Effect of fluctuation electron pairs on the absorption of sound in metals above the superconducting transition point

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It is shown that the existence of fluctuation Cooper electron pairs in the normal state near the critical temperature appreciably affects the sound absorption in metals. The effect increases in films and filamentary samples. In dirty films of thickness $d \ll [\xi_0 lT_c/(T-T_c)]^{1/2}$, the fluctuation correction to the absorption coefficient can be of the order of the leading term at temperature $T-T_c \sim 10^{-2}-10^{-3}$ °K. It is shown that, near the transition temperature and for large sound intensities (attainable in experiment), the fluctuation correction to the absorption coefficient depends on the sound intensity.

1. The absorption of sound in metals both in the normal and in the superconducting state has been studied in a large number of researches (see[1,2]). Works have appeared in recent years in which nonlinear effects in the sound absorption in normal[3] and superconducting[4] metals have been taken into account.

As is known, the absorption of sound in metals in the superconducting state at $\hbar\omega_0 < 2\Delta$ (ω_0 is the frequency of sound, Δ the energy gap in the electron spectrum) differs appreciably from the absorption in the normal state, because of the presence of a gap in the spectrum of the electrons. This is connected with the fact that in the normal metal phonons of suitably low frequency can decompose into pairs of quasiparticles. This is possible in the superconductor only at sound frequencies above the threshold.

The purpose of this research was the investigation of the effect of fluctuation electron pairs[5] on the absorption of low-frequency sound in "dirty" metals ($l \ll \xi_0$; l is the mean free path, ξ_0 the coherence length) above T_c. Because of the formation of fluctuation electron pairs above T_c, the density of states of electrons in "dirty" metals, in the narrow range $\sim(T-T_c)$ close to the Fermi surface, undergoes significant change in comparison with the density of states of the normal metal far from the transition temperature,[6-8] which should have an important effect on the law of absorption of low-frequency sound ($\hbar\omega_0 < T-T_c$) in the region $T \gtrsim T_c$.

In the present paper we study nonlinear effects in the sound absorption of "dirty" metals, due to the fluctuation electron pairs. The parameter of nonlinearity in this case, as in the case of fluctuation conductivity,[9,10] depends strongly on the closeness of the temperature to the temperature of the superconducting transition.

2. The interaction of the electron with low-frequency sound will be described in terms of the deformation potential.[4,11,12] The Hamiltonian of the interaction is of the form

$$\int H_{int}\,dt_1 = \int \Lambda_{ik}(x_1-x_2)\,u_{ik}(x_2)\,\psi^+(x_1)\,\psi(x_2)\,dx_1\,dx_2. \tag{1}$$

Here $x = (\mathbf{r}, t)$, ψ and ψ^+ are the electron creation and annihilation operators. The Fourier component $\Lambda_{ik}(\mathbf{p})$ of the spatial part of the tensor

$$\Lambda_{ik}(x_1-x_2) = \Lambda_{ik}(\mathbf{r}_1-\mathbf{r}_2)\,\delta(t_1-t_2), \tag{2}$$

is the so-called deformation potential, the components of which are of the order of the Fermi energy. It should also be noted that, because of the condition of electrical neutrality, the value of the tensor $\Lambda_{ik}(\mathbf{p})$, averaged over the Fermi sphere, is equal to zero. Further,

$$u_{ik}(\mathbf{r},t) = \frac{1}{2}\left(\frac{\partial u_i}{\partial r_k} + \frac{\partial u_k}{\partial r_i}\right) \tag{3}$$

is the deformation tensor, $\mathbf{u}$ the displacement vector of the medium. For convenience, we put the deformation tensor in the form

$$u_{ik}(\mathbf{r},t) = u_{ik}^0 \exp(i\mathbf{kr} - i\omega_0 t), \tag{4}$$

however, in the final results, we shall mean its real part only.

As in the work of Prokrovskiĭ and Savvinykh,[12] we shall calculate, for the determination of the sound absorption coefficient, the mean work performed on the electrons in the period of the oscillation, which gives the power of the sound losses; referred to unit volume, it is equal to

$$W = \langle \sigma_{ik}\dot{u}_{ik}(\mathbf{r},t)\rangle_t, \tag{5}$$

where $\langle\ldots\rangle_t$ denotes the average over the period of oscillation. The generalized force is

$$\sigma_{ik} = \partial\langle H\rangle/\partial u_{ik}, \tag{6}$$

where averaging over the state of the system is understood. Using (1), (5) and (6), we obtain

$$W = i\left\langle \int \Lambda_{ik}(x'-x)\,\dot{u}_{ik}(x)\,G^+(x,x')\,dx' \right\rangle_t, \tag{7}$$

where G^+ is the Green's function of the electron, defined in the Keldysh technique.[13]

3. The sound absorption far from the temperature of transition to the normal state is determined by an elementary graph in the lowest approximation in the interaction of the electron with the acoustic field. In the Keldysh technique, it is given by the expression

$$\delta\hat{G}_{(1)}(x,x') = \int \Lambda_{ik}(x_1-x_2)\,u_{ik}(x_2)\,\hat{G}(x,x_1)\,\sigma_z\hat{G}(x_2,x')\,dx_1\,dx_2, \tag{8}$$

where σ_z is the Pauli matrix. The Green's function is a matrix of second order, one of the components of which is the G^+ function. After simple transformations with the use of (4), we obtain

$$\delta G_{(1)}^{+}(x,x') = -\frac{1}{2}\int \frac{d\mathbf{p}\,d\varepsilon}{(2\pi)^4}\Lambda_{ik}(\mathbf{p})\,u_{ik}(\mathbf{r}',t') \times\left(\operatorname{th}\frac{\varepsilon}{2T} - \operatorname{th}\frac{\varepsilon-\omega_0}{2T}\right) G_\varepsilon^A(\mathbf{p})\,G_{\varepsilon-\omega_0}^R(\mathbf{p}-\mathbf{k})\,e^{i\mathbf{p}(\mathbf{r}-\mathbf{r}')-i\varepsilon(t-t')}. \tag{9}$$

In the derivation of this formula, we have taken into account only the "nonregular part"[14] of the Green's function; the "regular part" does not make a contribution to the absorption.

Substituting (9) in (7) and carrying out simple integration, we get for the power loss in the normal state

$$W_n = -i\nu\int\frac{d\Omega}{4\pi}\langle\varphi_{\mathbf{p}_0}{}^2(\mathbf{r},t)\rangle_t\left(\omega_0 - \mathbf{v}_0\mathbf{k} - \frac{i}{\tau}\right)^{-1}. \tag{10}$$

Here

$$\varphi_{p_0}(\mathbf{r},\ t)=\Lambda_{ik}(\mathbf{p}_0)u_{ik}(\mathbf{r},\ t);$$

$\mathbf{p}_0$ is the momentum, $\mathbf{v}_0$ is the Fermi velocity, $\nu = mp_0/2\pi^2$ is the density of states of the electrons, and τ the momentum relaxation time and is identical with the transport time in the case of isotropic scattering of electrons by impurities. In the derivation of (10), we used $\Lambda_{ik}(\mathbf{p}_0-\mathbf{k}) \approx \Lambda_{ik}(\mathbf{p}_0)$, which is quite natural for the case of low-frequency sound absorption. The expression (10) divided by the sound energy flux density gives the well-known result for the sound absorption in normal metals.

In what follows, we shall be interested in the case $kl \ll 1$ ($l = v_0\tau$). In this case,

$$W_n=\nu\tau\int\frac{d\Omega}{4\pi}\langle\varphi_{p_0}{}^2(\mathbf{r},t)\rangle_t\equiv\nu\langle A(\mathbf{r},t)\rangle_t. \tag{11}$$

4. In the study of the fluctuation correction to the sound absorption, we have limited ourselves to "dirty" metals, since the fluctuation effects in them are more significant. Estimates show that the fundamental contribution to the fluctuation correction for the absorption of low-frequency sound is due to the Maki graph.[15] Using the Keldysh technique, the nonregular part of the Maki graph for the Green's function of the electron in first order in the fluctuations can be represented in the form of Fig. 1. The solid lines with the labels R and A correspond to the Green's function of the free electron

$$G_\varepsilon{}^R(\mathbf{p})=(\varepsilon-\varepsilon(\mathbf{p})+\varepsilon_F+i/2\tau)^{-1}=[G_\varepsilon{}^A(\mathbf{p})]^*, \tag{12}$$

the wavy line corresponds to the thermodynamic Cooper vertex function[7]

$$K_\omega(\mathbf{q})=\operatorname{cth}\frac{\omega}{2T}[K_\omega{}^R(\mathbf{q})-K_\omega{}^A(\mathbf{q})],$$
$$K_\omega{}^R(\mathbf{q})=i\frac{8T}{\pi\nu}[\omega+iDq^2+i\eta]^{-1}=[K_\omega{}^A(\mathbf{q})]^*,\quad \eta=\frac{8}{\pi}(T-T_c), \tag{13}$$

where $D = v_0 l/3$ is the diffusion coefficient. The triangles on the graph correspond to averaging over the impurities of the product of the electron Green's functions, the broken line describes the sound wave. It is also necessary to point out that the entire nonlinear dependence of the interaction of the electron with the sound field is contained in these functions.

We denote by $\Gamma^{AR}_{\epsilon,\,\omega-\epsilon}$ and $\Gamma^{AR}_{\omega-\epsilon,\epsilon}$ expressions corresponding in Fig. 1 to the first and second triangles, respectively. The Green's function corresponding to this graph can be described in the following fashion:

$$\delta G_\Phi{}^+(x,x')=-\frac{i}{4}\int\frac{d\mathbf{p}\,d\varepsilon}{(2\pi)^4}\frac{d\mathbf{q}\,d\omega}{(2\pi)^4}\Lambda_{ik}(p)u_{ik}(r',t')K_\omega(\mathbf{q})$$
$$\times\left(\operatorname{th}\frac{\varepsilon}{2T}-\operatorname{th}\frac{\varepsilon-\omega_0}{2T}\right)G_\varepsilon{}^A(\mathbf{p})G^R_{\omega-\varepsilon}(\mathbf{q}-\mathbf{p})G^A_{\omega+\omega_0-\varepsilon}(\mathbf{q}+\mathbf{k}-\mathbf{p})G^R_{\varepsilon-\omega_0}(\mathbf{p}-\mathbf{k}) \tag{14}$$
$$\cdot\Gamma^{AR}_{\varepsilon,\omega-\varepsilon}\Gamma^{AR}_{\omega-\varepsilon,\varepsilon}\exp[i\mathbf{p}(\mathbf{r}-\mathbf{r}')-i\varepsilon(t-t')].$$

Substituting (14) in (7), setting

$$\operatorname{th}\frac{\varepsilon}{2T}-\operatorname{th}\frac{\varepsilon-\omega_0}{2T}\approx\frac{\omega_0}{2T}$$

and carrying out the integration over the momenta of the electrons, we obtain

$$W_f=i\frac{\pi\nu\tau^3}{2T}\int\frac{d\mathbf{q}\,d\omega}{(2\pi)^4}\frac{d\varepsilon}{2\pi}K_\omega(q)\left\langle\Gamma^{AR}_{\varepsilon,\omega-\varepsilon}\Gamma^{AR}_{\omega-\varepsilon,\varepsilon}\int\frac{d\Omega}{4\pi}\varphi_{p_0}{}^2(\mathbf{r},t)\right\rangle_t. \tag{15}$$

Proceeding to the calculation of the function Γ^{AR}, we first write down the expressions for the vertex functions γ^{AR}, which correspond to the zeroth order in the interaction of the electron with the sound field. As noted above, these functions appear because of averaging over the impurities of the products G^AG^R and have been calculated more than once (see, for example,[14]). We write down the result:

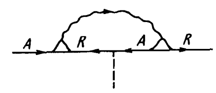

FIG. 1

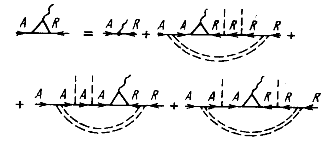

FIG. 2

$$\gamma^{AR}_{\varepsilon,\omega-\varepsilon}=-\frac{i}{2\tau}\left(\varepsilon-\frac{\omega}{2}-i\frac{Dq^2}{2}-\frac{i}{2\tau_0}\right)^{-1},$$
$$\gamma^{AR}_{\omega-\varepsilon,\varepsilon}=\frac{i}{2\tau}\left(\varepsilon-\frac{\omega}{2}+i\frac{Dq^2}{2}+\frac{i}{2\tau_0}\right)^{-1}, \tag{16}$$

where τ_0 is the homogeneous relaxation time,[16] with $\tau_0^{-1} \sim T_c^3/\Theta_0^2$ for the electron-phonon interaction and $\tau_0^{-1} \sim T_c^2/\epsilon_F$ for the electron-electron interaction (Θ_0 is the Debye temperature, ϵ_F the Fermi energy). In these formulas, τ_0 is introduced artificially in order to avoid divergence in the integral (15). We can neglect the wave vector $\mathbf{k}$ in (16) in comparison with q; in the same way, we impose the following limitation on the sound wavelength:

$$\lambda\gg\max\left[\left(\xi_0 l\frac{T_c}{T-T_c}\right)^{1/2},\ (l\xi_0\tau_0T_c)^{1/2}\right],$$

where ξ_0 is the coherence length.

The function $\Gamma^{AR}_{\epsilon,\omega-\epsilon}$ in any order in the interaction of the electron with the sound field can be obtained from the graph equation which is shown in Fig. 2. The first term on the right side is $\gamma^{AR}_{\epsilon,\omega-\epsilon}$. The pair of dashed lines without free ends corresponds to averaging over the impurities and is correlated with the expression

$$-\frac{i}{2\pi\tau^2}\left(\varepsilon_1-\varepsilon_2-iDq^2-\frac{i}{\tau_0}\right)^{-1}.$$

As noted above, the wavy lines with free ends are associated with $\varphi_p(\mathbf{r}, t)$. After all this, the graphic equation can be rewritten in the following form:

$$\Gamma^{AR}_{\varepsilon,\omega-\varepsilon}=\gamma^{AR}_{\varepsilon,\omega-\varepsilon}+\frac{1}{2\pi\tau}\gamma^{AR}_{\varepsilon,\omega+\omega_1+\omega_2-\varepsilon}\int\frac{d\Omega}{4\pi}\varphi_{p_0}(\mathbf{r},t)\varphi_{p_0}(\mathbf{r},t)\cdot$$
$$\times\left\{\Gamma^{AR}_{\varepsilon-\omega_1-\omega_2,\omega+\omega_1+\omega_2-\varepsilon}\int d\xi[G_\varepsilon{}^A(\xi)]^3G_{-\varepsilon}{}^R(\xi)+\Gamma^{AR}_{\varepsilon,\omega-\varepsilon}\int d\xi G_\varepsilon{}^A(\xi)\right. \tag{17}$$
$$\left.\times[G_{-\varepsilon}{}^R(\xi)]^3+\frac{1}{2}[\Gamma^{AR}_{\varepsilon-\omega_1,\omega+\omega_1-\varepsilon}+\Gamma^{AR}_{\varepsilon-\omega_2,\omega+\omega_2-\varepsilon}]\int d\xi[G_\varepsilon{}^A(\xi)]^2[G_{-\varepsilon}{}^R(\xi)]^2\right\},$$

where ω_1 and ω_2 denote the frequency of the sound wave. The last two terms in (17) represent the symmetrized form of the expression corresponding to the last graph in Fig. 2. Integration over ξ in (17) with subsequent expansion in ω_1 and ω_2 gives the equation

$$\Gamma^{AR}_{\varepsilon,\omega-\varepsilon}=\gamma^{AR}_{\varepsilon,\omega-\varepsilon}+\tau A(\mathbf{r},t)\gamma^{AR}_{\varepsilon,\omega-\varepsilon}d^2\Gamma^{AR}_{\varepsilon,\omega-\varepsilon}/d\varepsilon^2. \tag{18}$$

The solution of this equation, bounded on the entire real axis of ϵ, takes the form

$$\Gamma^{AR}_{\varepsilon,\omega-\varepsilon}=\frac{1}{2\tau}\int_0^\infty dx\exp\left\{-i\left(\varepsilon-\frac{\omega}{2}-i\frac{Dq^2}{2}-\frac{i}{2\tau_0}\right)x-\frac{1}{6}A(\mathbf{r},t)x^3\right\}. \tag{19}$$

Similar discussions show that

$$\Gamma^{AR}_{\omega-\varepsilon,\varepsilon}=[\Gamma^{AR}_{\varepsilon,\omega-\varepsilon}]^*. \tag{20}$$

Substituting (19) and (20) in (15), and carrying out integration over ϵ and ω, we get

$$W_f = -2T\int \frac{d\mathbf{q}}{(2\pi)^3}(Dq^2+\eta)^{-1} \times \left\langle A(\mathbf{r},t)\int_0^\infty dx \exp\left\{-(Dq^2+\tau_0^{-1})x - \frac{1}{3}A(\mathbf{r},t)x^3\right\}\right\rangle_t . \tag{21}$$

Integrating over the momentum of the fluctuations, we find, in the three-dimensional case,

$$W_f^{(3)} = -\frac{T}{2(\pi D)^{3/2}}\int_0^\infty dx\cdot x^{-3/2}\{1-\sqrt{\pi\eta x}[1-\Phi(\sqrt{\eta x})]e^{\eta x}\} \times \left\langle A(\mathbf{r},t)\exp\left(-\tau_0^{-1}x - \frac{1}{3}A(\mathbf{r},t)x^3\right)\right\rangle_t , \tag{22}$$

in the case of a thin film:

$$W_f^{(2)} = \frac{T}{2d\pi D}\int_0^\infty dx\,\mathrm{Ei}(-\eta x) \times \left\langle A(\mathbf{r},t)\exp\left[(\eta-\tau_0^{-1})x - \frac{1}{3}A(\mathbf{r},t)x^3\right]\right\rangle_t \tag{23}$$

and in the case of a thin filament:

$$W_f^{(1)} = -\frac{T}{S\sqrt{\eta D}}\int_0^\infty dx[1-\Phi(\sqrt{\eta x})] \times \left\langle A(\mathbf{r},t)\exp\left[(\eta-\tau_0^{-1})x - \frac{1}{3}A(\mathbf{r},t)x^3\right]\right\rangle_t . \tag{24}$$

In these formulas, d is the thickness of the thin film, S is the cross-section area of the thin filament, $\Phi(x)$ is the probability integral and $\mathrm{Ei}(x)$ is the integral exponential function.

For an estimate of the size of the effect, we first consider the absorption of low-intensity sound. As a criterion of smallness, as is seen from the last formula, we use

$$\langle A(\mathbf{r},t)\rangle_t = \tau\int\frac{d\Omega}{4\pi}\langle\varphi_{p_0}^2(\mathbf{r},t)\rangle_t \ll \min(\eta^3, \tau_0^{-3}). \tag{25}$$

Employing the relations (11) and (22)—(25), it is not difficult to see that the absolute value of the ratio of the fluctuation absorption coefficient to the normal coefficient is equal to

$$\left|\frac{\alpha_f^{(3)}}{\alpha_n}\right| = \frac{3\sqrt{3}\pi T_c}{4\varepsilon_F^2\tau^{3/2}(\eta^{1/2}+\tau_0^{-1/2})},$$
$$\left|\frac{\alpha_f^{(2)}}{\alpha_n}\right| = \frac{3\pi T_c}{2\varepsilon_F\tau p_0 d(\eta-\tau_0^{-1})}\ln\eta\tau_0, \tag{26}$$
$$\left|\frac{\alpha_f^{(1)}}{\alpha_n}\right| = \frac{2\sqrt{3}\pi^2\tau_0^{1/2}T_c}{Sp_0^2(\eta\tau)^{1/2}(\eta^{1/2}+\tau_0^{-1/2})}$$

We estimate the ratios of the absorption coefficients (26) in the three-dimensional and two-dimensional cases. For otherwise equal conditions, these ratios take on maximum values when the temperature of the sample is so close to critical that the condition $\eta \leq 1/\tau_0$ is satisfied. In real metals with a transition temperature of 10°K, we have the order-of-magnitude estimate $\tau_0^{-1} \sim T_c^2/\epsilon_F \sim 10^{-3}$ °K. Under these conditions, as seen from (26), the ratio of the absorption coefficients is of the order of unity for a bulk sample only in the extreme "dirty" case, $\epsilon_F\tau \sim 1$. In the case of a thin film of thickness 100 atomic layers, the ratio (26) is of the order of unity in the case $\epsilon_F\tau \sim 100$.

We now consider the absorption of sound of high intensity, satisfying the condition

$$\langle A(\mathbf{r},t)\rangle_t = \tau\int\frac{d\Omega}{4\pi}\langle\varphi_{p_0}^2(\mathbf{r},t)\rangle_t \gg \max(\eta^3, \tau_0^{-3}). \tag{27}$$

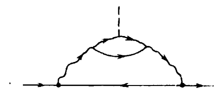

FIG. 3

In this case it is not difficult to obtain the following asymptotic values for the ratios of the absorption coefficients:

$$\left|\frac{\alpha_f^{(3)}}{\alpha_n}\right| \approx 4{,}7T_c/\varepsilon_F^2\tau^{3/2}\langle A(\mathbf{r}\ t)\rangle_t^{1/6},$$
$$\left|\frac{\alpha_f^{(2)}}{\alpha_n}\right| \approx \frac{5{,}4T_c}{\varepsilon_F\tau p_0 d\langle A(\mathbf{r},t)\rangle_t^{1/3}}\ln\frac{\langle A(\mathbf{r},t)\rangle_t^{1/3}}{\eta}, \tag{28}$$
$$\left|\frac{\alpha_f^{(1)}}{\alpha_n}\right| \approx 37T_c/Sp_0^2(\eta\tau)^{1/2}\langle A(\mathbf{r},t)\rangle_t^{1/3}.$$

The condition (27) can be rewritten in the more customary form if we introduce the sound intensity $I \sim \rho s\omega_0^2\mathbf{u}^2$ (s is the sound velocity, ρ the density of the crystal). Actually, recognizing that in order of magnitude

$$\langle A(\mathbf{r},t)\rangle_t \sim I\tau\varepsilon_F^2k^2/\rho s,$$

it is not difficult to rewrite (27) in the form of a condition imposed on the sound intensity:

$$I \gg I_c \sim \rho s\max(\eta^3, \tau_0^{-3})/\hbar\tau\varepsilon_F^2k^2. \tag{29}$$

Using typical values $l \sim 10^{-6}$ cm, $k \sim 10^3$ cm^{-1}, we get the following estimate of the critical intensity

$$I_c \sim 10^9\frac{\max(\eta^3, \tau_0^{-3})}{T_c^3}\quad \text{W/cm}^2.$$

It follows from (28) that the absolute value of the fluctuation correction to the sound absorption decreases with increase in intensity. The frequency dependence of this correction in the nonlinear region has the nonanalytic form

$$\alpha_f^{(3)} \sim \omega^{4/3},\quad \alpha_f^{(2)} \sim \omega^{2/3}\ln\omega.$$

Finally, we shall discuss briefly the contribution of the graph of Aslamazov and Larkin (Fig. 3) to the fluctuation absorption of low-frequency sound in the linear approximation in the sound intensity. Direct estimates show that, although the temperature dependence of the absorption coefficient of long wave sound ($Dk^2 \ll T - T_c$) which corresponds to this graph (for example, in the three-dimensional case $\sim (T - T_c)^{-3/2}$) is much stronger than for graph 1, this contribution is nevertheless smaller than the contribution from graph 1 for sufficient smallness of the parameter $Dk^4/(T - T_c)p_0^2 \ll 1$.

In conclusion, I wish to express my gratitude to A. F. Andreev and B. T. Geĭlikman for interest in the research and discussion of the results.

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