The possibility of the laser effect in stellar atmospheres

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The possibility of the existence population inversions in the atmospheres of hot (Be) stars is considered on the example of the OI oxygen. The condition for self-excitation of a medium with a population inversion as a result of resonance scattering at the amplifying transition is found. The generation-line width due to the contribution of the spontaneous radiation is found for such a laser, which pertains to the class of lasers with nonresonant feedback. It is shown that the generation-line width may be significantly narrower than the gain-line width. Methods for the experimental observation of the laser effects in stellar atmospheres are considered.

1. INTRODUCTION. FORMULATION OF THE PROBLEM

Radio astronomical observations of the microwave absorption spectra of molecules in the interstellar medium have led to the discovery of the emission lines of the radical OH, the molecule H_2O, etc., with anomalously high intensities and extremely small angular dimensions[1]. The brightness temperature of the radiation attains values of the order of 10^{13}°K, which can be explained only with the aid of the effect of stimulated emission at the corresponding lines[2].

It is natural to attempt to investigate the possibility of the appearance of the effects of stimulated emission in astrophysical objects in the optical frequency region. The idea that the laser effect is possible in stellar atmospheres was put forward earlier[3], and certain stellar-spectrum anomalies qualitatively corroborating it have been considered. There exist in stellar atmospheres conditions reminiscent of those that exist in a low-pressure gas laser. There occur in stellar spectra, besides absorption lines, bright emission lines whose appearance is explained[4] by the excitation of atoms during collisions, by the recombination of ions, as well as by fluorescent excitation based on the accidental coincidence of a bright emission line of one element with the absorption line of another and the subsequent successive transition of the excited atom.

However, in a number of cases these mechanisms do not give a satisfactory account of the anomalies observed in stellar spectra. Furthermore, it can be seen from the analysis of the anomalous emission lines presently known in the astrophysics literature[5] that certain schemes used by astrophysicists in attempts to explain these anomalies are essentially astrophysical analogs of the three- and four-level optical-pumping schemes used at present in lasers. Such an analogy suggests the possible existence in certain stellar atmospheres of conditions that facilitate the creation of population inversion at the corresponding transitions.

As an example of one of the still unexplained anomalies, we can point out the behavior of the 7774- and 8446-Å OI-oxygen lines in the spectra of the Be stars[5,6]. The ratio of the intensities of the 8446-Å ($3p^3P - 3s^3S$) and 7774-Å ($3p^5P - 3s^5S$) lines differs from the laboratory value. Bowen[7] has suggested that this is connected with the phenomenon of fluorescence caused by the coincidence of the bright HI-hydrogen line L_β 1025.72 Å with the 1025.77-Å ($2p^3P - 3d^3D$) OI absorption line. On the other hand, it can be seen from the transition scheme (Fig. 1) under consideration that the fluorescence process coincides with a four-level generator scheme and that, owing to the successful correlation of the probabilities A_{ij} of spontaneous transitions between the levels 3 – 2 and 2 – 1, a population inversion can arise at the 3 – 2 transition. The considered example is of even greater interest in that generation in the continuous regime has been realized under laboratory conditions at the 8446-Å line[8].

In the present paper, using the 8446-Å OI-oxygen line as an example, we investigate the problem of the possibility of the existence of population inversions in stellar atmospheres and the possibility of generation in such a medium owing to the return of part of the radiation as a result of the resonance scattering of photons at the amplifying transition back into the amplifying medium. Then we consider the question which characteristics of the emission lines due to stimulated emission will differ from the characteristics of the emission lines due to the spontaneous processes, and discuss possible methods for the detection of the spectrum-narrowing effect for the emission generation-line spectrum.

2. POPULATION INVERSION AT THE OI LEVELS IN A STELLAR ATMOSPHERE

Let us consider the specific case of the appearance of inversion under stellar atmosphere conditions at the 3–2 OI transition owing to optical pumping, according to the Bowen scheme, of the level 4 with a subsequent cascade transition to the level 3 (Fig. 1).

Let us assume that a particle gas of density N_0 and

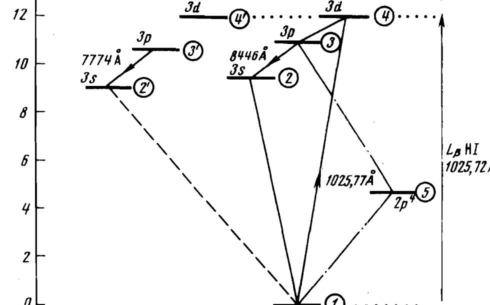

FIG. 1. Energy-level diagram of OI oxygen. The spontaneous-transition probabilities, computed by the method used in [20], are equal (in sec^{-1}) to: $A_{21} = 5 \times 10^8$, $A_{32} \approx A_{43} \approx A_{41} \approx A_{4'1'} \approx A_{3'2'} = 3 \times 10^7$; $A_{2'1'} = 10^5$; $A_{4'3'} = 2 \times 10^6$. The multiplicities of the degeneracies of the levels are: $g_1 = 9$, $g_2 = 3$, $g_3 = 9$, and $g_4 = 15$.

temperature T_0 is located in the field of radiation of spectral density U and frequency $\omega \approx \omega_{41}$. The balance equations for the populations in the steady-state case can be written in the general form:

$$n_i \sum_{j\neq i} p_{ij} - \sum_{j\neq i} n_j p_{ji} = 0,$$

$$\sum_{j=1}^{4} n_j = N_0; \quad i, j = 1, 2, 3, 4. \tag{1}$$

The probabilities of the transitions between the levels i and j can be represented with the aid of the Einstein coefficients A_{ij} and B_{ij} in the form

$$p_{ij} = A_{ij} + B_{ij}U_{ij}^0 + d_{ij} + B_{ij}U_{ij} = p_{ij}^0 + B_{ij}U_{ij},$$
$$p_{ji} = B_{ji}U_{ij}^0 + d_{ji} + B_{ji}U_{ij} = p_{ji}^0 + B_{ji}U_{ij}.$$

Here the d_{ij} are the nonradiative-transition probabilities,

$$p_{ji}^0/p_{ij}^0 = (g_i/g_j)\exp(-\hbar\omega_{ij}/kT_0),$$

g_i is the multiplicity of the degeneracy of the level i, and U_{ij}^0 is the equilibrium radiation density corresponding to the temperature T_0.

The solution to the system (1) has the form

$$n_1 = \frac{N_0}{D}[p_{21}p_{32}(p_{43}+p_{41}) + p_{41}p_{34}(p_{21}+p_{23})],$$
$$n_2 = \frac{N_0}{D}[p_{43}p_{32}(p_{12}+p_{14}) + p_{41}p_{12}(p_{32}+p_{34})],$$
$$n_3 = \frac{N_0}{D}[p_{12}p_{23}(p_{43}+p_{41}) + p_{14}p_{43}(p_{21}+p_{23})],$$
$$n_4 = \frac{N_0}{D}[p_{34}p_{23}(p_{14}+p_{12}) + p_{14}p_{21}(p_{32}+p_{34})]. \tag{2}$$

Here D is the sum of all the terms enclosed by the square brackets, and the 4–2 and 3–1 transitions have been neglected on the grounds that they are less probable.

The difference between the populations of the levels 3 and 2 can be represented in the form

$$\Delta n = \frac{n_3}{g_3} - \frac{n_2}{g_2} = \frac{N_0}{g_3}\frac{\mu(p_{14}-p_{14}^{\text{thr}})}{\Delta}\left[1 - \frac{1}{p_{21}}\left(\frac{g_3}{g_2}p_{32} - p_{23}\right)\right], \tag{3}$$

where

$$p_{14}^{\text{thr}} = \frac{p_{12}}{p_{21}}\frac{(p_{32}g_3/g_2 - p_{23})(p_{41}+p_{43}) + p_{41}p_{43}g_3/g_2}{p_{43}[1 - p_{21}^{-1}(p_{32}g_3/g_2 - p_{23})]}, \tag{4}$$

$$\Delta = \mu p_{14}\left(1 + \frac{p_{32}+p_{34}}{p_{43}}\right) + \left(p_{32} + \mu p_{41}\frac{p_{34}}{p_{43}}\right)\left(1 + \frac{p_{12}}{p_{21}}\right) + \frac{p_{12}}{p_{21}}p_{23}\left(1 + \mu\frac{p_{34}}{p_{43}}\right),$$
$$\mu = p_{43}/(p_{41}+p_{43}).$$

Besides the quantity Δn, we shall need below the quantity

$$\Sigma = n_2 + n_3 = \frac{N_0}{\Delta}\left[\mu p_{14}\left(1 + \frac{p_{32}+p_{23}}{p_{21}}\right) + \frac{p_{12}}{p_{21}}\left(p_{23} + p_{32} + \mu p_{41}\frac{p_{34}}{p_{43}}\right)\right]. \tag{5}$$

It should be noted that in four-level generators a significant influence on the threshold pump level is exerted by the population of the level 2, and that this population depends on the temperature of the radiation actuating the 1–2 transition (the nonradiative transitions contribute little to the case under consideration). If the gas were in equilibrium with the radiation, then the radiation temperature would coincide with the temperature T_0 of the gas. However, there exists in stellar atmospheres hotter radiation emanating from the photosphere and characterized by a temperature T_* considerably exceeding T_0. Therefore, below we shall assume that the 1–2 transition is actuated by radiation of temperature T_* [1)].

Notice that optical excitation by the continuous radiation of the photosphere is possible without resonance coincidence. But for population inversion to be obtained in this case at some transition, such continuous radiation must have a nonequilibrium spectrum. Mustel'[9] has shown that the continuous spectrum of the photosphere can be very different from the Planck distribution. Although in the concrete OI example under consideration the indicated spectral nonequilibrium turns out to be insufficient for the appearance of population inversion at the 3–2 transition, it must be borne in mind when computing other cases.

Let us now estimate the possible value of the amplification factor α_ω for the 8446-Å OI line in the atmospheres of the Be stars. The amplification factor per unit length in a medium with a given inverse-population value Δn is equal to

$$\alpha_\omega = \frac{\lambda^2}{2\pi}\frac{(\pi\ln 2)^2}{2}\frac{\gamma}{\Delta\omega_D}\exp\left[-\frac{(\omega-\omega_0)^2}{\Delta\omega_D^2}\ln 2\right]g_3\Delta n;$$
$$\Delta\omega_D = \left[\frac{2kT_0}{M_{OI}}\ln 2\right]^{1/2}\frac{\omega_0}{c} \tag{6}$$

is the Doppler halfwidth of the line and $\gamma = A_{32}$.

The density of the OI atoms in the region of the atmosphere close to the outer boundary of the photosphere (i.e., in the inverting layer) is $N_0 \sim 10^5$–$10^6\,\text{cm}^{-3}$, the gas temperature here is $T_0 \sim 1.5\times10^4\,{}^\circ$K, and the effective temperature of the photosphere is $T_* \sim 3\times10^4\,{}^\circ$K[10]. If we assume that the temperature of the radiation emitted in the 1–2 transition is of the order of T_*, then, as follows from (3) and (4), the requisite brightness temperature of the fluorescent pump at the threshold should be equal to $4\times10^\circ$K, while, in order to obtain an inverse population of, say, $\Delta n \sim 10^{-6}N_0$, it must exceed this value (i.e., it should be $\approx 4.1\times10^4\,{}^\circ$K).

Besides optical pumping, the contribution made by collisions with electrons to the inversion of the levels 3 and 2 (the excitation of the level 3 via the metastable level 5) was estimated. Inversion is not attainable through this mechanism when the 1–2 transition is stimulated by radiation of temperature T_* in the absence of stimulation by radiation of the other transitions.

The line widths necessary for the estimate are: $\gamma \approx 3\times10^7\,\text{sec}^{-1}$, $\Delta\omega_D \approx 5\times10^{10}\,\text{sec}^{-1}$. Substituting these values and $\Delta n = 10^{-6}N_0$ into (6), we obtain the following value for the amplification factor at the line center: $\alpha_0 \equiv \alpha_{\omega_0} \sim 10^{-12}\,\text{cm}^{-1}$.

3. NONCOHERENT FEEDBACK DUE TO RESONANCE SCATTERING

It was shown earlier[11] that in cosmic sources of radio-frequency radiation of anomalously high brightness temperature a generation regime maintained by part of the radiation returning to the amplifying volume as a result of resonance scattering at the amplifying transition is, in principle, possible. In the optical region resonance scattering can be play a more important role because of the larger value of the factor γ/Γ (Γ is the homogeneous line width).

The coefficient per unit length of resonance scattering into a solid angle of 4π in the medium has (for a Doppler line shape) the form[11]

$$\sigma_\omega = \frac{\lambda^2}{2\pi}\frac{(\pi\ln 2)^{1/2}}{2}\frac{\gamma^2}{\Delta\omega_D\,\Gamma}\exp\left[-\frac{(\omega-\omega_0)^2}{\Delta\omega_D^2}\ln 2\right](n_2+n_3). \tag{7}$$

For the conditions considered at the end of the preceding section, $n_2 + n_3 \approx 10^{-2} N_0$ and $\Gamma \approx 5 \times 10^8\ \mathrm{sec}^{-1}$, and we obtain the estimate: $\sigma_0 \equiv \sigma_{\omega_0} \approx 10^{-10}\ \mathrm{cm}^{-1}$.

The attractiveness of the generator model lies in the fact that a high gain per pass is not required when the scattering is sufficiently effective, since above the threshold, when the gain per pass exceeds the loss per pass, the intensity grows in the course of many passes.

The problem consists in the determination of the so-called threshold or (in the spherically symmetric model) of the critical radius r_0 at which the system is on the threshold of self-excitation. For the case when $\alpha_\omega \ll \sigma_\omega$ the solution of the problem is known[12]; however, as can be seen from (6) and (7), depending on the values of the quantities determining α_ω and σ_ω, the relation between the latter quantities can be arbitrary, and it is necessary to consider the problem in its general form. We can, in determining the threshold, restrict ourselves to the linear approximation, neglecting saturation effects.

The equation satisfied by the radiation intensity J_ω at the frequency ω in the case of low particle densities N_0[2)] and coherent light scattering (scattering without a change in frequency) with a spherical indicatrix has the form[13]

$$\cos\theta \frac{\partial J_\omega}{\partial r} - \frac{\sin\theta}{r}\frac{\partial J_\omega}{\partial\theta} + \frac{1}{c}\frac{\partial J_\omega}{\partial t} = (\alpha_\omega - \sigma_\omega) J_\omega + \frac{\sigma_\omega}{4\pi}\int J_\omega d\Omega, \tag{8}$$

where $d\Omega = \sin\theta d\theta d\varphi$ and c is the velocity of light.

Equation (8) admits of separation of variables. Denoting the separation constant by S, and representing the solution to the equation in the form $J_\omega = J_{0\omega}(t) J_{1\omega}(r, \theta, \varphi)$, we obtain for $J_{0\omega}(t)$ the equation

$$\partial J_{0\omega}/\partial t = cSJ_{0\omega}, \tag{9}$$

whose solution is: $J_{0\omega}(t) = J_{0\omega}(0)\exp(cSt)$. For $S > 0$ this solution corresponds to the appearance of temporal instability—the exponential growth of the intensity in time or of the generation. To the threshold regime corresponds the value $S = 0$. We then have the following equation for $J_{1\omega}$:

$$\cos\theta \frac{\partial J_{1\omega}}{\partial r} - \frac{\sin\theta}{r}\frac{\partial J_{1\omega}}{\partial\theta} = (\alpha_\omega - \sigma_\omega) J_{1\omega} + \frac{\sigma_\omega}{4\pi}\int J_{1\omega}\, d\Omega. \tag{10}$$

The solution of Eq. (10) is a complex problem, and, since one of the most serious difficulties is the necessity for us to take into account the dependence of $J_{1\omega}$ on the angle θ (in the present case the problem is symmetric with respect to φ), while we are primarily interested not in the form of the function $J_{1\omega}$ itself, but only in the relation between α_ω, σ_ω, and r_0 that arises from the solution to the equation, we shall average $J_{1\omega}$ over the directions (i.e., over θ). The method of averaging adopted here is called Eddington's approximation. Carrying through a procedure similar to the one used in[14], we obtain instead of (10) the following approximate equation:

$$\frac{d^2(rJ)}{dr^2} + 3\alpha(\sigma-\alpha)(rJ) = 0, \tag{11}$$

$$J = \frac{1}{2}\int_0^\pi J_{1\omega} \sin\theta\, d\theta.$$

It is assumed that no radiation is incident on the medium from outside. Then $J_{1\omega}(r_0, \theta) = 0$ for $\pi/2 < \theta \le \pi$, and the approximate boundary condition can be written in the form

$$J(r_0) = 2H(r_0), \quad H = \frac{1}{2}\int_0^\pi J_{1\omega}\sin\theta\cos\theta\, d\theta. \tag{12}$$

Furthermore, it is necessary to impose on J the condition $J \to \mathrm{const}$ as $r \to 0$, which corresponds to the requirement of finiteness of the solution at $r = 0$.

The solution to Eq. (11) has, under the indicated boundary conditions, the form

$$J = \frac{\mathrm{const}}{r}\sin qr, \quad q^2 = 3\alpha(\sigma-\alpha), \quad 0 \le r \le r_0.$$

The condition (12) leads to the following relation between r_0 and q:

$$\mathrm{tg}\, qr_0 = 2\alpha q r_0/(2\alpha - q^2 r_0). \tag{13}$$

This relation is essentially a threshold condition. In principle, all the solutions q_n for which $\alpha > \alpha_{\mathrm{thr}}(r_0)$ (where $\alpha_{\mathrm{thr}}(r_0)$ is the threshold value) are possible. Physically, however, it is clear that the solution possessing the least threshold value $\alpha_{\mathrm{thr}}(r_0)$ does not allow the appearance of the other possible solutions (at least before the onset of saturation). Figure 2 shows a qualitative plot of the behavior of r_0 as a function of α at fixed σ for the solution possessing the least threshold. In the limiting case when $\alpha \ll \sigma$ (the region I) the result coincides with the form of the solution in the diffusion approximation[12].

Let us now estimate the critical dimension r_0 for the OI laser, using the estimates for α_0 and σ_0 obtained in Secs. 2 and 3:

$$r_0 \approx \pi(3\alpha_0\sigma_0)^{-1/2} = \frac{2\pi^2}{\lambda^2}\frac{\Delta\omega_D}{\gamma} \times \frac{2}{(\pi\ln 2)^{1/2}}\left[3\frac{\Gamma}{\gamma}g_3\Delta n\Sigma\right]^{-1/2} \approx 10^{11}\ \mathrm{cm}.$$

Taking into account the fact that the dimension of the atmosphere of a Be star is 10^{13}–10^{14} cm, we can conclude that if there exists a population inversion in the stellar atmosphere, then the appearance of the generation regime is more practicable than that of the amplification regime.

The considered generator could have been called a generator with a nonresonant feedback, but such a term would have been a poor one in the present case, since the scattering cross section has a resonance character. Therefore, we employ the term "incoherent feedback," which reflects the fact that the radiation emitted by such a generator does not possess spatial coherence[15].

4. THE SPECTRAL WIDTH OF A LASER WITH AN INCOHERENT FEEDBACK

Let us now consider the difference between the stimulated radiation of a generator with an incoherent feedback and the spontaneous radiation emitted by the same medium as a result of the cyclic process connected with the fluorescence mechanism. In the case when generation occurs at the 8446-Å line the relation between the integral intensities in the 7774- and 8446-Å

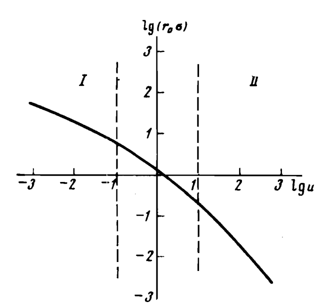

FIG. 2. The dependence of r_0 on α at fixed σ for the solution possessing the least threshold. In the region I $\alpha/\sigma \equiv u \ll 1$ and $r_0\sigma \sim \pi(3u)^{-1/2}$; in the region II $u \gg 1$ and $r_0\sigma \sim u^{-1}$.

lines does not change in comparison with the fluorescence mechanism. This happens because the ratio of the number of transitions giving rise to the 8446-Å line to the number of transitions giving rise to the 7774-Å line depends only on the rates of occupation of the $3p^3P$ and $3p^5P$ levels—rates which are determined only by the external conditions and the values of the probabilities of transition from the $3d^3D$ and $3d^5D$ levels respectively to the $3p^3P$ and $3p^5P$ levels. However, the emission lines due to the stimulated-emission mechanism and those due to spontaneous emission should differ in their spectral line widths. A narrowing of the spectrum occurs during generation because the gain at the line center is different from the gain at the edges. Such a narrowing can serve as a criterion for the unambiguous selection of the mechanism responsible for the appearance of the emission lines. Notice that the contribution of spontaneous emission in generators with incoherent feedbacks operating in the large solid-angle regime (in the present case $\Omega_{gen} = 4\pi$) is greater by many orders of magnitude than in ordinary generators, in which $\Omega_{gen} \lesssim 10^{-6}$ sr.

Let us consider generation by a system of two-level atoms of density N_0 contained in a spherical volume of diameter d. The feedback is effected by resonance scattering at the amplifying transition. Owing to the scattering, the conventional "mode" concept is not applicable to such a generator[12]. Therefore, below by the word "mode" will be meant the normal mode of the closed volume V corresponding to the volume occupied by the generator. The number of "modes" coupled by scattering is determined by the expression[12]

$$L=\Omega_{gen}(d/\lambda)^2. \tag{14}$$

The system of equations describing such a generator can be represented in the form

$$\frac{1}{c}\frac{d\langle n_i\rangle}{dt}=\sigma(\nu_i)(N_2-N_1)\langle n_i\rangle+\sigma(\nu_i)N_2-\gamma(\nu_i)\langle n_i\rangle, \tag{15a}$$

$$\frac{dN_1}{dt}=(N_2-N_1)\sum_{i=1}^{L}K(\nu_i)\langle n_i\rangle+AN_2-PN_1, \tag{15b}$$

$$N_1+N_2=N_0. \tag{15c}$$

Here $\langle n_i\rangle$ is the mean number of photons in the i-th mode (here and below the word "mode" has the meaning defined above) of frequency ν_i; $\sigma(\nu_i)$ is the cross section for radiative transition between the levels 1 and 2; $\gamma(\nu_i)$ is the loss factor per unit length for all the losses distributed over the entire volume V of the laser; $K(\nu_i) = \sigma(\nu_i)c/V$; A is the rate of spontaneous relaxation of the excited level; P is the rate of pumping of the upper level.

Without the term $\sigma(\nu_i)N_2$, which describes the spontaneous emission, Eq. (15a) does not have a steady-state solution, i.e., it describes an endless narrowing of the spectrum. With allowance, however, for the spontaneous radiation, we obtain the following steady-state solution to this equation:

$$\langle n_i\rangle=\sigma(\nu_i)N_2/[\gamma(\nu_i)-\sigma(\nu_i)(N_2-N_1)]. \tag{16}$$

The denominator in this expression is a small positive quantity, i.e., at the threshold the losses $\gamma(\nu_i)$ exceed somewhat the amplification $(N_2 - N_1)\sigma(\nu_i)$. This, as has already been mentioned, is connected with the contribution of the spontaneous emission to the radiation intensity. The radiated power is maximum at the center of the amplification line ν_0:

$$\langle n_0\rangle=\sigma_0N_2/[\gamma_0-\sigma_0(N_2-N_1)], \tag{17}$$

where $\langle n_0\rangle = \langle n_{\nu_0}\rangle$; $\sigma_0 = \sigma(\nu_0)$; $\gamma_0 = \gamma(\nu_0)$. The power decreases by a factor of two at the halfwidth level $\Delta\nu/2$ of the radiation spectrum:

$$\frac{1}{2}\langle n_0\rangle=\frac{\sigma(\Delta\nu/2)N_2}{\gamma(\Delta\nu/2)-\sigma(\Delta\nu/2)(N_2-N_1)}. \tag{18}$$

The spectral width $\Delta\nu$ of the generated radiation is obviously much smaller than the amplification-line width $\Delta\nu_0$, and therefore we can use the approximation

$$\sigma(\Delta\nu/2)=\sigma_0[1-(\Delta\nu/\Delta\nu_0)^2] \tag{19}$$

and, similarly,

$$\gamma(\Delta\nu/2)=\gamma_0[1+(\Delta\nu/\Delta\nu_0)^2], \tag{20}$$

where the amplification peak is approximated by a Lorentz contour of width $\Delta\nu_0$. It follows from the relations (17) and (18) with allowance for (19) and (20) that the spectral width of the radiation emitted in the continuous generation regime is equal to

$$\Delta\nu=\Delta\nu_0\left[\frac{\gamma_0-\sigma_0(N_2-N_1)}{\sigma_0(N_2-N_1)}\right]^{1/2}=\Delta\nu_0\left[\frac{N_2}{N_2-N_1}\frac{1}{2\langle n_0\rangle}\right]^{1/2}. \tag{21}$$

The last relation can be expressed in terms of such parameters as the pumping rate P, the number L of modes, etc. For this purpose, we must solve the system (15). In doing this the sum in Eq. (15b) can be simplified, since it has the meaning of the total number $\langle n\rangle$ of photons in all the modes, and, furthermore, the dependence of the cross section $\sigma(\nu_i)$ on the frequency can be neglected, since $\Delta\nu \ll \Delta\nu_0$:

$$\sum_{i=1}^{L}K(\nu_i)\langle n_i\rangle\approx K_0\langle n\rangle\approx K_0L\langle n_0\rangle. \tag{22}$$

The average number of photons in a mode is, when the spontaneous radiation is taken into account, given by the expression

$$\langle n_0\rangle=\frac{\rho(P-A+K_0L)-(P+A)}{4K_0L}+\left\{\frac{[\rho(P-A+K_0L)-(P+A)]^2}{(4K_0L)^2}+\frac{\rho P}{2K_0L}\right\}^{1/2}, \tag{23}$$

where $\rho = \sigma_0N_0/\gamma_0$ is the ratio of the maximum possible amplification in the laser (for $N_2 = N_0$; $N_1 = 0$) to the losses. Usually, in lasers, the quantity $\rho \gg 1$. The threshold pumping rate is determined by the expression

$$P_{thr}=A\frac{\rho+1}{\rho-1}-\frac{\rho}{\rho-1}K_0L=P^0_{thr}-\delta P. \tag{24}$$

The threshold pumping rate is smaller by the quantity δP than the threshold value $P^0_{thr} = A(\rho + 1)/(\rho - 1)$ obtained without allowance for the contribution of the spontaneous radiation to the L modes. For $P - P^0_{thr} \gg \delta P$, the number of photons in each mode increases linearly with the pump power:

$$\langle n_0\rangle=\frac{\rho-1}{2}\frac{P-P_{thr}}{K_0L}=(\eta-1)\frac{\rho+1}{2}\frac{A}{K_0L}, \tag{25}$$

where $\eta = P/P_{thr}$ is the excess of the pumping rate over the threshold value, and it is assumed that $\delta P \ll P^0_{thr}$. With the aid of the obtained relations, we can show that

$$\frac{N_2}{N_2-N_1}=\frac{P+K_0L\langle n_0\rangle}{P-A}=\frac{\rho+1}{2}. \tag{26}$$

Substituting the expressions (25) and (26) into (21), we obtain the following expression for the spectral width:

$$\Delta\nu=\Delta\nu_0[K_0L/(\eta-1)A]^{1/2}. \tag{27}$$

In the particular case of the OI laser under consideration, the analogous—to (27)—expression for the spectral width assumes (when the nonradiative relaxation of the level 3 is neglected) the form

$$\Delta\nu=\Delta\nu_0[K_0LH/2(\eta-1)A_{32}]^{1/2}, \quad (28)$$

where

$H\approx[1+(\eta-1)\rho\exp(-\hbar\omega_{21}/kT_{21})]\sim 1$ for $\eta-1\leqslant 1$.

Substituting into (28) the explicit expressions

$$K_0=\frac{\lambda^2}{2\pi}\frac{A_{32}}{2\pi\Delta\nu_0}\frac{3c}{4\pi r_0^3},\quad L=16\pi\frac{r_0^2}{\lambda^2},$$

we obtain the following expression for the spectral width:

$$\Delta\nu=\Delta\nu_0[3c/2\pi^2(\eta-1)\Delta\nu_0 r_0]^{1/2}. \quad (29)$$

The line width is, up to a constant factor, the geometric mean of the amplification-line width $\Delta\nu_0$ and the reciprocal of the time it takes light to pass through the medium (without allowance for scattering):

$$\Delta\nu=b(\Delta\nu_0 c/2r_0)^{1/2},\quad b=[3/\pi^2(\eta-1)]^{1/2}. \quad (30)$$

For the estimate of $\Delta\nu$, let us take the values: $\eta-1\approx 3.5\times 10^{-4}$ (a 0.035% excess pump power over the threshold value); $\Delta\nu_0\approx 10^{10}\ \text{sec}^{-1}$; $r_0\approx 10^{11}$ cm. We then find that $\Delta\nu=10^{-4}\Delta\nu_0$, i.e., that the generation line should be 10^4 times narrower than the spontaneous-radiation line.

Because of the narrowing of the spectrum, the brightness temperature T_b of the line can significantly differ from the temperature of the spontaneous radiation. Thus, if for the spontaneous line the brightness temperature $T\approx 3\times 10^{4}$°K, then for the line of width $\Delta\nu$ it will be equal to

$$T_b\approx T\frac{\Delta\nu_0}{\Delta\nu}\frac{\hbar\omega_{32}/kT}{\exp[\hbar\omega_{32}/kT]-1}\approx T\frac{\Delta\nu_0}{\Delta\nu}(\hbar\omega_{32}\ll kT_b). \quad (31)$$

5. OBSERVATION OF THE LASER EFFECT IN STELLAR ATMOSPHERES

The best proof of the appearance of the laser effect at certain spectral lines in stellar atmospheres would be the observation of the generation-induced spectrum narrowing. Since the spectral-line width in this case may be considerably smaller than the Doppler width, such measurements can be carried out only with the aid of high-resolution spectral instruments (e.g., the Fabry-Perot etalon). Let us also point out the possibility of measuring spectrum constriction by a correlation method with the aid of the Brown-Twiss effect[15,16].

The statistics of the fluctuations in the radiation emitted into a small solid angle by a laser with an incoherent feedback coincides with the statistics of an equilibrium (Gaussian) radiation. This has been theoretically and experimentally shown in a number of papers[17,18]. Therefore, in using the correlation method, we can consider the radiation of the stellar laser to be incoherent (Gaussian) light.

For such light the correlation function for the intensity fluctuations

$$k(\tau)=\langle\Delta J(t+\tau)\Delta J(t)\rangle,\quad \Delta J(t)=J(t)-\langle J(t)\rangle$$

(the brackets indicate averaging over the time) is connected with the spectral-line width $\Delta\nu$ by the relation

$$k(\tau)=\langle J(t)\rangle^2|\gamma(\tau)|^2;\quad |\gamma(\tau)|=\exp(-\tau\pi\Delta\nu). \quad (32)$$

By measuring the intensity-intensity correlation function $k(\tau)$, we can obtain information about the spectral-line width $\Delta\nu$. The spectrum constriction should manifest itself in the appearance of correlations with a characteristic time $\tau_{cor}\approx 1/\pi\Delta\nu \gg (\Delta\nu_D)^{-1}$, where $\Delta\nu_D$ is the Doppler line width. The correlation method is suitable for such measurements for a number of other reasons.

First, the spectrum constriction leads to a corresponding increase in the brightness temperature of the radiation and, consequently, to an increase in the signal-to-noise ratio S/N. Indeed, in the case of the observation of the intensity-intensity correlation of an incoherent source having an area of Σ_s and located at a distance R from the receiver, the ratio S/N is equal to[19]

$$\frac{S}{N}=\varkappa(t_m\tau_0)^{1/2}\Delta\nu\frac{\Sigma_s\Sigma_d}{\lambda^2R^2}\left[\exp\left(\frac{\hbar\omega_{32}}{kT_b}\right)-1\right]^{-1}, \quad (33)$$

where χ is the quantum yield of the photodetector, t_m is the measurement time, τ_0 is the lag in the autocorrelation measurement, and Σ_d is the area of the aperture of the telescope receiving the radiation. It can be seen from (33) that an increase in the brightness temperature significantly increases S/N. The measurement of the absolute value of S/N is also experimentally feasible. Indeed, for example, for $R=10^{20}$ cm (10^2 light years), $\chi=10^{-1}$, $t_m\approx 10^3$ sec, $\Sigma_s=4\pi r_0^2\approx 10^{23}$ cm^2, $\Sigma_d\approx 10^4$ cm^2, $\Delta\nu=10^6\ \text{sec}^{-1}$, $\hbar\omega_{32}/kT_b\approx 10^{-4}$, and $\tau_0\approx 10^{-7}$ sec, we obtain $S/N\sim 100$.

Secondly, the correlation method can, when the radiation is detected at two independent points in space, be used to measure the dimension of the generation region, i.e., the critical radius r_0 (the Brown-Twiss method[15]). The minimum detector spacing b_0 (i.e., the distance between the observation points) necessary for the resolution of a source of linear dimension r_0 located at a distance $R\gg r_0$ from the receivers is determined by the expression[15]

$$b_0=3.8\lambda R/r_0. \quad (34)$$

For example, for $\lambda=8446$ Å, $R=10^{20}$ cm, and $r_0=10^{11}$ cm, the required spacing $b_0\approx 5\times 10^5$ cm = 5 km.

Since the critical radius r_0 of the generation region is much smaller than the diameter of the stellar atmosphere, there will obviously arise in the steller atmosphere a large number of independent generation regions. Because of the rotation of the star, the components of the velocities of the individual generation regions along the observation direction will be different, i.e., there will arise a variable Doppler shift of the center of the generation line. The magnitude of the shift, $\Delta\nu_{sh}\approx\nu_0 v_{eff}/c\sqrt{2}$ (where v_{eff} is the component of the velocity at the equator along the observation direction), can be much larger than the Doppler width of the spectral line, thereby masking the narrowing of the spectrum in the individual generation regions. Because of this effect, it is impossible to obtain, in the observation of the spectrum with the aid of the Fabry-Perot etalon, information about the generation-induced spectrum constriction. The correlation technique, however, is insensitive to the spectral-line shift, and should contain the correlations of the intensity fluctuations owing to the interference of the radiation from each generation region.

[1)]Allowance for the continuous radiation of temperature T_* in the 3–2 and 4–3 transitions will not turn out to have a significant effect on the estimates made below, since the contribution made by this radiation to the probabilities $p_{32(43)}$ coincides in order of magnitude with the values of the spontaneous probabilities $A_{32(43)}$; therefore, it will not be considered.

[2)]If t_1 is the average time spent by a quantum in the absorbed state during the scattering process and t_2 is the mean interval between successive collisions, then the degree of smallness of N_0 is determined by the inequality $t_1 \ll t_2$. For greater details, see [13], p. 318.

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