Contribution to the theory of parametric phenomena in antiferromagnetic substances

V. A. Kolganov

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Stationary states of a parametrically excited spin system of an antiferromagnet with anisotropy of the "easy plane" type, which correspond to excitation of magnons of different spectral branches, are investigated in the spin-wave formalism. The behavior of the high frequency complex susceptibility beyond the excitation threshold is calculated in the stationary state. The stability of the stationary states is analyzed. Expressions are obtained for the frequencies and lifetimes of the collective oscillations corresponding to the additional degrees of freedom of the parametrically excited spin system, and also for the susceptibility of the homogeneous oscillations.

In an ideal antiferromagnetic dielectric with easy-plane anisotropy, situated in a constant magnetic field H and a perpendicular alternating field h(t), both lying in the basal plane, there occurs, besides direct excitation of pairs of spin waves corresponding to the oscillations of the ferromagnetism vector **M** and the antiferromagnetism vector **L**, also excitation of these pairs via nonresonant quasi-homogeneous oscillations of the ferromagnetism vector M. At sufficiently large amplitude of the alternating field, $h > h^c$, the spin-wave excitation frequency becomes larger than the damping frequency, and this leads to an intensive growth of the imaginary part of the complex susceptibility of the spin system. Similar phenomena were first observed by Suhl[3] and by Schlomann[2] and were investigated in $CsMnF_3$ by Seavey[3] and in $CsMnF_3$ and $MnCO_3$ by Borovik-Romanov, Prozorova, Kotyuzhanskiĭ, and Kveder[4-6].

The Hamiltonian of an antiferromagnet with anisotropy of the easy plane type and with a Dzyaloshinskiĭ interaction will be written in the form[7]

$$\mathscr{H}=\sum_{\mathbf{R}_1,\Delta}[-J(\Delta)\mathbf{S}_{\mathbf{R}_1}\mathbf{S}_{\mathbf{R}_1+\Delta}+2D(\Delta)(S^z_{\mathbf{R}_1}S^y_{\mathbf{R}_1+\Delta}-S^y_{\mathbf{R}_1}S^z_{\mathbf{R}_1+\Delta})]$$
$$+\sum_{\mathbf{r},\mathbf{R}\neq\mathbf{r}}Q(\mathbf{R}-\mathbf{r})S_\mathbf{r}^xS_\mathbf{R}^x+\sum_\mathbf{R}[P(S_\mathbf{R}^x)^2-\mu_Bg(HS_\mathbf{R}^z+hS_\mathbf{R}^y)].$$

At low temperatures, going over in the Holstein-Primakoff formalism to the second-quantization representation[8] in the proper coordinate frame of each spin, we express the operators of the spin components in the laboratory frame in terms of the operators of the spin deviations[9]. Considering the spin system in the absence of external alternating fields, we go over to the Fourier components and diagonalize that part of the Hamiltonian which is quadratic at a_{jk} with the aid of the transformation

$$a_{j\mathbf{k}}=\sum_l(u_{jl}c_{l\mathbf{k}}+v_{jl}^*\overset{+}{c}_{l-\mathbf{k}}),\quad j,l=1,2,$$

where at $\epsilon_{jk}\ll\epsilon_E$[9] we have

$$u_{lj}=\frac{1}{2}\left(\frac{\varepsilon_E}{\varepsilon_{jk}}\right)^{1/2}\left(1+\frac{\varepsilon_{jk}}{2\varepsilon_E}\right)(-1)^{(1+l)\delta_{l,2}},$$
$$v_{lj}=-\frac{1}{2}\left(\frac{\varepsilon_E}{\varepsilon_{jk}}\right)^{1/2}\left(1-\frac{\varepsilon_{jk}}{2\varepsilon_E}\right)(-1)^{l\delta_{l,2}}$$

and when account is taken of the magnetic-dipole interaction we have

$$\varepsilon_{j\mathbf{k}}^2\approx(\varepsilon_{j0}^2+\alpha_\mathbf{k}^2)\left\{1+\frac{\varepsilon_M}{\varepsilon_E}\left[\zeta_j\left(\frac{1}{3}-N_{zz}\right)+\delta_{j,2}\left(\frac{k_z^2}{k^2}-\frac{1}{3}\right)\right.\right.$$
$$\left.\left.+\delta_{j,1}\left(\left(\frac{k_x^2}{k^2}-\frac{1}{3}\right)+\zeta_j\left(\frac{k_y^2}{k^2}-\frac{1}{3}\right)\right)\right]\right\},$$

where N_{zz} is the component of the tensor of the demagnetizing coefficients,

$$\zeta_j=(\varepsilon_H+\varepsilon_D)^2/\varepsilon_{j\mathbf{k}}^2,\ \varepsilon_E=\mu H_E,\ \varepsilon_H=\mu H,\ \varepsilon_D=\mu H_D,$$
$$\varepsilon_M=4\pi\mu M,\ \varepsilon_{10}^2=\varepsilon_H(\varepsilon_H+\varepsilon_D),$$
$$\varepsilon_{20}^2=2\mu^2H_EH_A+\varepsilon_D(\varepsilon_H+\varepsilon_D),\ \alpha_\mathbf{k}\sim\varepsilon_Ek\Delta.$$

Allowance for the magnetic dipole interaction in the expression for the magnon energy makes it possible to estimate the excited-state interval, which is quite small, unlike in ferromagnets.

The Hamiltonian in the spin-wave representation, with allowance for interactions not higher than fourth order in c_{jk}, is reduced to the following form:

$$\mathscr{H}=\sum_\mathbf{k}(\varepsilon_{1\mathbf{k}}c_{1\mathbf{k}}^+c_{1\mathbf{k}}+\varepsilon_{2\mathbf{k}}c_{2\mathbf{k}}^+c_{2\mathbf{k}})+\left[A_{10}^fc_{10}^+(e^{-i\omega t}+e^{i\omega t})\right.$$
$$+\sum_\mathbf{k}(A^f_{1\mathbf{k},2-\mathbf{k}}\overset{+}{c}_{1\mathbf{k}}\overset{+}{c}_{2-\mathbf{k}}e^{-i\omega t}+\Psi^{10}_{1\mathbf{k},2-\mathbf{k}}\overset{+}{c}_{1\mathbf{k}}\overset{+}{c}_{2-\mathbf{k}}c_{10}$$
$$\left.+\Psi_{1\mathbf{k},2-\mathbf{k},10}\overset{+}{c}_{1\mathbf{k}}\overset{+}{c}_{2-\mathbf{k}}\overset{+}{c}_{10})+\text{э. c.}\right]+\mathscr{H}^{(3)}+\mathscr{H}^{(4)},$$

where at $\epsilon_M\epsilon_{jk}\ll\epsilon_H\epsilon_E$ we have

$$A^f_{10}=-i\left(\frac{\varepsilon_{10}\varepsilon_MV(\varepsilon_H+\varepsilon_D)}{32\pi\varepsilon_E\varepsilon_H}\right)^{1/2}h,\quad A^f_{1\mathbf{k},2-\mathbf{k}}=-\frac{\varepsilon_{2\mathbf{k}}-\varepsilon_{1\mathbf{k}}}{4(\varepsilon_{2\mathbf{k}}\varepsilon_{1\mathbf{k}})^{1/2}}\mu h,$$
$$\Psi_{1\mathbf{k},2-\mathbf{k}}=-i\left(\frac{2\pi\varepsilon_E}{\varepsilon_MV}\right)^{1/2}\frac{\mu\varepsilon_H(\varepsilon_{10}+\varepsilon_{2\mathbf{k}}-\varepsilon_{1\mathbf{k}})}{2(\varepsilon_{10}\varepsilon_{2\mathbf{k}}\varepsilon_{1\mathbf{k}})^{1/2}}$$
$$\Psi_{1\mathbf{k},2-\mathbf{k},10}=\frac{1}{2}\frac{\varepsilon_{10}-\varepsilon_{2\mathbf{k}}+\varepsilon_{1\mathbf{k}}}{\varepsilon_{10}+\varepsilon_{2\mathbf{k}}-\varepsilon_{1\mathbf{k}}}\Psi^{10}_{1\mathbf{k},2-\mathbf{k}},$$

$\mathscr{H}^{(3)}$ includes three-magnon interactions of the type 2p → 1k, 1q; 1q → 1k, 2p; 1k, 1q, 2p → 0; 2p → 2k, 2q; 2k, 2q, 2p → 0, which make a contribution, in the second order of perturbation theory, to the four-magnon interactions described by $\mathscr{H}^{(4)}$: 1k, 1p → 1q, 1s; 1k, 2p → 1q, 2s; 2k, 2p → 2q, 2s (see the Appendix).

To describe the dynamic properties of the spin system, we use the Liouville theorem, which yields for the mean value of the operator A of the physical quantity the equation

$$\frac{d\langle A\rangle}{dt}=\left\langle\frac{\partial A}{dt}\right\rangle+\frac{1}{i\hbar}\langle[A,\mathscr{H}]\rangle,$$

where $\langle A\rangle\equiv\mathrm{Tr}(\rho A)$, and $\rho(t)$ is the state-density matrix.

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We assume further that the specific heat of the system of the thermal magnons is infinitely large, and that the occupation numbers are small. We call attention also to the fact that the most important of the interactions of parametrically excited magnons, in addition to those renormalizing the frequency of the excited pair of spin waves $\mathbf{1k}, \mathbf{1q} \to \mathbf{1k}, \mathbf{1q}$; $\mathbf{1k}, \mathbf{2q} \to \mathbf{1k}, \mathbf{2q}$; $\mathbf{2k}, \mathbf{2q} \to \mathbf{2k}, \mathbf{2q}$, is the interaction of the magnon pairs $\mathbf{1k}, \mathbf{2{-}k} \to \mathbf{1q}, \mathbf{2{-}q}$[10]. The latter makes it possible to decouple the chain of equations for the correlation functions with the aid of the relation

$$\langle c_{\mathbf{k}}^+ c_{\mathbf{q}}^+ c_{\mathbf{p}} c_{\mathbf{s}}\rangle = \langle c_{\mathbf{k}}^+ c_{\mathbf{q}}^+\rangle\langle c_{\mathbf{p}} c_{\mathbf{s}}\rangle \Delta(\mathbf{k}+\mathbf{q})\Delta(\mathbf{p}+\mathbf{s}) + \langle c_{\mathbf{k}}^+ c_{\mathbf{p}}\rangle\langle c_{\mathbf{q}}^+ c_{\mathbf{s}}\rangle \Delta(\mathbf{k}-\mathbf{p})\Delta(\mathbf{q}-\mathbf{s}) + \langle c_{\mathbf{k}}^+ c_{\mathbf{s}}\rangle\langle c_{\mathbf{q}}^+ c_{\mathbf{p}}\rangle \Delta(\mathbf{k}-\mathbf{s})\Delta(\mathbf{q}-\mathbf{p}).$$

A linearly polarized field $h_y(t)$ can be represented as a super-position of fields having right- and left-hand circular polarizations. In the general case, an important role is played by the interaction of the quasihomogeneous oscillations of $\mathbf{M}$ with both components of the linearly polarized field, and accordintly we put

$$\langle c_{10}\rangle = l_{10}^{(-)} e^{-i\omega t} + l_{10}^{(+)} e^{i\omega t}.$$

At the same time, the parametrically excited oscillations interact only with the right-polarized component of the field.

Using the Liouville theorem, we obtain for the correlators

$$m_{\mathbf{k}} = \langle c_{1\mathbf{k}} c_{2-\mathbf{k}}\rangle, \quad n_{j\mathbf{k}} = \langle c_{j\mathbf{k}}^+ c_{j\mathbf{k}}\rangle, \quad j=1,\ 2$$

the system of equations

$$i\hbar \dot{l}_{10}^{(-)} = -i\hbar[\gamma_{10} + i(\omega_{10}-\omega)]l_{10}^{(-)} + A_{10}{}^{f} + \sum_{\mathbf{k}} \dot{\Psi}{}^{10}_{1\mathbf{k},2-\mathbf{k}} m_{\mathbf{k}},$$

$$i\hbar \dot{l}_{10}^{(+)} = -i\hbar[\gamma_{10} + i(\omega_{10}+\omega)]l_{10}^{(+)} + A_{10}{}^{f} + \sum_{\mathbf{k}} \Psi_{1\mathbf{k},2-\mathbf{k},10} m_{\mathbf{k}}^*,$$

$$i\hbar \dot{n}_{j\mathbf{k}} = -i\hbar 2\gamma_{j\mathbf{k}} n_{j\mathbf{k}} + (B_{\mathbf{k}} m_{\mathbf{k}}^* + \text{c.c.}),$$

$$i\hbar \dot{m}_{\mathbf{k}} = -i\hbar[\gamma_{1\mathbf{k}} + \gamma_{2\mathbf{k}} + i(\tilde{\omega}_{\mathbf{k}}-\omega)]m_{\mathbf{k}} + B_{\mathbf{k}}(n_{1\mathbf{k}} + n_{2\mathbf{k}});$$

$$B_{\mathbf{k}} = A^{f}_{1\mathbf{k},2-\mathbf{k}} + \Psi^{10}_{1\mathbf{k},2-\mathbf{k}} l_{10}^{(-)} + 2\Psi_{1\mathbf{k},2-\mathbf{k},10} l_{10}^{*(-)} + \sum_{\mathbf{q}} \left[\Psi^{1\mathbf{q},2-\mathbf{q}}_{1\mathbf{k},2-\mathbf{k}} + \frac{|\Psi^{10}_{1\mathbf{k},2-\mathbf{k}}|^2}{\hbar(\omega_{10}-\omega)} + \frac{4|\Psi_{1\mathbf{k},2-\mathbf{k},10}|^2}{\hbar(\omega_{10}+\omega)}\right] m_{\mathbf{q}},$$

$$\tilde{\omega}_{\mathbf{k}} = \sum_j \left[\omega_{j\mathbf{k}} + \frac{1}{\hbar}\sum_{i,\mathbf{q}} \Psi^{i\mathbf{k},j\mathbf{q}}_{i\mathbf{k},j\mathbf{q}} n_{j\mathbf{q}}(1+\delta_{i,j})\right],$$

where in the amplitudes of the four-magnon interactions account is taken of the three-magnon interactions in the second order of perturbation theory.

In the considered nonresonant case, when $|\omega_{10} - \omega| \gg \gamma_{10}$, the right-hand sides of the equations for $l_{10}^{(\pm)}$ are proportional to the large quantities $|\omega_{10} \pm \omega| \gg \gamma_{10}$, γ_{jk}, so that we can write down the equation in the form

$$\varepsilon_\pm \dot{l}_{10}^{(\pm)} = F_\pm(l_{10}^{(\pm)}; \{m_{\mathbf{k}}\}), \qquad \varepsilon_\pm \sim \frac{\gamma_{10}}{|\omega_{10} \pm \omega|} \ll 1.$$

Since the remaining equations do not have the indicated property, we can put

$$F_\pm(l_{10}^{(\pm)}; \{m_{\mathbf{k}}\}) = 0$$

and reduce the investigation of the complete system of equations to an investigation of a reduced system[11]. By using the notation

$$\Phi = \frac{1}{\hbar(\gamma_{1\mathbf{k}}+\gamma_{2\mathbf{k}})}\left[A^{f}_{1\mathbf{k},2-\mathbf{k}} - \frac{A_{10}{}^{f}\Psi^{10}_{1\mathbf{k},2-\mathbf{k}}}{\hbar(\omega_{10}-\omega)} - \frac{2A_{10}{}^{*f}\Psi_{1\mathbf{k},2-\mathbf{k},10}}{\hbar(\omega_{10}+\omega)}\right],$$

$$\beta_{\mathbf{kq}} = \frac{\Psi^{1\mathbf{q},2-\mathbf{q}}_{1\mathbf{k},2-\mathbf{k}}}{\hbar(\gamma_{1\mathbf{k}}+\gamma_{2\mathbf{k}})}, \qquad \beta^{(i)}_{\mathbf{kq}} = \sum_j \frac{\Psi^{i\mathbf{k},j\mathbf{q}}_{i\mathbf{k},j\mathbf{q}}}{\hbar(\gamma_{1\mathbf{k}}+\gamma_{2\mathbf{k}})}(1+\delta_{i,j}),$$

$$\eta_{\mathbf{k}} = \frac{\tilde{\omega}_{\mathbf{k}}-\omega}{\gamma_{1\mathbf{k}}+\gamma_{2\mathbf{k}}}, \qquad \varkappa_j = \frac{2\gamma_{j\mathbf{k}}}{\gamma_{1\mathbf{k}}+\gamma_{2\mathbf{k}}}, \qquad p = (h/h^c)^2,$$

$$\delta_{\mathbf{k}} = \frac{\omega_{1\mathbf{k}}+\omega_{2\mathbf{k}}-\omega}{\gamma_{1\mathbf{k}}+\gamma_{2\mathbf{k}}}$$

and by recognizing that the dependence of the magnon lifetime on the propagation direction is negligibly weak at $\epsilon_M \epsilon_{jk} \ll \epsilon_H \epsilon_E$, on the basis of which it is convenient to introduce the dimensionless time $\tau = (\gamma_{1k} + \gamma_{2k})t$, we can express the reduced system in the form

$$\dot{n}_{1\mathbf{k}} = -\varkappa_j n_{j\mathbf{k}} + \left[i m_{\mathbf{k}}\left(\Phi^* + \sum_{\mathbf{q}} \beta_{\mathbf{kq}} m_{\mathbf{q}}^*\right) + \text{c.c.}\right],$$

$$\dot{m}_{\mathbf{k}} = -(1+i\eta_{\mathbf{k}})m_{\mathbf{k}} - i(n_{1\mathbf{k}}+n_{2\mathbf{k}})\left(\Phi + \sum_{\mathbf{q}} \beta_{\mathbf{kq}} m_{\mathbf{q}}\right),$$

$$\eta_{\mathbf{k}} = \delta_k + \sum_{j\mathbf{q}} \beta^{(j)}_{\mathbf{kq}} n_{j\mathbf{q}}.$$

Such an approximation is valid for the investigation of stationary states, particularly their stability, but is not valid for the description of the motion of a system near stationary states.

The necessary condition for the existence of a stationary state of a spin system as such is its stability with respect to a small perturbation of the amplitudes. It must be recognized here that the stability requirement extends both to excited and unexcited modes[12]. We note further that the amplitudes of the four-magnon interactions in the interval of the excited states depend only on the angle ϑ between the moment of the interacting magnons (see the Appendix), on the basis of which we obtain, in the stationary states defined by the relation $\eta_k = 0$,

$$\beta_0 \sum_{j\mathbf{q}} n^{\text{st}}_{j\mathbf{q}} = (p-1)^{1/2},$$

where β_0 is the value of the function $\beta_{\mathbf{kq}}$ averaged over ϑ. The expression for β_0 is in the general case rather complicated, therefore in the calculation of the complex high-frequency susceptibility we confine ourselves to the condition $\epsilon^2_{2k} \gg 4\epsilon^2_{1k}$:

$$\chi_\omega = \frac{M}{H_E}\left[1 + \frac{\varepsilon_{20}{}^2}{\varepsilon_{20}{}^2 + 16\varepsilon_H{}^2}\,\frac{(p-1) + i(p-1)^{1/2}}{2p}\right]$$

It is of interest to investigate in greater detail the stability of the stationary state, in particular an analysis of the roots of the characteristic equation, from which it follows that additional degrees of freedom appear in the parametrically excited spin system[1)]. The corresponding frequencies and reciprocal lifetimes, referred to $\gamma_{1k} + \gamma_{2k}$, are given by

$$\Omega_s = p^{1/2}\left(1 + \varkappa_1\varkappa_2 \frac{\beta_s^{(1)} + \beta_s^{(2)}}{2\beta_s}\right)^{1/2} \frac{\beta_s}{\beta_0}$$

$$\Gamma_s = \frac{4\beta_s + \varkappa_1\varkappa_2(\varkappa_1\beta_s^{(1)} + \varkappa_2\beta_s^{(2)} - 2\beta_s)}{4\beta_s + 2\varkappa_1\varkappa_2(\beta_s^{(1)} + \beta_s^{(2)})}$$

where $p \gg 1$ and

$$\beta_s = \frac{2}{\pi}\int_0^\pi \beta_{\mathbf{kq}} \cos(s\vartheta)\,d\vartheta, \qquad \beta_{\mathbf{kq}} = \frac{1}{2}\beta_0 + \sum_{s=1}^{2} \beta_s \cos(s\vartheta)$$

takes a simple form if $\epsilon^2_{2k} \gg 4\epsilon^2_{1k}$:

$$\Omega_0 = p^{1/2}\left(1 + \frac{\varkappa_1\varkappa_2}{2}\,\frac{\varepsilon_{20}{}^2 + 26\varepsilon_H{}^2}{\varepsilon_{20}{}^2 + 16\varepsilon_H{}^2}\right)^{1/2}$$

The resonant susceptibility of a homogeneous mode can be expressed in terms of the susceptibility relative to the principal pumping

$$\chi''_{\omega\pm\Omega_0}=\chi_\omega''\frac{p}{4\Gamma_0}\left(1\mp\frac{\Omega_0}{p^{1/2}}\right)\left(1\mp\frac{p^{1/2}}{\Omega_0}\right),\quad p\gg 1.$$

The resonant excitation of the homogeneous mode of collective oscillations can be effected by a field of frequency $\omega \pm \Omega_0$, the polarization of which corresponds to the principal pumping field.

As noted above, the character of the excitation of the spin system is determined by four-magnon interactions of the type 1**k**, 2 −**k** → 1**q**, and 2 −**q**, which by renormalization of the pump establish the stationary level of the excitation of the spin waves that are isotropically distributed over a sphere $\tilde{\omega}_k = \omega$ in k-space, and i**k**, j**q** → i**k**, and j**q** (i, j = 1, 2), which renormalize the total frequency of the magnon pairs. The collective behavior of the excited magnons, particularly the stability of the stationary state, is governed to a considerable degree by the ratio of the interactions of the two types, which depend significantly on the external conditions and on the parameters of the spin system. This is indeed the difference between the investigated effect and excitation of spin waves by one branch of the spectrum, for in the latter case a fixed type of four-magnon interaction (for example i = j = 1) leads not only to a relatively simple dependence of this interaction on the system parameters, but also to a corresponding change due to the absence of the singularities that are inherent in the considered phenomenon.

The aforementioned singularities, which manifest themselves in an increase of the activation energy of the magnons of the first branch, are due to variation in the contribution of the three-magnon interactions to the four-magnon interactions in fourth order of perturbation theory, having the meaning of an interaction of spin waves via virtual magnons, including via nonresonant homogeneous oscillations of the ferromagnetism vector. Assuming $\epsilon_D \ll \epsilon_H$, we find in the limit of small wave vectors that the reversal of the sign of the amplitude of the interaction 1**k**, 2 − **k** → 1**q**, 2 − **q**, and the ensuing discontinuity in the phase shift of the pair of the spin waves relative to the pump at $\epsilon_H \approx 1.8\epsilon_{20}$, give rise to a maximum of finite width, on the order of $\gamma_{1k} + \gamma_{2k}$, of the imaginary part of the susceptibility, together with a corresponding reversal in the sign of the increment of the real part of the susceptibility, in analogy with the behavior of the spin system in the region of the spin-resonance saturation when magnons of one branch are excited[1]. With increasing Dzyaloshinskiĭ field, the position of the indicated singularity shifts towards weaker external magnetic fields.

The change of the amplitudes of the interactions as a result of the appreciable increase in ϵ_H leads also to violation of the conditions of the stability of the stationary state, which is directly connected with the vanishing of the frequency of the collective oscillations. At $\epsilon_D \ll \epsilon_H$, $\alpha_k \ll \epsilon_{j0}$, and $\gamma_{1k} = \gamma_{2k}$, the region of homogeneous (s = 0) instability is limited to the values $1.8 \lesssim \epsilon_H\epsilon_{20}^{-1} \lesssim 3.5$. The difference between the relaxation frequencies causes a narrowing of the instability region towards the lower limit of the interval mentioned above. The same results also from an increase of ϵ_D, which at the same time shifts the region of unstable ϵ_H towards lower values. In the limit of small **k**, an instability is also possible relative to the second mode (s = 2), a fact ensured by the condition

$$\varepsilon_{10}>2\varepsilon_{20}[1+(\gamma_{1k}-\gamma_{2k})^2/8\gamma_{1k}\gamma_{2k}]^{1/2}.$$

The existence of instability regions is a remarkable feature of the considered effect, the advantage of which over parametric phenomena in ferromagnets consists in the relatively small integral of excited states of the magnons, so that there are grounds for hoping to obtain a more detailed picture of the instabilities by comparing the experimental data and the calculation results.

The region of higher activation energies of the magnons of the first branch is of interest also because the three-magnon damping mechanism 1**k**, 1**p** → 2**q**, the contribution of which to the relaxation frequencies increases significantly with increasing temperature, is forbidden at $2\epsilon_{10} > \epsilon_{20}$ by virtue of the conservation laws. The corresponding decrease of the relaxation frequencies lowers the threshold of the parametric excitation

$$h_y^c=\frac{4(\gamma_{1k}\gamma_{2k}\omega_{1k}\omega_{2k})^{1/2}}{\gamma(\omega_{1k}+\omega_{2k})}\frac{(\omega_{2k}+\omega_{1k})^2-\omega_{10}^2}{\omega_{20}^2},\quad \omega=\omega_{1k}+\omega_{2k}.$$

However, the effect of decreasing the excitation threshold as a result of turning off the three-magnon damping mechanism should not be regarded as too appreciable, because in real crystals the lower limit of the relaxation frequencies is determined by the scattering of the spin waves by the inhomogeneities. In particular, the contribution of the scattering by a macroscopic paramagnetic (surface or volume) defect increases with decreasing temperature like the square of the Brillouin function, which determines the temperature dependence of the magnetization of the paramagnet.

Another possibility of lowering the threshold, which is afforded by the singularities of the excitation of magnons from different branches of the spectrum, is a decrease of ω_{1k}. The structure of the expression for the threshold field, which includes the ratio $(\omega_{1k}\omega_{2k})^{1/2}/\omega$, makes possible a gradual lowering of the threshold in a wide range of constant fields or pump frequencies, with an eventual decrease of the threshold field to $4\gamma_{10}\gamma_{20}^{1/2}/\gamma\omega^{1/2}$ near $\omega = \omega_{20}$, which is characteristic of the "saturation of the fundamental resonance." We note that for $MnCO_3$ the wavelength corresponding to the minimum excitation frequency $\omega = \omega_{20}$ is 2 mm. The indicated possibilities of lowering the necessary power level of the electromagnetic radiation make experimental observation of the effects realistic, in spite of the difficulties of low-temperature investigations in the millimeter band.

APPENDIX

We present expressions for the amplitudes of the fundamental interactions of the spin waves in an antiferromagnetic ellipsoid at $\epsilon_M\epsilon_{jk} \ll \epsilon_H\epsilon_E$:

$$\Psi_{1k,1q}^{2p}=(\varepsilon_{1k}+\varepsilon_{1q}+\varepsilon_{2p})\sigma_3,\quad \Psi_{1k,2p}^{1q}=2(\varepsilon_{1q}+\varepsilon_{2p}-\varepsilon_{1k})\sigma_3,$$

$$\Psi_{1k,1q,2p}=(\varepsilon_{1k}+\varepsilon_{1q}-\varepsilon_{2p})\sigma_3,\quad \Psi_{2k,2q}^{2p}=-(\varepsilon_{2k}+\varepsilon_{2q}-\varepsilon_{2p})\sigma_3,$$

$$\Psi_{2k,2q,2p}=-{}^1/_3(\varepsilon_{2k}+\varepsilon_{2q}+\varepsilon_{2p})\sigma_3,$$

where for simplicity we have left out the generalized Kronecker symbols that enture the momentum conservation law,

$$\sigma_3=-i\frac{\varepsilon_H}{4}\left(\frac{2\pi\varepsilon_E}{V\varepsilon_M}\right)^{1/2}\mu\prod_j\varepsilon_j^{-1/2}.$$

The condition $\epsilon_M\epsilon_{jk} \ll \epsilon_H\epsilon_E$, which make it possible to neglect the contribution of the magnetic dipole interaction, follows in obvious fashion from the more exact expressions for the interaction amplitudes, which differ from the expressions given above by factors similar to the following:

$$1-\frac{\varepsilon_M}{\varepsilon_E\varepsilon_H(\varepsilon_{1\mathbf{k}}+\varepsilon_{1\mathbf{q}}+\varepsilon_{2\mathbf{p}})}\left\{\frac{1}{4}(\varepsilon_H+\varepsilon_D)(\varepsilon_{1\mathbf{q}}-\varepsilon_{1\mathbf{k}}+3\varepsilon_{2\mathbf{p}})\sum_{j=y,z}\left(N_{jj}-\frac{1}{3}\right)\right.$$

$$\left.+i[\varepsilon_{1\mathbf{k}}(\varepsilon_{1\mathbf{q}}+\varepsilon_{2\mathbf{p}})\varphi_{\mathbf{p}-\mathbf{q}}+\varepsilon_{1\mathbf{q}}(\varepsilon_{1\mathbf{k}}+\varepsilon_{2\mathbf{p}})\varphi_{\mathbf{k}-\mathbf{p}}]\right\},$$

corresponding to the interaction **1k**, **1q** → **2p**, with

$$\varphi_{\mathbf{k}-\mathbf{p}}=\frac{(\mathbf{k}-\mathbf{p})_x(\mathbf{k}-\mathbf{p})_y}{|\mathbf{k}-\mathbf{p}|^2}.$$

The amplitudes of the four-magnon interactions with allowance for the three magnon interactions in second order of perturbation theory are

$$\Psi_{1\mathbf{k},1-\mathbf{q}}^{1\mathbf{k},1-\mathbf{q}}\approx-\left(\varepsilon_{10}^2+3\varepsilon_H^2+\frac{16\varepsilon_H^2\varepsilon_{1\mathbf{k}}^2}{\varepsilon_{20}^2-4\varepsilon_{1\mathbf{k}}^2-\alpha_s^2}\right)\sigma_4,$$

$$\Psi_{2\mathbf{k},2\mathbf{q}}^{2\mathbf{k},2\mathbf{q}}\approx-(\varepsilon_{20}^2+3\varepsilon_H^2)\sigma_4,$$

$$\Psi_{1\mathbf{k},2-\mathbf{q}}^{1\mathbf{k},2-\mathbf{q}}\approx 2\left[\varepsilon_{20}^2+\varepsilon_{10}^2+8\varepsilon_H^2\varepsilon_{2\mathbf{k}}^2\sum_{j=1,2}\frac{1}{(\varepsilon_{2\mathbf{k}}+(-1)^j\varepsilon_{1\mathbf{k}}]^2-\varepsilon_{10}^2-\alpha_s^2}\right]\sigma_4,$$

$$\Psi_{1\mathbf{k},2-\mathbf{k}}^{1\mathbf{q},2-\mathbf{q}}\approx 2\left[\varepsilon_{20}^2+\varepsilon_{10}^2+2\alpha_s^2+8\varepsilon_H^2\varepsilon_{2\mathbf{k}}^2\sum_{j=1,2}\frac{1}{[\varepsilon_{2\mathbf{k}}+(-1)^j\varepsilon_{1\mathbf{k}}]^2-\varepsilon_{10}^2-\alpha_s^2\delta_{1,j}}\right]\sigma_4,$$

where $\mathbf{s} = \mathbf{k} - \mathbf{q}$, $|\mathbf{q}| \approx |\mathbf{k}|$, and

$$\sigma_4=\frac{\pi\mu^2\varepsilon_E}{4V\varepsilon_M}\prod_j \varepsilon_j^{-1/2}.$$

[1)]See also the investigation of "parallel pumping" in ferromagnets [[13]].

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