Effect of pressure on magnetoacoustic resonance in uniaxial antiferromagnets

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The effect of pressure and magnetic field on the magnetoacoustic oscillation spectrum is considered for uniaxial antiferromagnets with positive or negative anisotropy constants. The dependences of the propagation velocity of quasielastic waves on field strength and pressure are calculated for a wide range of variation of these quantities. It is shown that, with respect to both magnetic field strength and pressure, the strongest (about 100%) variation of the propagation velocity of transverse, long-wave, quasielastic oscillations occurs in the vicinity of the phase transitions. The ratio of the damping to the frequency of the magnetoelastic wave is maximal at the phase-transition points, but is always less than unity.

1. INTRODUCTION

A strong dependence of the velocity of transverse sound on the value of the magnetic field applied in the basal plane of the crystal has been discovered in experimental study of magnetoacoustic resonance in hematite α-Fe_2O_3 and iron borate $FeBO_3$ [1-4]. The possibility of a similar strong dependence of the velocity of transverse sound in ferromagnetics has been pointed out in theoretical papers.[5-9] The case of a ferrite with collinear magnetic moments of the sublattices has been studied by Bar'yakhtar and Yablonskiĭ.[10] On the other hand, as follows from the work of Maksimenkov and Ozhogin,[1] the resonance properties of hematite are very sensitive to the influence of directional pressure.

In the present study we consider magnetoacoustic resonance in uniaxial antiferromagnets over wide ranges of magnetic field and pressure. It is shown that the coupling between the elastic and spin waves depends significantly on the value of the pressure. In phase transitions of the first and second order in the field and the pressure, singularities arise in magnetic crystals in the velocity of propagation of magnetoelastic waves.

2. MAGNETOELASTIC WAVES IN ANTIFERROMAGNETS WITHOUT WEAK FERROMAGNETISM

We use the following Hamiltonian to calculate the dispersion equation of coupled acoustic and spin waves in antiferromagnets of the considered type:

$$\mathcal{H}=\mathcal{H}_{m}+\mathcal{H}_{me}+\mathcal{H}_{e},$$

$$\mathcal{H}_{m}=\int dv\left\{\frac{1}{2}\alpha\left[\left(\frac{\partial \mathbf{M}_1}{\partial x_i}\right)^2+\left(\frac{\partial \mathbf{M}_2}{\partial x_i}\right)^2\right]+\alpha'\frac{\partial \mathbf{M}_1}{\partial x_i}\frac{\partial \mathbf{M}_2}{\partial x_i}+\delta\mathbf{M}_1\mathbf{M}_2\right.$$
$$\left.-\frac{1}{2}\beta(M_{1z}^2+M_{2z}^2)-\beta' M_{1z}M_{2z}-(\mathbf{H},\mathbf{M}_1+\mathbf{M}_2)\right\}, \quad (1)$$

$$\mathcal{H}_{me}=\frac{1}{4}\int dv\,\{\gamma(\mathbf{M}^2-\mathbf{L}^2)u_{ii}+2\gamma_1 L_iL_ku_{ik}+2\gamma_2 M_iM_ku_{ik}\},$$

$$\mathcal{H}_{e}=\frac{1}{2}\int dv\{\rho[\dot{\mathbf{u}}^2+(s_l^2-s_t^2)u_{ii}^2+2s_t^2u_{ik}^2]-2\sigma u_{xx}\}.$$

Here α, α' and δ are exchange integrals, β and β' are the anisotropy constants, γ, γ_1, γ_2 the magnetostriction constants, $\mathbf{L}=\mathbf{M}_1-\mathbf{M}_2$ and $\mathbf{M}=\mathbf{M}_1+\mathbf{M}_2$ are the antiferromagnetic and ferromagnetic vectors, respectively, $\mathbf{M}_1$ and $\mathbf{M}_2$ are the magnetizations of the sublattices, H the external magnetic field, ρ the density of the medium, s_l and s_t the velocities of longitudinal and transverse sound in the antiferromagnet, $\mathbf{u}(\mathbf{r}, t)$ the elastic displacement vector, and σ the tensor of the external elastic stresses.

For simplicity we assume the antiferromagnet to be isotropic in its elastic and magnetoelastic properties. A pressure applied along the x axis leads to the appearance of an effective anisotropy in the basal plane. It follows from the minimum of the ground-state energy that the vector L is established in the following way in the absence of a magnetic field (H = 0):

$$\mathbf{L}\|z, \quad \text{if} \quad H_{A1}=H_A-H_p\Theta(H_p)>0, \quad (2a)$$
$$\mathbf{L}\|x, \quad \text{if} \quad H_{A1}<0 \text{ and } H_p>0, \quad (2b)$$
$$\mathbf{L}\|y, \quad \text{if} \quad H_{A1}<0 \text{ and } H_p<0, \quad (2c)$$

where $H_A=(\beta-\beta')M_0$, $H_p=\gamma_1PM_0/\rho s_t^2$, $M_0=|\mathbf{M}_1|=|\mathbf{M}_2|$, $P=-\sigma$ is the external pressure ($P>0$ corresponds to compression), and

$$\Theta(\xi)=\begin{cases}1, & \text{at } \xi>0\\ 0, & \text{at } \xi<0.\end{cases}$$

A. If the field is directed along the z axis and the inequality (2a) is satisfied, then the collinear phase $\mathbf{L}\parallel\mathbf{H}$ is stable in the range of fields $0<H<H_2$ and the noncollinear phase is stable in the range $H_1<H<H_E$. Here $H_1\approx H_2\approx\sqrt{H_{A1}H_E}$, $H_E=2\delta M_0$.

In the analysis of the oscillations of the magnetic moments and of the lattice, we represent $\mathbf{M}_1(\mathbf{r}, t)$, $\mathbf{M}_2(\mathbf{r}, t)$ and $\mathbf{u}(\mathbf{r}, t)$ in the form

$$\mathbf{M}_j=\mathbf{M}_{j0}+\mathbf{m}_j, \quad u_{ik}=u_{ik}^{(0)}+u_{ik}^{(1)}, \quad |\mathbf{m}_j|\ll|\mathbf{M}_{j0}|, \quad u_{ik}^{(1)}\ll u_{ik}^{(0)}, \quad (3)$$

where $\mathbf{M}_{j0}$ and $u_{ik}^{(0)}$ are the equilibrium values determined from the energy minimum of (1). From the equations of motion of the magnetic moments of the sublattices with the relaxation term in the form $\mathbf{R}_j=-\delta_S[\mathbf{M}_j, \dot{\mathbf{M}}_j]/M_0$, where δ_S is the quantity which determines the relaxation of the spin waves, and from the equations of elasticity, we can easily obtain the dispersion equation of the coupled magnetoelastic waves of the antiferromagnet.

We first consider the case of the noncollinear phase ($H_1<H<H_E$). For waves propagating parallel to the antiferromagnetic vector $\mathbf{L}$ ($\mathbf{L}\parallel\mathbf{x}$ if $H_p>0$, and $\mathbf{L}\parallel\mathbf{y}$ if $H_p<0$), we have the following dispersion equation:

$$(\omega^2-\omega_{tk}^2)^2(\omega^2-\omega_{lk}^2)(\omega^2-\omega_{1sk}^2)(\omega^2-\omega_{2sk})-\omega_1^2\omega_{tk}^2(\omega^2-\omega_{lk}^2)^2(\omega^2-\omega_{1sk}^2)$$
$$-\omega_2^2\omega_{tk}^2(\omega^2-\omega_{lk}^2)(\omega^2-\omega_{tk}^2)(\omega^2-\omega_{1sk})-\omega_3^2\omega_{lk}^2(\omega^2-\omega_{tk}^2) \quad (4)$$
$$\times(\omega^2-\omega_{tk}^2)(\omega^2-\omega_{2sk})\approx 0.$$

The quantities ω_{tk} and ω_{lk} are the frequencies of transverse and longitudinal sound,

$$\omega_{1sk}=\omega_{1sk}'-i\omega_{1sk}''=\sqrt{\varepsilon_1\varepsilon_2}, \quad \omega_{2sk}=\omega_{2sk}'-i\omega_{2sk}''=\sqrt{\varepsilon_3\varepsilon_4}$$

are the frequencies of the first and second branches of the spin oscillations,

$$\varepsilon_1 \approx \omega_E - i\delta_s\omega,$$
$$\varepsilon_2 = g[(\alpha+\alpha'\cos 2\vartheta)k^2M_0 + H_E\cos^2\vartheta + H_{me} - H_{A1}] - i\delta_s\omega,$$
$$\varepsilon_3 = g[(\alpha-\alpha')k^2M_0 + H_{me}\sin^2\vartheta + |H_p|] - i\delta_s\omega,$$
$$\varepsilon_4 = g[(\alpha-\alpha'\cos 2\vartheta)k^2M_0 + H_E\sin^2\vartheta] - i\delta_s\omega,$$

g is the gyromagnetic ratio, k the quasimomentum, $\omega_E = gH_E$, $H_{me} = 2\gamma_1^2M_0^3/\mu s_t^2$, $\vartheta = \vartheta_1 = \vartheta_2$, $\cos\vartheta = H/H_E$, ϑ_1 and ϑ_2 are the polar angles of the magnetic moments of the sublattices,

$$\omega_1^2 = gH_{me}\varepsilon_3\left(\frac{\gamma}{\gamma_1} - 1\right)^2\sin^2 2\vartheta,$$

$$\omega_2^2 = gH_{me}\varepsilon_4\sin^2\vartheta, \quad \omega_3^2 = gH_{me}\varepsilon_1\left(\sin^2\vartheta - \frac{\gamma_2}{\gamma_1}\cos^2\vartheta\right)^2.$$

In the derivation of the dispersion equation, we started out from the approximation

$$H_E \gg H_A,\ |H_p|,\ H_{me}. \tag{5}$$

Analysis of the dispersion equation (4) shows that the magnetostriction interaction removes the degeneracy of the transverse sound. The transverse sound, polarized along the z axis, interacts most effectively with the first branch of the spin oscillations in the vicinity of the phase transition point of the first order from the noncollinear phase to the collinear. In the vicinity of the phase transition point ($H = H_1$), at small values of the wave vector $k(\omega_{tk} \ll \omega_{1s0})$, the following expressions are obtained for the corresponding frequencies of the coupled magnetoelastic waves:

$$\omega_{Ik} \approx \omega_{1sk},$$
$$\omega'_{IIk} \approx \frac{\omega_{tk}}{\omega'_{1sk}} g[(\alpha-\alpha')k^2M_0H_E + H^2 - H_1^2]^{1/2}, \tag{6}$$
$$\omega''_{IIk} \approx \frac{gH_{me}\omega_E^2\omega_{tk}^2}{2\omega'^4_{1sk}}\delta_s.$$

At the phase transition point $H = H_1$ the dispersion law of the quasielastic oscillations changes—it becomes quadratic. The velocity of these oscillations s_t' decreases to zero as $k \to 0$ (Fig. 1, curve 1). The ratio $Q_{II}^{-1} = 2\omega_{IIk}''/\omega_{IIk}'$ is at maximum at $H = H_1$ and is expressed as

$$Q_{II}^{-1} = \frac{\delta_s s_t}{g[(\alpha-\alpha')M_0H_{me}]^{1/2}}. \tag{7}$$

If $H_{me} \sim 1$ Oe, $s_t \sim 10^5$ cm/sec, $gM_0 \sim 10^{10}$ sec^{-1}, $\alpha - \alpha' \sim 10^{-12}$ cm^2, $\delta_s \sim 10^{-5}$, then $Q_{II}^{-1} \sim 10^{-2}$.

Transverse sound polarized in the basal plane turns out to be strongly coupled with the second branch of the spin waves throughout the entire range of fields of the noncollinear phase, with the exception of the vicinity of the point H_E. The corresponding frequencies of the coupled magnetoelastic waves in the approximation $\omega_{tk} \ll \omega_{2s0}$ have the following form:

$$\omega_{Ik} \approx \omega_{2sk},$$
$$\omega'_{IIk} = \frac{\omega_{tk}}{\omega'_{2sk}} g\{H_E[(\alpha-\alpha')k^2M_0 + |H_p|]\}^{1/2}\sin\vartheta, \tag{8}$$
$$\omega''_{IIk} = \frac{\delta_s gH_{me}\omega_E^2\omega_{tk}^2\sin^6\vartheta}{2(\omega'_{2sk})^4}.$$

In the range of fields $H_1 < H < H_E$, the velocity of the longwave quasielastic transverse oscillations s_t' is very sensitive to the value of the pressure applied in the basal plane of the crystal. With increase in pressure, s_t' increases from zero to s_t:

$$s_t' = s_t\left(\frac{|H_p|}{|H_p| + H_{me}\sin^2\vartheta}\right)^{1/2}. \tag{8'}$$

This is shown in Fig. 1, curves 2a and 2b. The point $H_p = 0$ is the point of a phase transition of the first order in the pressure (vanishing of the anisotropy in the basal plane).[1)]

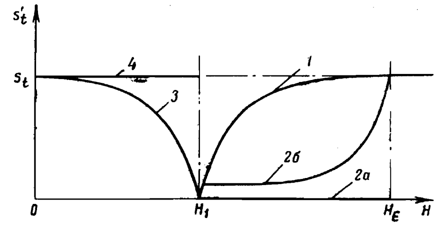

FIG. 1. Dependence of s_t' on H for $H_{A1} > 0$. 1) k ∥ L, polarization of the sound e ∥ z; 2) k ∥ L, e ⊥ z; a) P = 0, b) P ≠ 0; 3) k ∥ L, e ∥ x for $H_p > 0$ or e ∥ y for $H_p < 0$; 4) k ∥ L, e ∥ g for $H_p > 0$ or e ∥ x for $H_p < 0$; k ⊥ L, e ⊥ L.

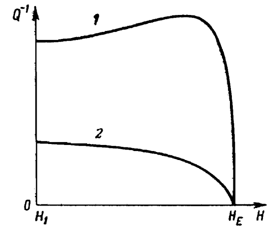

FIG. 2. Dependence of Q^{-1} on H for $H_{A1} > 0$, k ∥ L, e ⊥ z, k. 1) $|H_p| + (\alpha-\alpha')k^2M_0 < 1/2H_{me}$, $Q^{-1}_{max} = \delta_s\omega_{tk}\{3\sqrt{3g}[|H_p| + (\alpha-\alpha')k^2M_0]\}^{-1}$; 2) $|H_p| + (\alpha-\alpha')k^2M_0 > 1/2H_{me}$, $Q_{H=H_1} = \delta_s\omega_{tk}H_{me}\{2g[|H_p| + (\alpha-\alpha')k^2M_0]^{1/2}[|H_p| + (\alpha-\alpha')k^2M_0 + H_{me}]^{3/2}\}^{-1}$.

In the case considered,

$$Q_{II}^{-1} = \frac{\delta_s s_t}{g\sqrt{(\alpha-\alpha')M_0H_{me}}\sin\vartheta} \quad \text{for}$$
$$|H_p| \ll (\alpha-\alpha')k^2M_0 \ll H_{me}\sin^2\vartheta, \tag{9}$$
$$Q_{II}^{-1} = \frac{\delta_s H_{me}\omega_{tk}\sin^2\vartheta}{g|H_p|^{1/2}(|H_p| + H_{me}\sin^2\vartheta)^{3/2}} \quad \text{for}$$
$$(\alpha-\alpha')k^2M_0 \ll H_{me}\sin^2\vartheta \sim |H_p|.$$

Figure 2 shows the dependence of Q_{II}^{-1} on H for this case at various values of the pressure.

At the point of collapse of the magnetic moments of the sublattices $H \approx H_E$, the magnetoelastic gap ω_{2s0} vanishes. As a result the strong interaction of the acoustic and spin waves also disappears.

The interaction of the spin system with the longitudinal sound is maximal for k directed at an angle $\pi/4$ to L and lying in the basal plane (for $H_1 < H < H_E$) or in the plane (L ∥ H) (for $H \approx H_1$). If $P = 0$, then $s_l' = (s_l^2 - s_t^2)^{1/2}$.

The results are, of course, invariant under permutation of the directions of propagation and polarization of the elastic wave.

If the magnetic moments of the sublattices are collinear ($0 < H < H_2$), the dispersion equation for coupled magnetoelastic waves propagating along the z axis ($k = k_z$) has the form

$$(\omega^2-\omega_{tk}^2)^2(\omega^2-\omega_{lk}^2)(\omega^2-\omega_{1sk}^2)(\omega^2-\omega_{2sk}^2) - gH_{me}\omega_{tk}^2(\omega^2-\omega_{tk}^2)(\omega^2-\omega_{lk}^2)(B_1+B_2) \approx 0, \tag{10}$$

where

$$\omega_{1,2sk}^2 \approx {}^1/_2\{\Omega_1^2+\Omega_2^2+2g^2H^2 \pm [(\Omega_1^2-\Omega_2^2)^2 + 8g^2H^2(\Omega_1^2+\Omega_2^2)]^{1/2}\},$$
$$\Omega_1^2 = \{g[(\alpha-\alpha')k^2M_0 + H_{me} + H_{A1} + |H_p|] - i\omega\delta_s\}\omega_E,$$
$$\Omega_2^2 = \{g[(\alpha-\alpha')k^2M_0 + H_{me} + H_{A1}] - i\omega\delta_s\}\omega_E,$$
$$B_{1,2} = \omega_E(\omega^2 - \Omega_{1,2}^2).$$

Under the condition $\omega_{tk} \ll \omega_{js0}$, the frequencies of the coupled magnetoelastic waves are determined by the following expressions:

$$\omega_{Ik}\approx\omega_{1sk},\quad \omega_{IIk}\approx\omega_{2sk},\quad \omega_{IIIk}\approx\omega_{lk},\quad \omega_{IVk}\approx\omega_{tk},$$
$$\omega_{Vk}=\frac{\omega_{tk}}{(\Omega_2^2-g^2H^2)^{1/2}}g\sqrt{(\alpha-\alpha')k^2M_0H_E+H_2^2-H^2}. \tag{11}$$

Transverse sound polarized along the easy (for L) direction in the basal plane (along the **x** axis if $H_p>0$, or along the **y** axis if $H_p<0$) interacts most strongly with the spin system. The corresponding variation of the velocity s_t^l as a function of H is shown schematically by Fig. 1, curve 3.

B. We consider further the case in which $\mathbf{H}\parallel\mathbf{x}$ and the condition (2b) is satisfied. In an antiferromagnet of the easy-plane type ($H_A<0$), the collinear phase is stable in the range of fields $0<H<H_2'$ and the noncollinear phase, in which $\mathbf{L}\parallel\mathbf{y}$, is stable in the field range $H_1'<H<H_E$; $H_1'\approx H_2'\approx\sqrt{H_pH_E}$. The frequencies of magnetoelastic resonance are determined from Eqs. (4)–(11) (for the case $H_p>0$), in which we make the substitutions $\mathbf{x}\to\mathbf{y}$, $\mathbf{y}\to\mathbf{z}$, $\mathbf{z}\to\mathbf{x}$, $\vartheta\to\varphi=\varphi_1=-\varphi_2$, $H_{A1}\to H_p$, $|H_p|\to|H_A|$, $H_1\to H_1'$, $H_2\to H_2'$, (φ_1 and φ_2 are the azimuthal angles of the magnetic moments of the sublattices).

In an antiferromagnet of the easy-axis type (condition (2b) is satisfied in this case for $0<H_A<H_p$), the collinear phase $\mathbf{L}\parallel\mathbf{H}$ is stable in the range of fields $0<H<H_2''$ and the noncollinear phase $\mathbf{L}\parallel\mathbf{z}$ is stable in the range of fields $H_1''<H<H_E$; $H_1''\approx H_2''\approx\sqrt{|H_{A1}|H_E}$. The frequencies of the coupled magnetoelastic waves can be obtained from Eqs. (4)–(11) (for $H_p>0$) with the help of the substitutions $\mathbf{x}\to\mathbf{z}$, $\mathbf{z}\to\mathbf{x}$, $\vartheta\to\vartheta'=(\vartheta_1-\vartheta_2)/2$, $H_{A1}\to|H_{A1}|$, $|H_p|\to|H_A|$, $H_1\to H_1''$, $H_2\to H_2''$.

In the vicinity of the easy-plane-antiferromagnet ⇌ easy-axis-antiferromagnet phase transition, i.e., upon change of the sign of the magnetic anisotropy H_A, and in magnetic fields that exceed the field of reversal of the magnetic moments of the sublattices, H_1', the low-frequency branch of spin waves interacts effectively with transverse sound which propagates along the z(y) axis and is polarized along the **y**(**z**) axis.

3. MAGNETOELASTIC WAVES IN ANTIFERROMAGNETS WITH WEAK FERROMAGNETISM

We now consider the features of magnetoacoustic resonance in an easy-plane antiferromagnet with weak ferromagnetism. The Hamiltonian of such an antiferromagnet can be written in the form (1), where

$$\mathscr{H}_m=\int dv\left\{\alpha\left[\left(\frac{\partial\mathbf{M}_1}{\partial x_i}\right)^2+\left(\frac{\partial\mathbf{M}_2}{\partial x_i}\right)^2\right]+\alpha'\frac{\partial\mathbf{M}_1}{\partial x_i}\frac{\partial\mathbf{M}_2}{\partial x_i}+\delta\mathbf{M}_1\mathbf{M}_2+\frac{1}{2}\beta(M_{1z}^2+M_{2z}^2)+\beta'M_{1z}M_{2z}+d[\mathbf{M}_1\times\mathbf{M}_2]_z-(\mathbf{H},\mathbf{M}_1+\mathbf{M}_2)\right\},$$

$$\mathscr{H}_{me}=\frac{1}{4}\int dv\{\mathbf{L}^2[\gamma_{12}(u_{xx}+u_{yy})+\gamma_{13}u_{zz}]-L_z^2[(\gamma_{12}-\gamma_{31})(u_{xx}+u_{yy})+(\gamma_{13}-\gamma_{33})u_{zz}]+2\gamma_{66}(L_x^2u_{xx}+L_y^2u_{yy}+2L_xL_yu_{xy})+4\gamma_{44}L_z(L_xu_{xz}+L_yu_{yz})\} \tag{12}$$

$$\mathscr{H}_e=\frac{1}{2}\int dv[\rho\dot{\mathbf{u}}^2+c_{11}(u_{xx}^2+u_{yy}^2)+c_{33}u_{zz}^2+2c_{13}(u_{xx}+u_{yy})u_{zz}+2c_{12}u_{xx}u_{yy}+4c_{66}u_{xy}^2+4c_{44}(u_{xz}^2+u_{yz}^2)-2\sigma u_{xx}].$$

Here d is the Dzyaloshinskiĭ constant, γ the magnetostriction constant, and c the elastic modulus, and the remaining designations correspond to those introduced earlier. We shall not take account of the anisotropy in the basal plane of the crystal, and shall assume that $L^2\gg\mathbf{M}\cdot\mathbf{L}$, M^2. We consider the case that was realized experimentally in [1] in measurement of the antiferromagnetic resonance, when the magnetic field is applied in the basal plane of the crystal parallel to the pressure ($\mathbf{H}\parallel\mathbf{x}$). As before, in examining the oscillations of the system, we represent $\mathbf{M}_1$, $\mathbf{M}_2$ and u_{ik} in the form (3). For equilibrium values of the polar angles ϑ_1 and ϑ_2 and the azimuthal angles φ_1 and φ_2 of the magnetic moments of the sublattices, we obtain

1) for $H_p>0$ and $H\leqslant H_c\equiv\sqrt{\frac{H_D^2}{4}+H_pH_E}-\frac{H_D}{2}$

$$\cos\varphi=\frac{HH_D}{H_pH_E-H^2},\quad \Phi\equiv\frac{\pi}{2}-\Delta\Phi,\quad \Delta\Phi=\frac{H_pH_D}{H_pH_E-H^2}, \tag{13}$$

2) for $H_p>0$ and $H\geqslant H_c$, or for $H_p<0$

$$\cos\varphi=1,\quad \Delta\Phi=(H+H_D)/H_E. \tag{14}$$

Here $\varphi=\frac{1}{2}(\varphi_1+\varphi_2)$, $\Phi=\frac{1}{2}(\varphi_1-\varphi_2)$, $H_p=\gamma_{66}PM_0/c_{66}$, and $H_D=dM_0$. In the derivation of these formulas, we have assumed H_D, $|H_A|$, H_p, H all $\ll H_E$. It can be shown that the low-frequency branch of spin waves ω_{1sk} interacts most strongly with sound propagating along the **x** axis. In the approximation $s_1k\ll\omega_{1s0}$, we introduce the corresponding frequencies of the coupled magnetoelastic waves:

I. In the case $H_p>0$ and $H\leq H_c$:

$$\omega_{Ik}\approx\omega_{1sk}=g\left\{(\alpha-\alpha')k^2M_0H_E+H_{me}H_E+\frac{[(H_c+H_D)^2-H^2](H_c^2-H^2)}{H_c(H_c+H_D)-H^2}\right\}^{1/2}$$
$$\omega_{IIk,\,IIIk}\approx\frac{k}{\sqrt{2}}\{s_1^2+s_6^2(1-\zeta)\pm[(s_1^2+(1-\zeta)s_6^2)^2-4s_1^2s_6^2+4\zeta s_6^2(s_6^2\sin^2 2\varphi+s_1^2\cos^2 2\varphi)]^{1/2}\}^{1/2}, \tag{15}$$
$$\omega_{IVk}\approx s_4k,$$

where

$$s_1^2=c_{11}/\rho,\quad s_4^2=c_{44}/\rho,\quad s_6^2=c_{66}/\rho,\quad H_{me}=2\gamma_{66}^2M_0^3/c_{66},\quad \zeta=g^2H_{me}H_E/\omega_{1sk}^2.$$

Analysis of expressions (15) shows that in the vicinity of the phase transition of second order $H\to H_c$, $\varphi\to0$ (the magnetic moment lies along the direction of the magnetic field H), the transverse sound polarized along the **y** axis and the low-frequency branch of the magnon spectrum interact effectively. At the phase-transition point,

$$\omega_{IIk}=s_1k,\quad \omega_{IIIk}=s_6k^2\sqrt{\frac{(\alpha-\alpha')M_0}{H_{me}}}. \tag{16}$$

Thus, the dispersion of the quasielastic wave changes character at the phase-transition point $H=H_c$ and the velocity of this wave vanishes.

The damping is maximal at the phase-transition point, and the ratio $Q_{II}^{-1}=2\omega_{IIk}''/\omega_{IIk}'$ is determined by Eq. (7). As in the case of an antiferromagnet without weak ferromagnetism, estimates give $Q_{II}^{-1}\sim10^{-2}$.

The coupling of longitudinal sound with the low-frequency branch of the spin waves is maximal at $H=(H_D^2/2+H_pH_E)^{1/2}-H_D/\sqrt{2}$ ($\varphi=\pi/4$). Here the frequencies of the quasielastic waves are equal to

$$\omega_{IIk}\approx ks_6,\quad \omega_{IIIk}=k(s_1^2-\zeta_1s_6^2)^{1/2},$$
$$\zeta_1=\frac{H_{me}H_E}{(\alpha-\alpha')k^2M_0H_E+H_{me}H_E+HH_D/\sqrt{2}}. \tag{17}$$

II. For $H_p>0$ and $H\geq H_c$ (or for $H_p<0$ and $H\geq0$), the transverse sound polarized along the **y** axis interacts strongly with the low-frequency branch of the spin waves. The frequencies of the coupled magnetoelastic waves ($\omega_{tk}\ll\omega_{1s0}$) have the following form

$$\omega_{Ik}\approx\omega_{1sk}=g[(\alpha-\alpha')k^2M_0H_E+H_{me}H_E+H(H+H_D)-H_pH_E]^{1/2},$$
$$\omega_{IIk}=s_6k\sqrt{1-\zeta},\quad \omega_{IIIk}\approx s_1k,\quad \omega_{IVk}\approx s_4k. \tag{18}$$

It follows from Eqs. (18) that the maximum magneto-

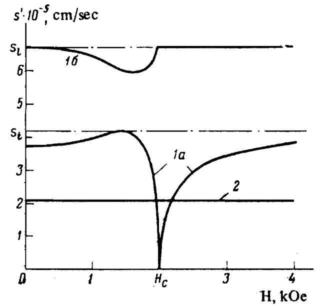

FIG. 3. Dependence of s_t' and s_l' on H in hematite with L ⊥ z. 1) H ∥ x, $H_p > 0$, P ≈ 1 kbar, k ∥ x: a) for s_t' (e ∥ y), b) for s_l'; 2) H ∥ z, k ∥ L, e ∥ y for $H_p > 0$ or e ∥ x for $H_p < 0$, $|H_p| = (1/3)H_{me}$ for s_t'.

elastic coupling for $H_p > 0$ takes place in the vicinity of the phase transition $H = H_c$ (or $H = 0$ for $H_p < 0$) on the approach to which (from the high-field side) $\zeta \to [1-(\alpha - \alpha')k^2M_0/H_{me}]$ and consequently the velocity of the coupled magnetoelastic wave $s_t' \to 0$ as $k \to 0$.

The longitudinal sound is weakly coupled with the magnetic subsystem in this case. The dependence of the velocity of the longwave transverse and longitudinal vibrations on H in hematite is shown in Fig. 3.

We turn our attention to another interesting case: an easy-plane antiferromagnet (with or without weak ferromagnetism) in a field H ∥ z. Here, over the entire range of fields from zero to H_E, a situation will exist that is similar to what occurs in an easy-axis antiferromagnet in the noncollinear phase, i.e., in the range of high fields $H_1 < H < H_E$ (curves 2 on Fig. 1). In this case, strong interaction of transverse sound with k ∥ **L**, polarized in the basal plane of the crystal, with the low-frequency oscillations of the magnons should be observed near the phase transition point in the pressure $H_p = 0$ over a wide range of magnetic fields $0 < H < H_E$ (see (8′), footnote [1)] and curve 2 in Fig. 3).

The presence of strong singularities in the quasi-elastic-oscillation spectrum at first and second order phase-transition points is in correspondence with the general principle according to which the character of the dispersion of at least one of the branches of the oscillations of the system should change in the phase transitions. Inasmuch as the frequency of neither of the branches of magnon oscillations vanishes at the phase-transition points, because of the presence of the spontaneous magnetoelastic gap,[11] the dispersion law of one of the quasielastic waves changes at these points (from linear to quadratic).

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[1)]To account for the crystallographic anisotropy in the basal plane, H_A' in Eqs. (8) and (8′) should be replaced by $|H_p| \to |H_p + H_A'|$. Here the phase transition will take place at $H_p = -H_A'$.

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