Injection of a relativistic electron beam into a plasma with strong spatial dispersion

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A solution is reported for the problem of injection of a monoenergetic electron beam into a plasma, taking into account spatial dispersion. It is shown that, when the mean free path of the plasma electron is much less than the radius of the beam, and spatial dispersion is weak, the diffusion relaxation length of the fields and currents induced by the beam in the plasma is determined by collisional dissipation and is given by the well-known expression (1) which remains valid even for ultrarelativistic temperatures when Eq. (16) is taken into account. In the opposite limit of strong spatial dispersion in the plasma, when the electron free path is greater than the beam radius, the diffusion length is determined by Cerenkov dissipation and is given by Eqs. (8) and (17).

The reaction of "cold" plasma during injection of a low-density electron beam is investigated in a number of papers.[1-3] It is shown in[2] that a monoenergetic beam induces fields and currents in the plasma which compensate the beam magnetic field and current at distances

$$z<z_0=\frac{u}{\nu}\left(\frac{\omega_p}{c}r_0\right)^2 \tag{1}$$

from the leading front of the beam, and this facilitates the injection of the beam into the plasma. In this expression, u is the beam velocity, ω_p is the Langmuir frequency of the plasma electrons, and ν is their collision frequency. This result is valid when the plasma is sufficiently dense, so that the mean free path of the electrons is less than the beam radius, i.e., $\nu \gg v_{Te}/r_0$, where v_{Te} is the thermal velocity of the electrons, and the spatial dispersion can be neglected.

In this paper we consider the injection of an electron beam into plasma with arbitrary (including relativistic) temperatures when the electron distribution function is

$$f(\mathbf{p})=\frac{N_e}{4\pi(mc)^3}\exp\left[-\frac{c(m^2c^2+p^2)^{1/2}}{T_e}\right]\Big/\frac{T_e}{mc^2}K_2\left(\frac{mc^2}{T_e}\right), \tag{2}$$

where N_e is the plasma electron density, T_e is the electron temperature, and $K_2(x)$ is the Macdonald function.

The distribution given by (2) leads to the following expressions for the longitudinal (l) and transverse (tr) permittivities of isotropic electron plasma:[4]

$$\begin{gathered}\varepsilon^l(\omega,k)=1-\frac{2\pi e^2N_e}{\omega k^3cT_e}K_2^{-1}\left(\frac{mc^2}{T_e}\right)\int_{-kc}^{kc}d\omega'\frac{\omega'^2}{\omega+i\nu-\omega'}\cdot\\ \times\exp\left\{-mc^2/T_e\left(1-\frac{\omega'^2}{k^2c^2}\right)^{1/2}\right\}\left[\left(1-\frac{\omega'^2}{k^2c^2}\right)^{-1}\right.\\ \left.+2\frac{T_e}{mc^2}\left(1-\frac{\omega'^2}{k^2c^2}\right)^{-1/2}+2\left(\frac{T_e}{mc^2}\right)^2\right],\\ \varepsilon^{tr}(\omega,k)=1-\frac{2\pi e^2N_e}{\omega kmc}K_2^{-1}\left(\frac{mc^2}{T_e}\right)\int_{-kc}^{kc}\frac{d\omega'}{\omega+i\nu-\omega'}\\ \times\exp\left\{-mc^2/T_e\left(1-\frac{\omega'^2}{k^2c^2}\right)^{1/2}\right\}\left[\frac{T_e}{mc^2}\left(1-\frac{\omega'^2}{k^2c^2}\right)+\left(1-\frac{\omega'^2}{k^2c^2}\right)^{1/2}\right].\end{gathered} \tag{3}$$

Following the method put forward in[2], we obtain the following spatial distribution for the magnetic field induced by the beam:

$$B_\varphi(r,z')=i\frac{B_0}{2\pi}\int_0^\infty dk_\perp\cdot k_\perp J_1(k_\perp r)J_1(k_\perp r_0)\int_{-\infty}^{\infty}\frac{dk_z}{k_z}\frac{\exp\{ik_zz'\}-\exp\{ik_zz_1\}}{k^2-(k_zu)^2c^{-2}\varepsilon^{tr}(k_zu,k)},$$

$$k^2=k_z^2+k_\perp^2,\quad B_0=4\pi en_0ur_0/c,\quad z'=z-ut,\quad z_1=z+u\tau, \tag{4}$$

where τ is the injection time and n_0 the beam density.

In the limit of nonrelativistic plasma temperatures, $T_e \ll mc^2$, and in the region in front of the beam ($z' > 0$), we have from (4) the expression for the magnetic field B_φ, which is identical with the previous result reported in[2]. In the region of the beam itself, ($z' < 0$, $z_1 > 0$) we observe the diffusion of the magnetic field into plasma, which is determined by the contribution of a diffusion-type pole to (4), in addition to disturbances which decay exponentially over distances $c/\gamma\omega_p$. This pole is found from the dispersion relation

$$k^2=(k_zu)^2\varepsilon^{tr}(k_zu,k)/c^2, \tag{5}$$

where for $\nu \gg v_{Te}/r_0$ and nonrelativistic temperatures the diffusion length is given by (1). In the opposite case, on the other hand, when $\nu \ll v_{Te}/r_0$, we can use the approximation of strong spatial dispersion

$$\omega=k_zu\ll kv_{Te}, \tag{6}$$

and the diffusion pole is given by

$$k_z=-i\left(\frac{2}{\pi}\right)^{1/2}\frac{c^2}{\omega_p^2}\frac{v_{Te}}{u}k_\perp^3, \tag{7}$$

which enables us to introduce the diffusion length

$$z_0'=\left(\frac{\pi}{2}\right)^{1/2}\frac{ur_0}{v_{Te}}\left(\frac{\omega_p}{c}r_0\right)^2. \tag{8}$$

We note that the approximation given by (4) is valid for the contribution determined by the pole (5) only in the case of dense plasma, when

$$\omega_p\gg c/r_0. \tag{9}$$

The expression for the magnetic field obtained from (4) in the nonrelativistic limit $T_e \ll mc^2$, and when $\nu \ll v_{Te}/r_0$, has the following form in the region of the beam ($z' < 0$, $z_1 > 0$):

$$B_\varphi=B_0\left\{-\frac{1}{2}[\Sigma^c_{1,1}(z')+\Sigma_{1,1}(z_1)]+\Psi_{1,1}(\infty)-S_{1,-1}(z')\right\}, \tag{10}$$

$$\Sigma^c_{nm}(z)=\int_0^\infty dk_\perp\cdot k_\perp^m\frac{J_n(k_\perp r)J_1(k_\perp r_0)}{k_\perp^2+\omega_p^2/c^2}\exp\left\{-|z|\gamma\left(k_\perp^2+\frac{\omega_p^2}{c^2}\right)^{1/2}\right\},$$

$$\Psi_{11}(\infty)=\int_0^\infty\frac{dk_\perp}{k_\perp}J_1(k_\perp r)J_1(k_\perp r_0)=\frac{1}{2}\begin{Bmatrix}r/r_0, & r<r_0\\ r_0/r, & r>r_0\end{Bmatrix}.$$

The new feature as compared with[2] is the diffusion contribution which is given by the pole (7):

$$S_{1,-1}=\int_0^\infty\frac{dk_\perp}{k_\perp}J_1(k_\perp r)J_1(k_\perp r_0)\exp\{-\alpha r_0^3k_\perp^3\}, \tag{11}$$

where $\alpha = |z'|/z_0'$.

The integral given by (11) can be evaluated in two opposite limiting cases. When $\alpha \gg 1$ (large distances from the leading front of the beam) we have from (10) and (11)

$$B_\varphi = B_0 \left\{ \begin{matrix} r/r_0, & r<r_0 \\ r_0/r, & r>r_0 \end{matrix} + \frac{1}{12}\Gamma\left(\frac{2}{3}\right)\left(\frac{z_0'}{|z'|}\right)^{2/3} \right\}. \qquad (12)$$

On the other hand, when $\alpha \ll 1$, in a region expanding toward the leading front

$$r_0\alpha^{1/3} \ll |r-r_0| \ll r_0 \qquad (13)$$

we have

$$B_\varphi = B_0 \frac{\alpha}{\pi}\left[1-\frac{r}{r_0}\right]^{-2}. \qquad (14)$$

Analysis of (13) and (14) shows that, when the field B_φ is a maximum near the beam surface and $r = r_0$, we have with good accuracy

$$B_\varphi = B_0 \frac{2}{\pi}\alpha^{1/3}\Gamma\left(-\frac{1}{3}\right). \qquad (15)$$

It is readily seen that, when $\alpha \ll 1$, the field outside the region $|r - r_0| \ll r_0$, is much lower than that given by (14). This means that the magnetic field is tubular and largely localized in the region $|r - r_0|$ r_0, i.e., we have magnetic beam neutralization over distances less than the diffusion length (8). With increasing distance from the leading front of the beam, the tubular nature of the magnetic field distribution is found to disappear, and magnetic neutralization weakens. It is clear from (12) that it is practically absent for $|z'| > z_0'$.

A similar situation occurs for ultrarelativistic temperatures $T_e \gg mc^2$. When $\nu \gg c/r_0$ the result given in[2] $[z_0 = (\omega_p r_0/c)^2 u/v]$ remains unaltered except that, instead of ω_p, we must substitute

$$\omega_p'^2 = 2\pi N_e e^2 c^2/T_e. \qquad (16)$$

When $\nu \ll c/r_0$, the expression for the diffusion length given by (8) is replaced by

$$z_0'' = \frac{\pi}{2}\frac{ur_0}{c}\left(\frac{\omega_p'}{c}r_0\right)^2. \qquad (17)$$

Here again

$$\omega_p'^2 = 2\pi N_e e^2 c^2/T_e.$$

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