^l$ (of the cross section for electron loss by unexcited ions $\sigma_{Z-2, Z-1}^0 \equiv \sigma_{1s}^{(2)}(1s^2)$ to the cross section $\sigma_{Z-2, Z-1}^l$ obtained in the experiments) we will have

$$K^0 = (1 + \epsilon) \left\{ 1 + \frac{rbc(1 + \gamma)}{a} + (1 - r)\epsilon \right\}^{-1}, \quad (3)$$

where

$$a = \frac{\sigma_{1s}^{(2)}(1s^2)}{\sigma_{1s}^{(2)}(1s^2 2s)}, \quad b = \frac{\sigma_{2s}^{(2)}(1s2s)}{\sigma_{2s}^{(2)}(1s^2 2s)}, \quad \epsilon = \frac{\sigma_{1s}^{(1)}(1s^2 2s)}{\sigma_{2s}^{(1)}(1s^2 2s)},$$

$$c = \frac{\sigma_{2s}^{(2)}(1s^2 2s) \sigma_{1s}^{(1)}(1s^2 2s)}{\sigma_{2s}^{(1)}(1s^2 2s) \sigma_{1s}^{(2)}(1s^2 2s)}, \quad \gamma = \frac{\sigma_{1s}^{(2)}(1s2s)}{\sigma_{2s}^{(2)}(1s2s)}.$$

The quantities a and b are the ratios of the cross sections for loss of one and the same $1s$ or $2s$ electrons, respectively, from helium-like and lithium-like ions, and

therefore the ratio b/a depends only weakly on assumptions regarding the values of the cross sections themselves and is close to unity. The quantity c is strictly unity when the target and collision chamber are filled with the same gas, and the values of γ and ϵ are small. Accordingly the coefficient K^0 turns out to be close to $1/2$, i.e., the cross sections for loss of an electron by unexcited ions $\sigma_{Z-2, Z-1}^0$ are roughly a factor of two smaller than the experimentally obtained cross sections $\sigma_{Z-2, Z-1}^l$. Thus, it turns out to be possible to determine from the experimental cross sections $\sigma_{Z-2, Z-1}^l$ the cross sections for electron loss $\sigma_{Z-2, Z-1}^0 \equiv \sigma_{1s}^{(2)}(1s^2)$ for unexcited particles.

Specific values of the coefficient K^0 in the present work were obtained on the assumption that the ratios of the cross sections for loss of $1s$ and $2s$ electrons, which make up the quantities a , b , c , γ , and ϵ , are identical with the corresponding ratios of cross sections for loss of $1s$ and $2s$ electrons by hydrogen-like and lithium-like ions with the same electron binding energy I as in the particles being considered.¹⁾ Here we have used the experimental dependence of the cross sections for electron loss by hydrogen-like and lithium-like ions on I , which as a rule is not very different from the theoretical or semiempirical dependences obtained in refs. 15–17. The values of I have been taken from the tables given in refs. 18 and 19.

Since the quantity

$$\epsilon = \frac{\sigma_{1s}^{(1)}(1s^2 2s)}{\sigma_{2s}^{(1)}(1s^2 2s)} = \frac{1}{a'} \frac{\sigma_{1s}^{(1)}(1s^2)}{\sigma_{2s}^{(1)}(1s^2 2s)}$$

($a' = \sigma_{1s}^{(1)}(1s^2) / \sigma_{1s}^{(1)}(1s^2 2s)$) which enters into Eq. (3) depends on the cross section $\sigma_{1s}^{(1)}(1s^2)$ which is, strictly speaking, unknown before calculation of the coefficient K^0 , values of K^0 were found by means of the following formula obtained by transformation of Eq. (3), in which the explicit dependence of K^0 on ϵ is reduced to a minimum:

$$K^0 = \left\{ 1 + \frac{rbc(1 + \gamma - \delta/r)}{a} + (1 - r)\epsilon \right\}^{-1}, \quad (4)$$

where

$$\delta = \frac{\sigma_{Z-2, Z-1}^l}{\sigma_{2s}^{(2)}(1s2s)} = \frac{1}{b} \frac{\sigma_{Z-2, Z-1}^l}{\sigma_{2s}^{(2)}(1s^2 2s)}.$$

A calculation shows that the ratio b/a for the ions

discussed is practically independent of the type of gas passed through the collision chamber and for all cases studied lies between 0.6 and 1.6. Since the shape of the curves of the electron-loss cross sections as a function of I are somewhat different in helium and nitrogen, then in the case in which the collision chamber is filled with helium and the target with nitrogen the values of c are close to 0.6–0.7, while for the collision chamber filled with nitrogen and the target with helium (in the experiments with C^{+4} and N^{+5} ions for $v = 8 \times 10^8$ cm/sec) we have $c \approx 1.7$. When the collision chamber and target are filled with the same gas, we have $c \equiv 1$. The values of δ are determined in practice directly from the experimental cross sections, since the values of $\sigma_{2s}^{(2)}(1s^2 2s)$ are close to the cross sections obtained experimentally for loss of an electron by unexcited lithium-like ions $\sigma_{Z-3, Z-2} = \sigma_{2s}^{(2)}(1s^2 2s) + \sigma_{1s}^{(2)}(1s^2 2s)$. The values of δ increase with increasing v from $\delta \approx 0.03$ for N^{+4} ions at $v = 8 \times 10^8$ cm/sec to $\delta \approx 0.3$ for the same ions at $v = 12 \times 10^8$ cm/sec. In all cases we have also $\gamma \leq 0.07$ and $(1-r)\epsilon \leq 0.02$.

The values of K^0 calculated in this way for the ions considered lie within the range 0.4 to 0.7. It should be noted that with the target filled with nitrogen the cross sections $\sigma_{Z-2, Z-1}^l$ are 20–30% lower than those obtained when the target gas is replaced by helium. However, the values of the coefficient K^0 are increased by the same amount, as a consequence of which and as shown by the experimental results for C^{+4} and N^{+5} ions, the values of $\sigma_{Z-2, Z-1}^0$ turn out to be practically identical in both cases (see Fig. 1). The error in the coefficient K^0 is determined mainly by the errors in determination of the ratios b/a and c and amount to the order of 10–20%. As a result the total error in the cross sections $\sigma_{Z-2, Z-1}^0$ obtained are 20–20%.

From the experimental values of $\sigma_{Z-2, Z-1}$ it is possible to determine also the cross sections for electron loss by metastable ions $\sigma_{Z-2, Z-1}^m$. Since the quantities $\sigma_{Z-2, Z-1}^m$ and $\sigma_{Z-2, Z-1}^0$ in Eq. (1) are equally justified, we can write for the ratio K^m

$$= \sigma_{Z-2, Z-1}^m / \sigma_{Z-2, Z-1}^l \text{ an expression similar to Eq. (3):}$$

$$K^m = \frac{1+\epsilon}{r\epsilon} \left\{ 1 + \frac{a}{rbc(1+\gamma)} [1 + (1-r)\epsilon] \right\}^{-1}. \quad (5)$$

It is evident from this expression that, in contrast to K^0 , the values of K^m depend substantially on the value of ϵ , which is known with poor accuracy prior to determination of the cross section $\sigma_{Z-2, Z-1}^0$. Therefore the values of K^m and consequently also the cross sections $\sigma_{Z-2, Z-1}^m$ also can be determined in practice only after finding the cross sections $\sigma_{Z-2, Z-1}^0$ and estimation on this basis of the values of ϵ . The error in the values of $\sigma_{Z-2, Z-1}^m$ obtained in this way is composed of the errors in the quantities $\sigma_{Z-2, Z-1}^l$ and ϵ and is close to 30%.

Since the cross sections $\sigma_{2s}(1s^2 2s)$ and $\sigma_{2s}(1s 2s)$ correspond to loss of 2s electrons by lithium-like and helium-like particles which differ only in the number of electrons in the inner shells, the values of $\sigma_{Z-2, Z-1}^m = (1+\gamma)\sigma_{2s}(1s 2s)$ can also be estimated from the experimental values of $\sigma_{Z-3, Z-2} = (1+\epsilon)\sigma_{2s}(1s^2 2s)$ for lithium-like particles:

$$\sigma_{Z-2, Z-1}^m = b \frac{1+\gamma}{1+\epsilon} \sigma_{Z-3, Z-2}. \quad (6)$$

The values of the coefficient $b(1+\gamma)/(1+\epsilon)$ obtained on the assumptions mentioned above lie within the limits 0.4 to 0.7. Thus, the cross sections for electron loss by metastable helium-like ions $\sigma_{Z-2, Z-1}^m$ are on the average a factor of two smaller than the cross sections for electron loss $\sigma_{Z-3, Z-2}^0$ by unexcited lithium-like ions of the same element. The error in the values of $\sigma_{Z-2, Z-1}^m$ calculated from Eq. (6) is close to 20%. The values of $\sigma_{Z-2, Z-1}^m$ obtained from Eqs. (5) and (6) agree within the accuracy indicated (see Fig. 1).

The cross sections obtained for the loss of an electron by unexcited and metastable helium-like ions $\sigma_{Z-2, Z-1}^0$ and $\sigma_{Z-2, Z-1}^m$ are shown in Fig. 1 together with the experimental values of $\sigma_{Z-2, Z-1}^j$. The dashed lines show the most probable values of $\sigma_{Z-2, Z-1}^0$ which follow from the entire set of data obtained on $\sigma_{Z-2, Z-1}^0$. For $Z \leq 5$, in accordance with the conclusions of Sec. 6, they are defined by the relation

$$\sigma_{Z-2, Z-1}^0 = 2[\sigma_{1s}(1s) - \sigma_{Z-2, Z-1}^0],$$

where $\sigma_{1s}(1s)$ is the cross section for electron loss by hydrogen-like ions with the same electron binding energy I as in the helium-like particle being considered. The dot-dash lines show the most probable values of $\sigma_{Z-2, Z-1}^m$ obtained by means of Eq. (6) from the smoothed experimental dependence of the cross sections for electron loss by lithium-like ions $\sigma_{Z-3, Z-2}^0$ on the electron binding energy I .

4. RELATIVE NUMBER OF METASTABLE PARTICLES IN BEAMS OF HELIUM-LIKE IONS

From Eqs. (1) and (2) we have for the relative number of metastable particles α_j in the ion beam

$$\alpha_j = \frac{1}{r} \frac{(\sigma_{Z-2, Z-1}^j / \sigma_{Z-2, Z-1}^0 - 1)}{(\sigma_{Z-2, Z-1}^m / \sigma_{Z-2, Z-1}^0 - 1)}. \quad (7)$$

Thus, if the cross sections for electron loss by unexcited ions $\sigma_{Z-2, Z-1}^0$ and metastable ions $\sigma_{Z-2, Z-1}^m$ are known, we can determine the values of α_j from the experimentally found average cross sections for electron loss $\sigma_{Z-2, Z-1}^j$. In calculation of the values of α_j , we took as the values of $\sigma_{Z-2, Z-1}^0$ and $\sigma_{Z-2, Z-1}^m$ in Eq. (7) their most probable values shown in Fig. 1 by the dashed and dot-dash lines. The relative numbers obtained in this way α_l , α_s , and α_a of metastable particles in beams of helium-like ions arising as the result of ionization of lithium-like particles in a thin gas target, produced in passage of a beam of ions through a solid film, and for helium-like ions obtained directly from an accelerator, are given in the table. The relative error in α_j in accordance with Eq. (7) is 2–3 times the relative error in the cross sections $\sigma_{Z-2, Z-1}^j$ and is on the average on the order of 30%.

5. CROSS SECTIONS FOR CAPTURE OF ONE AND TWO ELECTRONS AND LOSS OF TWO ELECTRONS HELIUM-LIKE IONS

From measurements of the average cross sections for any process for two ion beams with different but known content of metastable particles (for example, from measurements of σ^l and σ^s for known α_l and α_s) it is possible, generally speaking, to determine separately

Ion	$v, 10^8 \text{ cm/sec}$	α_s		α_l				α_a	
		—		Gas in target He		Gas in target N ₂		—	
		Gas in collision chamber							
		He	N ₂	He	N ₂	He	N ₂	He	N ₂
Li ⁺	4	—	0.3±0.2	—	—	—	—	—	—
	8	0.5±0.2	0.4±0.2	—	—	—	—	—	0.4±0.2
	12	0.16±0.05	—	—	—	—	—	—	—
Be ⁺²	4	—	0.23±0.10	—	—	0.04±0.03	0.04±0.03	—	—
	8	0.4±0.2	0.65±0.30	—	—	—	—	0.10±0.05	0.2±0.1
	12	—	—	—	—	—	—	—	—
B ⁺³	4	—	0.075±0.030	—	—	0.03±0.02	0.014±0.010	—	—
	8	0.37±0.10	0.31±0.07	—	—	0.15±0.05	0.09±0.03	—	—
	12	0.33±0.10	0.22±0.07	—	—	—	—	0.10±0.05	0.08±0.05
C ⁺⁴	8	0.20±0.05	0.17±0.05	0.06±0.02	0.07±0.02	—	0.034±0.015	—	—
N ⁺⁵	8	0.14±0.05	0.10±0.03	0.04±0.02	—	0.03±0.01	0.03±0.01	—	—
	12	0.15±0.10	0.12±0.10	—	—	0.13±0.04	0.07±0.05	—	—
O ⁺⁶	8	0.09±0.04	0.06±0.03	—	—	—	0.15±0.10	—	—

the cross sections for this process σ^0 and σ^m for unexcited and metastable particles. From Eq. (1), in particular, we have

$$\frac{\sigma^0}{\sigma^l} = \left(\frac{\alpha_s}{\alpha_l} - \frac{\sigma^l}{\sigma^l} \right) / \left(\frac{\alpha_s}{\alpha_l} - 1 \right),$$

$$\frac{\sigma^m}{\sigma^l} = \left(\frac{\sigma^l}{\sigma^l} \frac{1-\alpha_l}{\alpha_l} - \frac{1-\alpha_s}{\alpha_l} \right) / \left(\frac{\alpha_s}{\alpha_l} - 1 \right). \quad (8)$$

The cross sections obtained in the present work for capture of one and two electrons and for loss of two electrons lead to the following average values of σ^S/σ^l and their average deviations:

$$\frac{\sigma_{Z-2,Z-3}^0}{\sigma_{Z-2,Z-3}^l} = 0.98 \pm 0.15, \quad \frac{\sigma_{Z-2,Z-4}^0}{\sigma_{Z-2,Z-4}^l} = 1.1 \pm 0.3, \quad \frac{\sigma_{Z-2,Z}^0}{\sigma_{Z-2,Z}^l} = 0.8 \pm 0.2.$$

The experimental error in these ratios is practically identical to the average deviations given here.

From these values of σ^S/σ^l and the values of α_s and α_l given in the table, the following average values and average deviations are obtained for the ratio of the desired cross sections σ^0 and σ^m to the measured cross section σ^l :

For capture of one electron:

$$\frac{\sigma_{Z-2,Z-3}^0}{\sigma_{Z-2,Z-3}^l} = 1.01 \pm 0.03, \quad \frac{\sigma_{Z-2,Z-3}^m}{\sigma_{Z-2,Z-3}^l} = 0.7 \pm 0.5 (\pm 0.2),$$

for capture of two electrons:

$$\frac{\sigma_{Z-2,Z-4}^0}{\sigma_{Z-2,Z-4}^l} = 0.98 \pm 0.05, \quad \frac{\sigma_{Z-2,Z-4}^m}{\sigma_{Z-2,Z-4}^l} = 1.4 \pm 1.8 (\pm 0.6)$$

and for loss of two electrons:

$$\frac{\sigma_{Z-2,Z}^0}{\sigma_{Z-2,Z}^l} = 1.1 \pm 0.1, \quad \frac{\sigma_{Z-2,Z}^m}{\sigma_{Z-2,Z}^l} = 0.2 \pm 0.7 (\pm 0.2).$$

In view of the fact that the ratios σ^S/σ^l are close to unity and the values of α_s in most cases are several times greater than α_l , the error in the ratio σ^0/σ^l turns out to be substantially less than the error $\Delta(\sigma^S/\sigma^l)$ in the ratio σ^S/σ^l and close to $(\alpha_l/\alpha_s)\Delta(\sigma^S/\sigma^l)$, while the error in the ratio σ^m/σ^l is significantly larger than the error $\Delta(\sigma^S/\sigma^l)$ and close to $(1/\alpha_s)\Delta(\sigma^S/\sigma^l)$. Accordingly, for capture and loss of two electrons the error in the individual values of σ^m/σ^l turns out to be more than 100%. However, for the entire set of values of these ratios the errors amount to respectively $\sim 40\%$ and $\sim 100\%$. These errors are shown in parentheses.

From the ratios presented it is evident that the cross sections σ^0 for capture of one and two electrons and the

cross sections for loss of two electrons by unexcited helium-like ions agree within the experimental error with the corresponding experimental cross sections σ^l for helium-like ions produced by loss of an electron by lithium-like ions in a thin gas target. For the analogous cross sections σ^m which refer to metastable ions, we have obtained only rather rough estimates from which, however, we can conclude that the cross sections for capture of one electron $\sigma_{Z-2,Z-3}^m$ and loss of two electrons $\sigma_{Z-2,Z}^m$ by metastable particles are apparently somewhat smaller than the corresponding cross sections $\sigma_{Z-2,Z-3}^0$ and $\sigma_{Z-2,Z}^0$ for unexcited ions.

6. DISCUSSION OF RESULTS OBTAINED

A. Cross Sections for Electron Loss by Helium-Like Ions

It is interesting to compare the cross sections obtained for electron loss $\sigma_{Z-2,Z-1}^0$ by unexcited helium-like ions with the corresponding cross sections $\sigma_{Z-1,Z}$ for hydrogen-like particles. Since the numbers of electrons q in these ions are different, it is appropriate to compare, as shown in ref. 10, the cross sections for loss of an individual electron

$$\sigma_i = \frac{1}{q} \sum_s \sigma_{i,i+s},$$

i.e., the quantities $\sigma_{Z-2}^0 \equiv (1/2)\sigma_{Z-2,Z-1}^0 + \sigma_{Z-2,Z}^0$ (which in all our cases differ from $(1/2)\sigma_{Z-2,Z-1}^0$ by from 3 to 10%) and $\sigma_{Z-1} \equiv \sigma_{Z-1,Z}$. This comparison shows that for ions with $Z = 4$ and 5 the cross sections σ_{Z-2}^0 agree within experimental error with the electron-loss cross sections σ_{Z^*-1} for hydrogen-like particles with electron binding energy I_{Z^*-1} equal to the electron binding energy I_{Z-2} in the helium-like ions being considered, while for ions with $Z = 6-8$ the values of σ_{Z-2}^0 are approximately 1.5 times smaller than the cross sections σ_{Z^*-1} for hydrogen-like particles with $I_{Z^*-1} = I_{Z-2}$ and are close to the values of σ_{Z^*-1} for $I_{Z^*-1} \approx 1.15I_{Z-2}$ (see Fig. 3). The corresponding values of the effective charge of the hydrogen-like particles $Z^* = (I_{Z^*-1}/I_0)^{1/2}$, where $I_0 = 13.6 \text{ eV}$, turn out to be close to $Z - 0.6$ for ions with $Z = 4$ and 5, and close to Z for ions with $Z = 6-8$ (see Fig. 4).

Since the wave function of the electron in an unexcited helium-like ion is close to the Coulomb wave function in

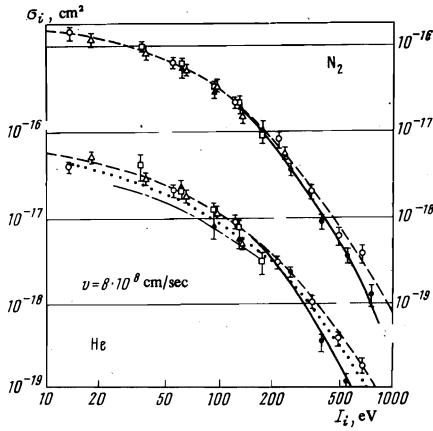


FIG. 3. Cross sections for loss of an individual electron σ_i as a function of its binding energy I , for various ions in helium and in nitrogen for $v = 8 \times 10^8$ cm/sec; \circ (and dashed line) — $\sigma_{1s}(1s)$ for hydrogen-like particles; \bullet (and solid curve) — $\sigma_{1s}(1s^2)$ for unexcited helium-like ions; \square and \blacktriangle — cross sections $\sigma_{2s}(1s2s)$ for metastable helium-like ions, obtained respectively from Eqs. (5) and (6); \triangle — $\sigma_{2s}(1s^22s)$ for lithium-like particles; dotted curve — theoretical value of $\sigma_{1s}(1s)$ from ref. 15; dot-dash curve — theoretical value of $\sigma_{1s}(1s^2)$ from ref. 22.

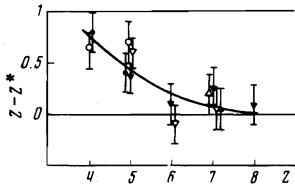


FIG. 4. Values of $Z - Z^*$ corresponding to the experimental cross sections for electron loss by unexcited helium-like ions in helium (hollow points) and in nitrogen (solid points), as a function of the nuclear charge of the ions Z : \bullet , \circ — $v = 4 \times 10^8$ cm/sec; ∇ , \triangleright — $v = 8 \times 10^8$ cm/sec; \blacktriangle , \triangle — $v = 12 \times 10^8$ cm/sec.

the field of a charge $Z - 0.3$, while the liberated electron at large distances from the nucleus is in the Coulomb field of a charge $Z - 1$, the correspondence of the values of σ_{Z-2}^0 (for $Z \leq 5$) to the cross sections for electron loss by hydrogen-like particles with charges $Z^* = Z - 0.6$ which are intermediate between the values $Z - 0.3$ and $Z - 1$ is quite natural. As a consequence it should be noted that the experimental cross sections for ionization of helium by protons^[20] for $v > 2v_0$ also correspond to the cross sections for ionization of hydrogen-like particles with nuclear charge $Z^* = Z - 0.65$ or, in other words, with the same electron binding energy as in the helium atom. However, the result for ions with $Z = 6-8$ from the same point of view is anomalous. It is inconsistent with the conclusions following from theoretical calculations of the cross sections for loss of a K electron by nonhydrogen-like particles^[21, 22] by the method of Merzbacher and Lewis,^[21] according to which in the region $Zv_0/v > 1$, where $v_0 = e^2/\hbar = 2.19 \times 10^8$ cm/sec, with increase of the parameter Zv_0/v the difference between the values of Z and Z^* should increase. The reduction of the cross sections for ions with $Z = 6-8$ apparently has the same nature as that noted in the review article by Garcia, Fortner, and Kavanagh^[23] for the cross sections for ejection of K electrons from atoms of light elements by protons.

For metastable helium-like particles the cross sections for loss of an individual electron σ_{Z-2}^m do not differ essentially from the cross sections for loss of one 2s electron $\sigma_{Z-2, Z-1}^m$ which, as already noted above (Sec. 3), agree with the cross sections for electron loss by unexcited lithium-like ions with the same electron binding energy I . It should be noted that, in the region of

I from 20 to 200 eV to which the cross sections $\sigma_{Z-2, Z-1}^m$ obtained belong, for identical I values the cross sections for loss of a 2s electron coincide with the cross sections for loss of a 1s electron for hydrogen-like and helium-like particles (Fig. 3).

The cross sections $\sigma_{Z-2, Z}^0$ obtained for loss of two 1s electrons by unexcited helium-like particles were compared with those taken from ref. 10 for loss of two electrons by beryllium-like ions, in whose outer L shell there are two 2s electrons. For ions with identical values of binding energy of the remote outer 1s and 2s electrons, these cross sections agree within experimental error.

In discussing the cross sections $\sigma_{Z-2, Z}^m$ for loss of two electrons by metastable helium-like particles, it is necessary to keep in mind that the binding energies of 1s and 2s electrons in these ions are close to the binding energies of 1s and 2s electrons in lithium-like ions with the same charge i , while the number of 1s electrons is a factor of two smaller. As a result the cross sections $\sigma_{i, i+2}^m$ for helium-like ions should be approximately a factor of two smaller than the cross sections $\sigma_{i, i+2}$ for unexcited lithium-like particles with the same charge i . When this is taken into account we obtain, from the values of $\sigma_{Z-2, Z}^0 \equiv \sigma_{i, i+2}(1s^2)$ found in the present work and the previously known^[10] cross sections $\sigma_{i, i+2}(1s^22s)$ for the loss of two electrons by lithium-like ions, for the ratio of the cross sections for loss of two electrons by metastable and unexcited helium-like ions

$$\frac{\sigma_{Z-2, Z}^m}{\sigma_{Z-2, Z}^0} \approx \frac{1}{2} \frac{\sigma_{i, i+2}(1s^22s)}{\sigma_{i, i+2}(1s^2)} = 0.7 \pm 0.3,$$

which agrees with the experimental estimate of this ratio obtained in Sec. 5.

B. Cross Section for Electron Captures by Very Simple Ionic Systems

Since the experimentally obtained cross sections for electron capture $\sigma_{Z-2, Z-3}^l$ are practically identical to the corresponding cross sections $\sigma_{Z-2, Z-3}^0$ for unexcited particles, it is reasonable to compare these cross sections with the cross sections $\sigma_{i, i-1}$ for electron capture by other simpler ionic systems: nuclei and hydrogen-like and lithium-like ions.

Such a comparison of the electron-capture cross sections for particles with a number of electrons $N = Z - i$ from 0 to 3 shows that for ions with large charges i for which the ratio $\eta = I(n_0)/(1/2)\mu v^2$ is greater than unity (here $I(n_0)$ is the electron binding energy in the unfilled electron shell closest to the nucleus with principal quantum number $n = n_0$; μ is the electron mass) the cross sections $\sigma_{i, i-1}$ are practically independent of the number N of electrons in the ion and are proportional to i^2 , while for $\eta \lesssim 1$ they are substantially different for ions with different values of N , the cross sections $\sigma_{i, i-1}$ for helium-like ions being smaller than for hydrogen-like and lithium-like particles (Fig. 5). However the cross sections divided by the number of vacancies $p(n_0)$ in the unfilled electron shell nearest the nucleus turn out to be practically identical for ions of all four types for the same values of $I(n_0)$ (Fig. 6). Here the quantities $\sigma_{i, i-1}/p(n_0)$ are proportional to $I^\alpha(n_0)$, where $\alpha \sim 2$ for $\eta \lesssim 0.2$ and $\alpha \approx 3/2$ for $\eta = 0.2-1$. Values of $\sigma_{i, i-1}$

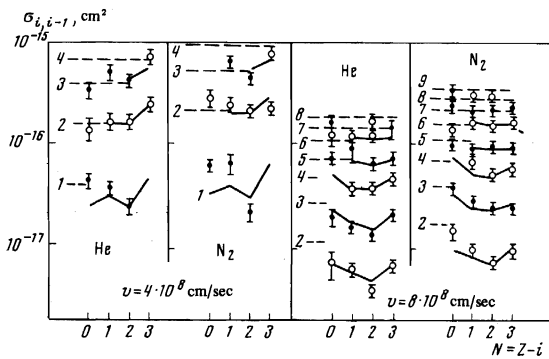


FIG. 5. Cross sections for electron capture $\sigma_{i,i-1}$ by the simplest atomic systems in helium and in nitrogen, as a function of the number of electrons in the ion N for $v = 4 \times 10^8$ and 8×10^8 cm/sec. The points show the experimental cross sections; the solid lines give the values of $\sigma_{i,i-1}$ corresponding to a dependence of $\sigma_{i,i-1}/p(n_0)$ on $I(n_0)$ common for all ions for $\eta < 1$; the dashed lines show values of $\sigma_{i,i-1}$ proportional to i^2 for $\eta > 1$. The numbers from 1 to 9 indicate the ionic charge i .

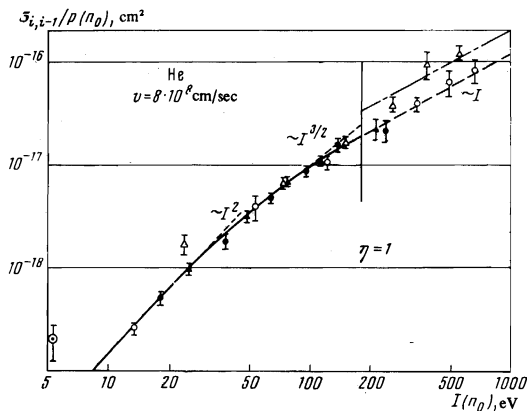


FIG. 6. Values of $\sigma_{i,i-1}/p(n_0)$ in helium for unexcited ions with various numbers of electrons N as a function of $I(n_0)$: $\circ, \Delta, \bullet,$ and \blacktriangle —for N equal to 0, 1, 2, and 3, respectively. The solid line shows a dependence $\sigma_{i,i-1}/p(n_0)$ common for all ions for $\eta < 1$. The short dashes show dependences of the type $I^2(n_0)$ and $I^{3/2}(n_0)$ for $\eta < 1$; the long dashes and the dot-dash line show values of $\sigma_{i,i-1}/p(n_0)$ proportional to $I(n_0)$ in the region $\eta > 1$ for ions with N equal to 0, 2, and 3, and for ions with $N = 1$, respectively. \ominus —cross sections σ_{i0}^5 for Li^+ ions.

corresponding to a dependence of $\sigma_{i,i-1}/p(n_0)$ on $I(n_0)$ which is the same for all ions, as shown in Fig. 6 by the solid line, satisfactorily describe the behavior of the electron-capture cross sections $\sigma_{i,i-1}$ for variation of N in the region $\eta < 1$ (see Fig. 5). For $\eta \sim 1$ the values of $\sigma_{i,i-1}/p(n_0)$ become proportional to $I(n_0)$, which corresponds to a quadratic dependence of $\sigma_{i,i-1}$ on i , which is characteristic of ions with large charges.

The experimental data obtained on the cross sections for electron capture by hydrogen-like ions in states with principal quantum number $n \geq 1$ and by helium-like ions in states with $n \geq 2$ provide the possibility of estimating the cross sections for electron capture only in states with $n = 1$. From these estimates it follows that for $\eta < 1/3$ the cross section for electron capture in states with $n \geq 2$ does not exceed 20–30% of the cross section for electron capture in states with $n = 1$. However, in the region $\eta > 1/3$ the relative probability of electron capture in states with $n \geq 2$ rises rapidly with increase of $I(n_0)$ and for ions with $\eta > 1$ the cross sections for electron capture in states with $n \geq 2$ are practically identical to the experimentally measured cross sections $\sigma_{Z,Z-1}$ and $\sigma_{Z-1,Z-2}$. As a result the dominant role of the

quantities $p(n_0)$ and $I(n_0)$ in the region $\eta \leq 1$ and the absence of a dependence of $\sigma_{i,i-1}$ on N in the region $\eta > 1$ become understandable.

It is interesting to note that these conclusions follow also from the very simple quantum-mechanical formula of Brinkman and Kramers^[24,25] for the cross sections for electron capture by nuclei with charge Z if in applying it to particles with a number of electrons $N > 0$ the quantity Z/n is replaced by $[I(n)/I_0]^{1/2}$, and the quantity n^2 by $(1/2)p(n)$, where $I(n)$ and $p(n)$ are the average electron binding energy and the number of vacancies in states with principal quantum number n . From this formula it follows that for $\eta > 1$ electron capture occurs preferentially in excited states with binding energy $I(n) \approx (1/4)\mu v^2 + I_c$ (I_c is the binding energy of the captured electrons in an atom of the medium), and therefore the electron-capture cross section is determined by the ion charge i and velocity v and does not depend on N . In the region $\eta \ll 1$ the values of $\sigma_{i,i-1}$ should be proportional to the sum $\sum_n p(n)[I(n)]^{5/2}$, and therefore for unexcited helium-like particles if the relation $I(n) = i^2 I_0/n^2$ is satisfied and in particular $I(n_0) = i^2 I_0/4$, the quantities $\sigma_{i,i-1}/p(n_0)$ would be 30% larger than for nuclei with the same value of $I(n_0)$. However, as a result of the fact that for helium-like ions the actual values of $I(n_0)$ are greater than $i^2 I_0/4$, the values of $\sigma_{i,i-1}/p(n_0)$ for ions with $n = 0$ and 2 differ by no more than 15%.

In discussing the electron-capture cross sections $\sigma_{Z-2,Z-3}^m$ of metastable helium-like ions we should keep in mind that, in capture of an electron in states with $n \geq 2$, lithium-like particles with K vacancies are formed, a large fraction of which are autoionized with a transition to the ground state of the helium-like ions before the particle leaves the collision chamber.^[26] Therefore the experimentally obtained cross sections for electron capture by metastable ions should be close to the cross section for electron capture only into a vacant state of the K shell. Hence it follows that for $\eta \geq 1$, when the metastable ions capture an electron mainly into states with $n \geq 2$, the cross sections $\sigma_{Z-2,Z-3}^m$ should be less than the values of $\sigma_{Z-2,Z-3}^0$ for unexcited particles, and for $\eta < 1$ they should be greater than $\sigma_{Z-2,Z-3}^0$. In particular, for $\eta \ll 1$ when the electron-capture cross sections are proportional to $p(n_0)[I(n_0)]^{5/2}$, the values of $\sigma_{Z-2,Z-3}^m$, in agreement with the known values^[18] of $I(n_0)$, should exceed the values of $\sigma_{Z-2,Z-3}^0$ by a factor $4[(Z-0.2)/(Z-1.75)]^5$, i.e., for ions with $Z = 7-10$ by an order of magnitude, and for ions with $Z = 3-4$ by two orders of magnitude.

The ratios $\sigma_{Z-2,Z-3}^m/\sigma_{Z-2,Z-3}^0$ can be estimated on the basis of the experimentally determined dependence of $\sigma_{i,i-1}/p(n_0)$ on $I(n_0)$. For those cases in which the average cross sections for electron capture by helium-like ions in our experiments were determined for two different means of formation, the values of η for metastable particles lie in the range from 0.8 (for B^{+3} ions at $v = 12 \times 10^8$ cm/sec) to ~ 7 (for the same B^{+3} ions at $v = 4 \times 10^8$ cm/sec) and therefore the values of $\sigma_{Z-2,Z-3}^m/\sigma_{Z-2,Z-3}^0$ should lie in the range from ~ 1 to ~ 0.1 , and the values of $\sigma_{Z-2,Z-3}^0/\sigma_{Z-2,Z-3}^j$ between 1.0 and 1.1. These values are in good agreement with the direct experimental evaluations of these ratios obtained in Sec. 5.

For singly charged lithium ions with velocities 4×10^8 , 8×10^8 , and 12×10^8 cm/sec, for which only values of σ_{10}^S were obtained in our experiments, the values of η are respectively 4.4, 0.6, and 0.3. As a result the ratios $\sigma_{10}^{II}/\sigma_{10}^0$ should be close to 3, 5, and 30, and the values of $\sigma_{10}^0/\sigma_{10}^S$ should be ~ 1.5 , ~ 0.3 , and ~ 0.2 , i.e., the cross sections for electron capture by unexcited Li^+ ions should be 1.6, ~ 3 , and ~ 5 times smaller than the experimentally determined average values of σ_{10}^S . This is confirmed by comparison of the values of σ_{10}^S with the values $\sigma_{i,i-1}^0$ found from the general curve for all particles of $\sigma_{i,i-1}^0/p(n_0)$ as a function of $I(n_0)$ (see Fig. 6).

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¹⁾From the results of the present work (see Sec. 6) it follows that for ions with $Z \geq 6$ the dependence of the cross sections $\sigma_{Z-2,Z-1}^0$ on the binding energy I will turn out to be somewhat more rapid than for hydrogen-like particles (see Fig. 3). However, allowance for this circumstance leads to a change in the value of b/a , and consequently, also in the coefficient K^0 , by no more than 1%.

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