Turbulent dispersion of a fast particle cloud in plasma

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(Submitted March 14, 1974)
Zh. Eksp. Teor. Fiz. 67, 969–978 (September 1974)

Nonstationary expansion of high-energy electron cloud in a cold plasma with a weak magnetic field is considered. It is shown that within a broad range of parameters the dispersion of fast particles occurs in a diffuse manner with a characteristic velocity that is much lower than the particle velocity. Effective isotropization of the directions of particle velocities occurs during the dispersion, while the energy spectrum itself is weakly deformed. A feature of turbulent dispersion is the "evaporation" of sharply anisotropic low-density particle streams from the cloud, with their velocities parallel to the dispersion direction. The velocity of these particles is close to the free dispersion velocity.

1. The expansion dynamics of a fast charged particle cloud in a plasma is of fairly general interest, especially in its application to solar and magnetospheric physics. The dispersion of fast particles from a local source also plays an important role in anomalous plasma heating when most of the energy is dissipated in a small volume, near the point of reflection of a microwave field for example. In such problems pair collisions are not significant to fast particle dispersion, while collective effects leading to plasma turbulence and anomalous collision frequency are decisive.

Turbulent dispersion of a fast electron cloud in a plasma with a strong magnetic field ($\omega_p^2 \ll \omega_H^2$) is considered in [1-3]. In a strong magnetic field, the plasma-wave spectrum and the diffusion of the particles in velocity space become one-dimensional. For a sufficiently dense cloud we can thus write a relatively simple system of "quasi-gasdynamic" equations describing the dispersion.

For the case of a weak magnetic field ($\omega_H^2 \ll \omega_p^2$) or for zero field the diffusion of the fast particles in velocity space and the spectrum of the plasma turbulence excited by the particles are no longer one-dimensional, making the problem more complicated. A simplifying factor in this case is the fact that fast particles excite plasma oscillations whose phase velocity is much lower than the average velocity of the fast particles ($v_{ph} \ll v_0$). As a result, the particles experience elastic collisions with the plasma oscillations and their velocity distribution function remains almost isotropic. Therefore the cloud expands diffusively with a characteristic velocity significantly lower than the free dispersion velocity. The plasma turbulence level is maintained by the weak anisotropy created by the expanding cloud. Typical of this case is the formation of a "halo" [1)] of low-density particles having an energy plateau within the interval $w_T < w \ll w_0 = (1/2)mv_0^2$ (w_T is the thermal energy of background-plasma particles); this halo controls the intensity of the plasma waves. A quantitative investigation of this problem is given below for the case of a simple model of one-dimensional symmetric dispersion.

2. We consider a one-dimensional (in coordinate space) symmetric dispersion of an electron cloud along the s_0 axis, controlled by a weak magnetic field, for example. The system of quasilinear equations describing such a dispersion is [2]:

$$\frac{\partial f}{\partial t}+vx\frac{\partial f}{\partial s}=\frac{1}{v^2}\frac{\partial}{\partial v}v^2\left(D_{vv}\frac{\partial f}{\partial v}+D_{vx}\frac{1}{v}\frac{\partial f}{\partial x}\right) \tag{1}$$

$$+\frac{1}{v}\frac{\partial}{\partial x}\left(D_{vx}\frac{\partial f}{\partial v}+D_{xx}\frac{1}{v}\frac{\partial f}{\partial x}\right),$$

$$\frac{\partial \varepsilon_k}{\partial t}+\mathbf{v}_{gr}\frac{\partial \varepsilon_k}{\partial \mathbf{r}}=2(\gamma_k-\nu_k)\varepsilon_k, \tag{2}$$

$$\left.\begin{matrix}D_{vv}\\ D_{vx}\\ D_{xx}\end{matrix}\right\}=\left(\frac{4\pi e}{m}\right)^2\int_0^\infty\int_{-1}^{+1}k^2dkdy\varepsilon_k$$

$$\times\operatorname{Re}\left\{kv\left[1-x^2-y^2-\frac{\omega}{kv}\left(\frac{\omega}{kv}-2xy\right)\right]^{1/2}\right\}^{-1}\left\{\begin{matrix}\dfrac{\omega^2}{k^2v^2}\\ \dfrac{\omega}{kv}\left(y-\dfrac{\omega x}{kv}\right),\\ \left(y-\dfrac{\omega x}{kv}\right)^2\end{matrix}\right. \tag{3}$$

$$\gamma_k=\frac{2\omega}{n_p k\partial\varepsilon/\partial\omega}\left(\frac{\omega_p}{k}\right)^3\int_0^\infty\int_{-1}^{\pm 1}dvdx\operatorname{Re}\left\{\left[\frac{\partial}{\partial v}-\frac{1}{v}\left(x-\frac{kv}{\omega}y\right)\frac{\partial}{\partial x}\right]f\right.$$
$$\left.\times\left[1-x^2-y^2-\frac{\omega}{kv}\left(\frac{\omega}{kv}-2xy\right)\right]^{-1/2}\right\}. \tag{4}$$

The kinetic equation for the electron distribution function f is written in terms of the variables of the velocity modulus $v = |\mathbf{v}|$ and the cosine x of the angle between the velocity and direction of the dispersion; ϵ_k is the spectral energy density of the plasma oscillations, γ_k is the growth rate of the plasma oscillations, ν_k is the linear decrement and includes the Landau damping in the background plasma and the collision damping, and y is the cosine of the angle between wave vector **k** and $\mathbf{s}_0$.

If the high-energy electron cloud is dense enough, the anisotropy due to the second term in the left-hand side of (1), which appears during the dispersion process, is quickly eliminated through scattering by the waves, and the particle distribution is kept quasi-isotropic. This condition permits us to write the distribution function in the form

$$f=F+\Phi,\quad F=\langle f\rangle\equiv\frac{1}{2}\int_{-1}^{+}dxf,\quad \Phi\ll F, \tag{5}$$

and to write separate equations for the isotropic F and anisotropic Φ parts of f (see also [4]):

$$\frac{\partial F}{\partial t}=\frac{v^4}{4}\frac{\partial}{\partial s}\left[\left\langle\frac{(1-x^2)^2}{D_{xx}}\right\rangle\frac{\partial F}{\partial s}\right]+\frac{1}{v^2}\frac{\partial}{\partial v}v^2\left[\langle D_{vv}\rangle-\left\langle\frac{D_{vx}^2}{D_{xx}}\right\rangle\frac{\partial F}{\partial v}\right]$$
$$+\frac{v^2}{2}\frac{\partial}{\partial s}\left[\left\langle\frac{(1-x^2)D_{vx}}{D_{xx}}\right\rangle\frac{\partial F}{\partial v}\right]-\frac{1}{2v^2}\frac{\partial}{\partial v}v^2\left[v^2\left\langle\frac{(1-x^2)D_{vx}}{D_{xx}}\right\rangle\frac{\partial F}{\partial s}\right]. \tag{6}$$

The anisotropic part Φ of the distribution function is expressed in terms of F in the following manner:

$$\Phi\approx\frac{v^3}{2}\frac{\partial F}{\partial s}\int_0^x dx'\frac{(x'^2-1)}{D_{xx}}. \tag{7}$$

According to (4) plasma oscillations satisfying condition (5) are excited in the region of low phase velocities

$$v_{\text{ph}} \ll v_0. \tag{8}$$

Using this condition and substituting (5) and (7) in (4), we obtain the following expression for the growth rate:

$$\gamma_k = \omega_p \left(\frac{\omega_p}{k}\right)^3 \frac{\pi}{n_p} \left[-F\left(\frac{\omega_p}{k}\right) - \frac{ky}{4\omega_p} \int_0^\infty \int_{-1}^{+1} dv\, dx\, \mathrm{Re} \frac{1}{(1-x^2-y^2)^{1/2}} \frac{v^3(1-x^2)}{D_{xx}} \frac{\partial F}{\partial s} \right], \tag{9}$$

where $\omega \approx \omega_p = (4\pi e^2 n_p/m)^{1/2}$ is the Langmuir frequency and n_p is the concentration of the background plasma.

We use one more simplifying assumption that is valid at sufficiently high values of γ, i.e., the intensity of the plasma waves is determined at the quasi-stationary level [2]

$$\gamma - \nu = 0, \quad \partial(\gamma-\nu)/\partial v_{\text{ph}} = 0, \quad \partial^2(\gamma-\nu)/\partial v_{\text{ph}}^2 < 0, \tag{10}$$

where ν is the total damping of the plasma waves by the background-plasma particles. The last two relations of (10) follow from the requirement that condition $\gamma = \nu$ be satisfied in the extremum point where $\gamma - \nu$ reaches a maximum.

Equations (10) enable us to write the intensity ϵ and the phase velocity v_{ph} of plasma waves as functionals of F. We can readily see that the first relation of (10), taking (9) into account, represents an Abelian integral equation in terms of y. Its solution is of the form

$$D_{xx} = \frac{1}{v}(1-x^2)^2 D(t,s). \tag{11}$$

The angular dependence of the energy density of the plasma waves is found from the solution of one more Abelian integral equation which follows from (3) and (11) when (8) and (9) are taken into account:

$$\varepsilon_k = \frac{m^2}{3\pi^3 e^2 k^4} y^2 D(t,s)\,\delta(k - k^*(t,s)), \quad \text{if} \quad y \int_0^\infty dv\, v^4 \frac{\partial F}{\partial s} < 0;$$
$$\varepsilon_k = 0, \quad \text{if} \quad y \int_0^\infty dv\, v^4 \frac{\partial F}{\partial s} > 0, \tag{12}$$

Here $k^* = \omega_p/v^*_{\text{ph}}$ (as in D(t, s)) is expressed in terms of F using (10). The diffusion coefficients present in (6) are readily computed for the obtained energy distribution (12) of the plasma oscillations. Using (6), (9), and (10), we obtain the following system of equations determining the behavior of turbulent dispersion of a fast-electron cloud:

$$\frac{\partial F}{\partial t} = \frac{0.27}{v^2} \frac{\partial}{\partial v} v^{-1} \left(v_{\text{ph}}^2 D \frac{\partial F}{\partial v} \right) - 0.44 \left(\operatorname{sgn} \int_0^\infty dv\, v^4 \frac{\partial F}{\partial s} \right) \frac{\partial}{\partial s} \left(v v_{\text{ph}} \frac{\partial F}{\partial v} \right)$$
$$+ 0.44 \left(\operatorname{sgn} \int^\infty dv\, v^4 \frac{\partial F}{\partial s} \right) \frac{1}{v^2} \frac{\partial}{\partial v} v^3 \left(v_{\text{ph}} \frac{\partial F}{\partial s} \right) + \frac{v^5}{4} \frac{\partial}{\partial s} \left(D^{-1} \frac{\partial F}{\partial s} \right), \tag{13a}$$

$$-\frac{n_p \nu}{\omega_p} - v_{\text{ph}}^3 F(v_{\text{ph}}) + \frac{\pi}{2} v_{\text{ph}}^2 \left| \int_0^\infty dv\, v^4 \frac{\partial F}{\partial s} \right| D^{-1} = 0, \tag{13b}$$

$$-\frac{n_p}{\omega_p} \frac{\partial \nu}{\partial v_{\text{ph}}} - \frac{\partial}{\partial v_{\text{ph}}} (v_{\text{ph}}^3 F) + \pi v_{\text{ph}} \left| \int_0^\infty dv\, v^4 \frac{\partial F}{\partial s} \right| D^{-1} = 0. \tag{13c}$$

The initial and boundary conditions take the form [2)]

$$t=0, \; F=F_0(s,v), \; D=D_0(s,v), v_{\text{ph}} = v_{\text{ph}0}(s); \tag{14a}$$

$$s \to \infty, \; F \to 0; \quad v \to \infty, \; F \to 0; \quad s=0, \; \frac{\partial F}{\partial s} = 0; \quad v = v_{\text{ph}}, \; \frac{\partial F}{\partial v} = 0. \tag{14b}$$

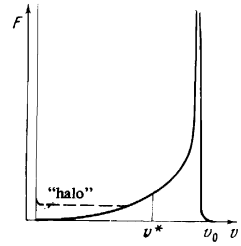

FIG. 1

The system of equations (13) enables us to find the distribution function F(t, s, v), the phase velocity $v_{\text{ph}}(t, s)$, and the intensity D(t, s) of plasma noise.

The analytic solution of (13) and (14) can be obtained from the following considerations. It follows from (13a) that velocity diffusion occurs primarily in the region of low velocities $v \ll v_0$. In this region the plasma waves cause a rapid smoothing of the energy distribution function. By virtue of (13b) a relatively small number of particles with $v \sim v_{\text{ph}}$ is sufficient to stabilize the instability. Consequently, in the low velocity region there forms an extended "halo" that is due in part to the cloud electrons and in part to electrons from the plasma itself (Fig. 1). The relaxation of the distribution function in the low-energy region is quantitatively described by the first term in the right-hand side of (13a).

A different behavior is shown by the cloud particles with velocity $v_0 \gg v_{\text{ph}}$. Under certain conditions discussed below we can neglect the energy diffusion of these particles and consider merely the last term of (13a), which describes the spatial expansion of the cloud.

The solution method described above is convenient if we start with a fast-particle cloud with a small energy scatter and with an extended "halo" of background electrons having a relatively low amplitude (Fig. 1).

As an example, we consider turbulent expansion of the cloud, given an initial fast-particle distribution function of the type

$$F_0 = \frac{n_0(s)}{4\pi v_0^2} \delta(v - v_0) + \frac{(\alpha+3)}{4\pi v_0^{\alpha+3}} n_h(s) M^\alpha(v) [1(v) - 1(v - v_0)]. \tag{15}$$

Here $\delta(\xi)$ and $1(\xi)$ are the Dirac delta function and the Heaviside unit function, n_0 is electron density in the cloud, n_h is particle density in the "halo," and $M^\alpha(v)$ is a polynomial of the α-th degree in v.

The evolution of F in the region of low energies is described by the equation (see (13a))

$$\frac{\partial F}{\partial t} = \frac{0.27}{v^2} \frac{\partial}{\partial v} v^{-1} \left(v_{\text{ph}}^2 D \frac{\partial F}{\partial v} \right), \tag{16}$$

$$t=0, \quad F = F_0(s,v); \quad v = v_{\text{ph}}, \quad \frac{\partial F}{\partial v} = 0; \quad v = v^*, \quad F = F^*(s,v).$$

The last boundary condition is due to the fact that the solution of (16) for $v = v^*$ ($v_{\text{ph}} \ll v^*$) should produce a fit with the total solution of (13a). For the case of low collision frequency the phase velocity of the waves is determined by Landau damping in the main plasma. In such a case $v_{\text{ph}} \sim 3v$ and varies little in the course of dispersion of the cloud.

The solution of (16) is derived in the Appendix. Provided that $v^{*5}\tau^{-1} \gg 1$ this solution is independent of v^* and F^* in the region $v \sim v_{\text{ph}}$ and is determined by the following expression (A6):

$$F(\tau, v_{ph}) = \frac{1.2}{\tau^{0.6}} \int_0^\infty dv\, v^2 \exp\left(-\frac{4v^5}{25\tau}\right) F_0(s, v), \qquad (17)$$

where

$$\tau = \int_0^t v_{ph}{}^2 D\, dt'. \qquad (18)$$

Substituting (15) into (17) we obtain a solution in the form [3)]

$$F(\tau, s, v_{ph}) = A(\tau + \tau_0)^{0.4\alpha}. \qquad (19)$$

Here

$$A = \frac{(2\alpha+3)}{4\pi v_0^{2\alpha+3}} \frac{\Gamma(0.6+0.4\alpha) n_h}{\Gamma(0.6)\cdot(0.4)^{0.8\alpha}},$$

and $\Gamma(\xi)$ is the gamma function.

We apply the obtained expression for the distribution function in the "low" velocity region (19) in order to determine the coefficient D. For this purpose we substitute (19) and (18) in (13b) and obtain a first-order differential equation relating τ to t. This equation is readily integrated and yields

$$\tau = \left(\frac{\pi}{2A\delta} v_{ph} \int_0^t dt' \left| \int_0^\infty dv\, v^4 \frac{\partial F}{\partial s} \right| + \tau_0^{-1/\delta} \right)^\delta - \tau_0, \qquad (20)$$

where $\delta = (0.4\alpha + 1)^{-1}$.

After the relationship between τ and t is established, the quantity D can be determined with the aid of (18):

$$D = \frac{1}{v_{ph}^2} \frac{\partial \tau}{\partial t}. \qquad (21)$$

Under certain conditions discussed below we can neglect the energy diffusion of the cloud particles. Retaining in such a case only the last term in the right-hand side of (13a) and changing over to the particle density in the cloud

$$n = 4\pi \int_0^\infty dv\, v^2 F,$$

we obtain the following equations describing the turbulent dispersion:

$$\frac{\partial n}{\partial t} = \frac{v_0^5}{4} \frac{\partial}{\partial s} \left(D^{-1} \frac{\partial n}{\partial s} \right). \qquad (22)$$

Differentiating (22) with respect to time and using (19)–(21) we obtain

$$\frac{\partial^2 n}{\partial t^2} = \frac{v_{ph}^2 v_0^5}{4} \frac{(1-\delta)}{\delta} \frac{\partial}{\partial s} \left[(\tau + \tau_0)^{-1} \frac{\partial n}{\partial s} \right]. \qquad (23)$$

Equation (23) yields a correct solution in the region of s values for which $\partial n/\partial s \neq 0$.

We now assume that $\partial n/\partial s = 0$ at a certain point $s = s^*$ ($s > 0$) [4)] and that the cloud expansion proceeds from the origin $s = 0$. Then in the region $s > s^*$ the solution is described by (23). In the region $s < s^*$ formally $D^{-1} \to \infty$ (see (20) and (21)) and $\partial n/\partial s \approx 0$. Actually in this region there is no plasma turbulence and the motion of the cloud particles follows the law of free dispersion. The solution for both regions must be "fitted" at $s = s^* : n^-(s^*) = n^+(s^*)$.

The simplest to solve is the case $\delta \equiv (0.4\alpha + 1)^{-1} \ll 1$. Here τ in (23) can be considered as independent of t and s, and equal to τ_∞. Thus the dispersion of the main cloud at $|s| > |s^*|$ is described by the equation

$$\frac{\partial^2 n^+}{\partial t^2} = v_d{}^2 \frac{\partial^2 n^+}{\partial s^2}, \quad v_d{}^2 = \frac{v_{ph}^2 v_0^5 (1-\delta)}{4(\tau_0 + \tau_\infty)\delta}, \qquad (24)$$

under the following initial and boundary conditions:

$$t=0, \quad n = n_0(s); \quad s = s^*, \quad n^+ = n^-. \qquad (25)$$

The density $n^-(t)$ is independent of s in the region $|s| < |s^*|$. To find $n^-(t)$ we can use the law of conservation of the total number of particles in the cloud

$$\int_{-\infty}^{+\infty} [n^-(t) + n^+(t, s)]\, ds = 2N_0 = \int_{-\infty}^{+\infty} n_0(s)\, ds. \qquad (26)$$

For sufficiently large $t \gg s_0/v_p$ (s_0 is the initial dimension of the cloud) the contribution from the region $|s| \geq |s^*|$ to the integral of (26) is small and therefore

$$n^-(t) = N_0/s^* = N_0/v_d t. \qquad (27)$$

The dispersion rate can be expressed, taking (20) into account, in the form

$$v_d = \alpha v_{ph} \left(\frac{3}{\alpha} \frac{v_d}{v_{ph}} \frac{n_h}{n_{0\,max}} \right)^{1/0.8\alpha}, \qquad (28)$$

where n_h is the particle density in the "halo" and $n_{0}max$ is the maximum initial particle density in the cloud.

Figure 2 illustrates the approximate form of the solution for $t \geq s_0/v_p$. The same figure shows the energy density ϵ of plasma waves as a function of s. Selecting as an example the initial distribution $n_0(s)$ in the form of a triangle $2s_0$ wide, we have

$$\tau = \frac{v_0^5}{\alpha} \left(0.3 \frac{n_0}{n_h} \frac{s_0}{s} \right)^{1/(0.4\alpha+1)},$$

$$D = \frac{s v_0^5}{s_0^2 v_{ph} (0.4\alpha+1)} \left(0.3 \frac{n_0}{n_h} \frac{s_0}{s} \right)^{1/(0.4\alpha+1)}, \quad s_0 + v_d t - \frac{s_0^2}{v_d t} \leq |s| < s_0 + v_d t;$$

$$D = 0, \quad |s| < s_0 + v_d t - \frac{s_0^2}{v_d t}, \quad s_0 + v_d t < |s|. \qquad (29)$$

We consider the limits of applicability of the obtained solution. A comparison of various terms in the right-hand side of (13a) readily shows that the condition $v^{*5}\tau^{-1} \gg 1$ under which solution (19) holds is satisfied through the entire cloud-dispersion phase.

Integrating (13a) with respect to s from $-\infty$ to ∞, we obtain an equation describing the "cooling" of the cloud. Using (29) we have

$$\frac{\partial \bar{F}}{\partial t} \approx \frac{0.6[0.3(n_0/n_h) s_0/s]^{1/(0.4\alpha+1)}}{\alpha^2 (t + s_0/v_p)} \frac{1}{u^2} \frac{\partial}{\partial u} \left(\frac{1}{u} \frac{\partial \bar{F}}{\partial u} \right), \qquad (30)$$

where

$$u = \frac{v}{v_0}, \quad \bar{F} = \int_{-\infty}^{+\infty} ds\, F.$$

According to (39), the characteristic time t* describing the change in the energy spectrum of the cloud is defined by the relation

$$\alpha^{-2} 0.6 \left(0.3 \frac{n_0}{n_h} \frac{s_0}{s} \right)^{1/(0.4\alpha+1)} \ln \frac{t^* v_d}{s_0} \sim 1; \qquad (31)$$

Therefore for $\alpha \gg 1$ the cloud cools down very slowly and the cooling effect can be neglected in the description of spatial diffusion.

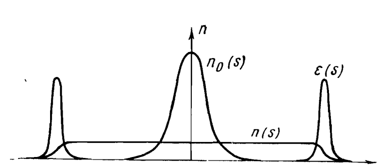

FIG. 2

The approximate picture of the dispersion appears to be valid also in the more general case of arbitrary initial conditions and parameters. However, the quantitative description based on (13) can differ significantly from the foregoing.

We now briefly consider the dispersion as a function of the external magnetic field. We note that even in the absence of external magnetic field there is a small fraction of particles that do not interact with plasma waves in the manner described above. These are particles whose velocities are subtended by a narrow cone about the dispersion axis. The existence of this cone is due to the fact that the energy density of the plasma oscillations vanishes at $\mathbf{k} \perp \mathbf{s}_0$ ($\mathbf{s}_0$ is the dispersion axis) and it is precisely these waves that interact with particles with $\mathbf{v} \parallel \mathbf{s}_0$. The number of such particles can be found by determining the anisotropic part Φ of the distribution function. According to (7) and (11) we have

$$\Phi \approx \frac{v_{ph}}{v_0} F \ln \left| \frac{1-x}{1+x} \right|, \quad x \equiv \cos(\widehat{sv}). \qquad (32)$$

Using this we find the region of transverse electron velocities for which the above analysis is invalid:

$$v_\perp \leqslant 2v_0 \exp(-v_0/2\eta v_{ph}), \quad \eta > 1. \qquad (33)$$

If the external magnetic field differs from zero, the fraction of these particles increases. In fact, for a weak magnetic field the nature of the dispersion is preserved if the particle effectively interacts with a large number of gyrofrequency harmonics, i.e., if the following condition is satisfied:

$$\frac{k_\perp v_\perp}{\omega_H} \gg 1 \quad \text{or} \quad \frac{v_\perp}{v_{ph}} \frac{\omega_p}{\omega_H} \gg 1. \qquad (34)$$

The particles for which the reverse inequality is satisfied interact with the oscillations only at the Cerenkov resonance $\omega = k_S v_S$, and the relaxation in velocity space is one-dimensional.

We note one more difficulty with the isotropization caused by the increasing external magnetic field. Isotropization is possible if passage through zero v_S is permitted in the quasilinear diffusion process. Passage through zero occurs when $\omega = n\omega_H$ ($n = 1, 2, \ldots$), i.e., the plasma wave spectrum contains frequencies that coincide with the gyrofrequency harmonic of the electrons in the cloud. This condition imposes the following limitation on the strength of the magnetic field:

$$\omega_H/\omega_p < v_T^2/v_{ph}^2, \quad \omega_H \ll \omega_p, \qquad (35)$$

where v_T is the thermal velocity of electrons in the main plasma.

We sum up the results:

1. Turbulent dispersion of a fast ($v_0 \gg v_T$) particle cloud in dense plasma with a sufficiently weak magnetic field has a diffusive character and the mean dispersion velocity is significantly lower than the characteristic velocity of the particles in the cloud. The turbulence is concentrated in thin "walls" at the edges of the cloud.

2. The dispersion process is accompanied by strong isotropization of the velocity directions of the particles in the cloud, while the initial energy spectrum is relatively weakly deformed.

3. Typical for turbulent dispersion is the formation, in velocity space, of an extended "halo" of low density particles having an energy plateau in the interval $w_T < w \ll w_0$.

4. Sharply anisotropic low-density particle streams whose velocities are directed along the dispersion axis of the cloud "evaporate" from the relatively slowly expanding cloud. The velocities of these particles are close to the free dispersion velocity.

APPENDIX

We make the following change of variables in (16)

$$\tau = 1.08 \int_0^t v_{ph}^2 D \, dt', \quad z = v^2. \qquad (A.1)$$

As a result, we have

$$\partial F/\partial \tau = z^{2-1/\nu} \partial^2 F/\partial z^2, \qquad (A.2)$$

where

$$\nu = 2/5, \quad z \in [ab], \quad \partial F/\partial s|_{z=a} = 0, \quad F|_{z=b} = F^*.$$

After Laplace transformation in time domain, Eq. (A.2) assumes the form

$$pF - F_0 = z^{2-1/\nu} d^2F/dz^2. \qquad (A.3)$$

Solving this inhomogeneous linear differential equation by the Euler method we find that

$$\begin{aligned} F(p, z) &= C_1(p) z^{1/2} I_\nu(2\nu p^{1/2} z^{1/2\nu}) + C_2(p) z^{1/2} K_\nu(2\nu p^{1/2} z^{1/2\nu}) \\ &- 2\nu z^{1/2} I_\nu(2\nu p^{1/2} z^{1/2\nu}) \int_b^z dz' F_0(z') z'^{(1/\nu - 3/2)} K_\nu(2\nu p^{1/2} z'^{1/2\nu}) \\ &+ 2\nu z^{1/2} K_\nu(2\nu p^{1/2} z^{1/2\nu}) \int_a^z dz' F_0(z') z'^{(1/\nu - 3/2)} I_\nu(2\nu p^{1/2} z'^{1/2\nu}). \end{aligned} \qquad (A.4)$$

The coefficients $C_1(p)$ and $C_2(p)$ are determined from the boundary conditions

$$\partial F(p, z)/\partial z = 0 \quad \text{at} \quad z = a,$$

$$F(p, z) = F^*(p) \quad \text{at} \quad z = b.$$

The solution $F(p, z)$ has the simplest form for $1 \ll (v^*)^5 \tau^{-1}$. In this case $F(p, a)$ does not depend on v^* or F^* and is written in the form

$$F(p, a) = \frac{1}{p^{1/2} a^{1/2\nu - 1} K_{1-\nu}(2\nu p^{1/2} a^{1/2\nu})} \int_a^b dz' F_0(z') z'^{(1/\nu - 3/2)} \times K_\nu(2\nu p^{1/2} z'^{1/2\nu}). \qquad (A.5)$$

The inverse Laplace transform in the limit of small values $a \to 0$ yields

$$\begin{aligned} F(t, a) &= \frac{\nu^{1-2\nu}}{\tau^{1-\nu}\Gamma(1-\nu)} \int_0^b dz \, F_0(z) z^{1/\nu - 2} \exp\{-\nu^2 z^{1/\nu}/\tau\} \\ &\approx \frac{\nu^{1-2\nu}}{\tau^{1-\nu}\Gamma(1-\nu)} \int_0^\infty dz \, F_0(z) z^{1/\nu - 2} \exp\{-\nu^2 z^{1/\nu}/\tau\}. \end{aligned} \qquad (A.6)$$

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1) An analogous situation occurs in the asymptotic regime of current instability [2].

2) The initial conditions (14a) cannot be arbitrary, of course, since (13b) and (13c) are algebraic for a given F_0.

3) Solution (19) in this form can always be obtained upon a suitable choice of the coefficients in $M^\alpha(\nu)$.

4) For symmetric initial conditions, the solution in the region $s < 0$ is symmetric to the solution for $s > 0$.

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Translated by S. Kassel
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