Effect of a magnetic field on the fluctuation conductivity of layered superconductors

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The effect of a constant magnetic field on the fluctuation conductivity of pure, layered superconductors with a Josephson interaction between the layers is investigated. The fluctuation correction to the conductivity is calculated with the paramagnetic effect taken into consideration, and it is shown that the latter may lead to an appreciable enhancement of the effect of the superconducting fluctuations. The role of the paramagnetic effect is large if the critical temperature T_c (H) is determined with the aid of excited Landau orbits.

Layered superconductors with a Josephson (weak) interaction between the layers were experimentally investigated in the articles by Morris and Coleman[1] and by Thompson, Gamble, and Koehler.[2] Such superconductors can be obtained upon intercalation by the molecules of layer compounds of the type TaS_2. Layered superconductors have been theoretically investigated in[3-6]. In particular, Bulaevskiĭ[4] has shown that the critical field H_{c2} for superconducting nucleation can be found without taking the motion of the electrons between the layers into consideration, i.e., to regard the motion of the electrons as purely two-dimensional.

In experiment[1] a slow increase of the resistance $\rho_{\parallel}$ toward the normal-state resistance was observed for small angles of inclination of the magnetic field H to the surface of the layer. It was conjectured[4] that this is connected with an increase of the superconducting fluctuations due to the paramagnetic effect, which facilitates the appearance of superconducting regions with dimensions of the order of $v_F/\mu H$ (v_F is the Fermi velocity of the electron, μ is the Bohr magneton). Impurities play an important role in this case. The analysis carried out by Bulaevskiĭ[4] indicates that an intermediate case in regard to purity may have been realized in the experiments of Morris and Coleman.[1]

An investigation of the effect of a magnetic field (without taking the paramagnetic effect into account) on the fluctuation conductivity of "dirty" superconductors was carried out in the series of articles[7-9]. In the present article the fluctuation conductivity of pure, layered superconductors with a Josephson interaction between the layers is calculated for the case when a constant magnetic field is applied to the sample. The influence of the paramagnetic effect on this conductivity is investigated in detail.

The dispersion law of an electron for such compounds in the normal state is determined by the expression[3,5]

$$\varepsilon(\mathbf{p}q)=\mathbf{p}^2/2m+2b\cos q, \tag{1}$$

where $0 \leq q \leq 2\pi$ is the quasimomentum for the motion of the electrons between the layers, **p** is the quasimomentum along the layers, m is the effective mass, and b determines the probability for electron transitions between the layers. This probability is assumed to be small. In what follows in this article, the interaction between the layers is not taken into consideration. One can obtain a concrete estimate for such an approximation by starting from formula (4) (see below) in which allowance for the interaction between the layers leads to the appearance of an additional term $\sim b^2/(\pi T)^2$ (T denotes the temperature). Therefore, the approximation being used is valid for $b \ll T \gtrsim T_c(H)$, where $T_c(H)$ is the critical temperature. If T is not too close to zero, then apparently this criterion can be satisfied for certain layer compounds.

The main contribution to the correction to the conductivity of pure superconductors, connected with the superconducting fluctuations σ', is determined by the Aslamazov-Larkin diagram.[10] In the case of a magnetic field H directed at an angle θ to the surface of the layer, the expression for $\sigma' = \sigma_{\parallel}$ can be written in the form[7]

$$\sigma_{\parallel}=\frac{1}{S}\frac{1}{i\omega_0}\int d^2r\,d^2r'\Delta Q(\mathbf{rr}'\omega_0),\qquad \omega_0\to 0;$$
$$\Delta Q(\mathbf{rr}'i\omega_\nu)=-\left(\frac{e^2}{2m}\right)^2\cdot 4T\sum_{\omega mn}\frac{j_{mn}(\mathbf{r}\omega_\nu\omega)j_{nm}(\mathbf{r}'-\omega_\nu\omega_+)}{N^2E_n(\omega_+)E_m(\omega)}, \tag{2}$$

where $\omega_\nu = 2\pi l T$, $\omega = 2\pi kT$; l and k are integers; S is the surface area; $\omega_+ = \omega + \omega_\nu$; $N = m/2\pi d$ is the effective density of states;[4] d is the thickness of the conducting layer ($\hbar = c = 1$).

The function $E_n(\omega)$ is related to the Cooper instability in the presence of a magnetic field.[11,12] It is the eigenvalue of the integral equation

$$\int K^\omega(\mathbf{rr}')\varphi_m(\mathbf{r}')\,d^2r'=NgE_m(\omega)\varphi_m(\mathbf{r}), \tag{3}$$

where

$$K^\omega(\mathbf{rr}')=\delta(\mathbf{r}-\mathbf{r}')-gT\sum_{\omega}G_{\omega'}(\mathbf{rr}')G_{\omega-\omega'}(\mathbf{rr}');$$

g is the coupling constant for the electron-electron interaction; $G_\omega(\mathbf{rr}')$ is the Green's function of an electron in a magnetic field described by the two-dimensional vector potential $\mathbf{A}(\mathbf{r})$ ($A_x = 0$, $A_y = Hx\sin\theta$); the $\varphi_m(x)$ are Landau wave functions describing the electron's motion in a magnetic field H.

For layered superconductors with a Josephson interaction between the layers, it follows from the last equation that (see[4,11])

$$E_n(\omega)=\ln\frac{T}{T_{c0}}+\lambda^2(-1)^n\int_0^\infty q\,dq\exp\left(-\frac{\lambda^2}{2}q^2\right)L_n(q^2\lambda^2)f_T(q^2), \tag{4}$$

$$f_T(q^2)=4\pi T\sum_{\omega_1>0}\left[\frac{1}{2\omega_1}-\frac{1}{[(qv_F)^2+4(\omega_1+|\omega|)^2]^{1/2}}\right], \tag{5}$$

where the $L_n(x)$ are Laguerre polynomials. Here T_{c0} is the critical temperature for $H = 0$;

$$\lambda^2=1/eH\sin\theta;\quad \omega_1=(2l+1)\pi T.$$

Since the entire singularity near the transition point is contained in the function $E_n(\omega)$ for small arguments, $E_n(\omega)$ can be expanded in powers of the frequency ω:

$$E_n(\omega)=E_n(0)+|\omega|E_n'$$

and in the expression for $j_{mn}(r\omega_\nu\omega)$ one can set $\omega_\nu = \omega = 0$. Then, if the equation for $G_\omega(rr')$ is utilized, we obtain $j_{mn}(r00)$ in the form

$$j_{mn}(\mathbf{r}00)=-\frac{im}{e}\int d^2r_1\,d^2r_2\varphi_m^*(\mathbf{r}_2)\cdot\left[\frac{\delta}{\delta A(\mathbf{r})}\frac{K^{\omega=0}(\mathbf{r}_1\mathbf{r}_2)}{Ng}\right]\varphi_n(\mathbf{r}_1). \tag{6}$$

In contrast to the limiting case of "dirty" superconductors, the kernel $K^{\omega=0}(rr')$ does not satisfy a simple differential equation.[7] Therefore, in order to vary explicitly with respect to the vector potential, we express $K^{\omega=0}(rr')$ in terms of the eigenfunctions $\varphi_m(r)$ and the eigenvalues $E_m(0)$ of Eq. (3):

$$K^{\omega=0}(\mathbf{rr}')=\sum_m NgE_m(0)\varphi_m(\mathbf{r})\varphi_m^*(\mathbf{r}'). \tag{7}$$

As a result, for the quantity

$$\Delta Q(i\omega_\nu)=\frac{1}{S}\int d^2r_1\,d^2r_2\Delta Q(\mathbf{r}_1\mathbf{r}_2 i\omega_\nu)$$

we obtain the expression

$$\Delta Q(i\omega_\nu)=\frac{2e^2T}{\pi d}\sum_{n=0}(E_{n+1}-E_n)^2(n+1)\sum_\omega\frac{1}{E_n(\omega_+)E_{n+1}(\omega)}, \tag{8}$$

where $E_n \equiv E_n(0)$. The summation over ω in this formula is carried out with the aid of analytic continuation,[10] and finally we obtain the following expression for the correction (2) to the conductivity:

$$\sigma_\parallel=\frac{2Te^2}{\pi d}\sum_{n=0}\frac{(n+1)(E_{n+1}-E_n)^2E_n'E_{n+1}'}{E_nE_{n+1}(E_nE'_{n+1}+E_{n+1}E_n')}. \tag{9}$$

Let us consider the limit of large fields, $T \ll T_{c0}$. Then, in the summation over n we can restrict our attention to the term with the maximum singularity. The critical temperature $T_c(H)$ is determined from the equation $E_0(T_c(H)) = 0$. Therefore, in the vicinity of $T_c(H)$ the largest singularity is associated with the function E_0 and $\sigma_\parallel$ is given by

$$\sigma_\parallel=\frac{2e^2T}{\pi d}\frac{E_1'}{E_0}. \tag{10}$$

Making use of the definition of $T_c(H)$, E_0 can be represented in the form

$$E_0=\eta_H+\lambda^2\int_0^\infty q\,dq\exp\left\{-\frac{\lambda^2}{2}q^2\right\}[f_T(q^2)-f_{T_c(H)}(q^2)],$$
$$\eta_H=\ln\frac{T}{T_c(H)}\approx\frac{T-T_c(H)}{T_c(H)}. \tag{11}$$

For temperature close to $T_c(H)$, the integral in this formula is evaluated exactly, and in the strong-field limit under consideration we obtain:

$$E_0=16\eta_H\frac{T_c^2(H)\lambda^2}{v_F^2}=8\eta_H\left(\frac{T_c(H)}{\Delta(0)}\right)^2\frac{H_{c20}}{H\sin\theta}, \tag{12}$$

where $H_{c20} = 2\Delta^2(0)/ev_F^2$ and $\Delta(0)$ denotes the energy gap at $T = 0$.

The condition $\eta_H \ll 1$ is utilized in the derivation of the last formula for E_0. One can evaluate the quantity E_0' by representing the sums over the frequency in terms of derivatives of the Euler digamma function $\psi(x)$ and using the asymptotic expansions of $\psi(x)$ for large arguments. One can however, immediately integrate over the momenta, as a result of which E_0' takes the form

$$E_0'=2\pi T\varepsilon^2\sum_{\omega>0}[1-\omega\varepsilon\pi^{1/2}\exp\{\omega^2\varepsilon^2\}(1-\Phi(\omega\varepsilon))]$$

$$\Phi(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt. \tag{13}$$

Here $\epsilon^2 = 2\lambda^2/v_F^2$ and $\omega = (2l + 1)\pi T$.

In the limit $T^2\epsilon^2 \ll 1$ (large field), we carry out the summation in the last formula according to the Euler-Maclaurin formula:

$$E_0'={}^1/_2\sqrt{\pi}\varepsilon-\pi^{5/2}T^2\varepsilon^3/6. \tag{14}$$

The values of E_n' (for $n > 0$) are obtained from a relation which follows from the properties of Laguerre polynomials:

$$E_{n+1}'=\frac{n}{n+1}E_{n-1}'+\frac{1}{n+1}E_n'-\frac{2\lambda^2}{n+1}\frac{dE_n'}{d\lambda^2}. \tag{15}$$

Thus, we find

$$E_1'=\frac{\pi^{5/2}}{3}\varepsilon\frac{T^2}{\Delta^2(0)}\frac{H_{c20}}{H\sin\theta}$$

and $\sigma_\parallel$ is given by

$$\sigma_\parallel=\frac{\pi^{5/2}e^2}{12d}\frac{T_c(H)}{\Delta(0)}\left(\frac{H_{c20}}{H\sin\theta}\right)^{1/2}\frac{1}{\eta_H}. \tag{16}$$

Now let us take the paramagnetic effect into consideration and show that in this case the fluctuations are more important. Allowance for paramagnetism is achieved by the substitution $\omega \to \tilde{\omega} = \omega - i\mu H$ in formulas (11) and (13) and by taking the real part of the corresponding expressions. As a result, after integrating with respect to q we obtain the following expression for E_0:

$$E_0=\eta_H(1-\chi);$$
$$\chi=\sqrt{\pi}\varepsilon\,\mathrm{Re}\,2\pi T\sum_{\omega>0}^\infty\left[\varphi(\tilde\omega\varepsilon)+2\varepsilon^2\frac{d}{d\varepsilon^2}\varphi(\tilde\omega\varepsilon)\right], \tag{17}$$
$$\varphi(\tilde\omega\varepsilon)=(1-\Phi(\tilde\omega\varepsilon))\exp(\tilde\omega^2\varepsilon^2).$$

The parameter

$$\delta^2=\mu^2H^2\varepsilon^2=\frac{1}{2}\frac{H_{c20}}{H_p\sin\theta}\frac{H}{H_p}=\frac{\gamma\alpha H}{\sqrt{2}H_p}.$$

will play an important role below. Here $\ln\gamma = C$ is Euler's constant and $H_p = \Delta(0)/\sqrt{2}\,\mu$. Let us consider the case when the paramagnetic effect is not very small, so that the inequality $\delta^2 \gg T^2\epsilon^2$ is valid. In addition, $\delta > \delta_{min}$, where δ_{min} is determined by the large field limit considered in this work ($H \lesssim H_{c2}(\theta, T = 0)$, $T \ll T_{c0}$).

The summation over ω in Eq. (17) can be carried out in the same way as in formula (13). Confining our attention to only the integral term, we find:

$$E_0=\eta_H\,\mathrm{Im}\,[\sqrt{\pi}\delta\Phi(-i\delta)e^{-\delta^2}]. \tag{18}$$

One can determine E_0' and E_1' in similar fashion:

$$E_0'=\frac{\sqrt{\pi}}{2}\varepsilon\exp(-\delta^2),\quad E_1'=2\delta^2E_0'. \tag{19}$$

Let us investigate various limiting cases. If $\alpha < 1.25$, then $\delta^2 \ll 1$ is possible for fields $H < H_{c2}(\theta, T = 0)$. The value of $T_c(H)$ is found from the condition[4]

$$E_0(T_c(H))=0,\quad E_0=\eta_H(2\delta^2+8T^2\varepsilon^2)$$

(see Eqs. (12) and (18)); and $\sigma_\parallel$ is determined from formula (10). The fluctuation contribution to the conductivity differs slightly from the result which follows from Eq. (16). At sufficiently small angles θ, α may be greater than 1.25, and $T_c(H)$ is found with the aid of excited Landau orbits.[4] If the n-th orbit is essential, then by keeping the n-th and (n + 1)-st terms of the summation over n in Eq. (9), we obtain the following expression for $\sigma_\parallel^n$:

$$\sigma_{\parallel}^{(n)}=\frac{2e^2}{\pi d}\frac{T}{E_n}(nE'_{n-1}+(n+1)E'_{n+1}). \tag{20}$$

As before let $\delta^2 \ll 1$ and $T_c(H)$, for example, is determined from the equation $E_1(T_c(H)) = 0$. Then $\sigma_{\parallel}^{(1)}$ takes the form

$$|\sigma_{\parallel}^{(1)}|=\frac{e^2}{\sqrt{\pi}d}\frac{T_c(H)}{\Delta(0)}\left(\frac{H_{c20}}{H\sin\theta}\right)^{1/2}\cdot\frac{e^{-\delta^2}}{\delta^2+4T_c^2(H)\varepsilon^2}\frac{1}{\eta_H}. \tag{21}$$

The appearance of the small factor $\delta^2 + 4T_c^2(H)\epsilon^2$ in the denominator substantially increases the fluctuation contribution to the conductivity. For small values of n, the excitation of even orbits gives smaller values of $\sigma_{\parallel}^{(n)}$ ($\sigma_{\parallel}^{(2)} = (1/4)|\sigma_{\parallel}^{(1)}|$). We note, however, that for $H \lesssim H_{c2}(\theta, T = 0)$ we have $\delta^2 \sim 1$[4] even for $n = 1$, and the last formula is not applicable.

In the region of small angles, $\alpha \gg 1$ and $\delta^2 \gg 1$. In this limit we find

$$E_n=\eta_H,\quad E_n'=\frac{1}{n}\left(\frac{\pi}{2}\right)^{1/2}\frac{\lambda}{v_F}(2\delta^2)^n e^{-\delta^2},$$
$$\sigma_{\parallel}^{(n-1)}=\frac{e^2}{2\sqrt{\pi}d}\frac{T_c(H)}{\Delta(0)}\left(\frac{H_{c20}}{H\sin\theta}\right)^{1/2}(2\delta^2)^n e^{-\delta^2}\frac{1}{\eta_H}. \tag{22}$$

Since here $n - 1$ is the order of the integer part of $\alpha/1.8 \gtrsim \delta^2/3.6$,[4] the coefficient associated with $1/\eta_H$ may be large and the fluctuations substantial. The last expression for $\sigma_{\parallel}^{(n-1)}$ retains its form if the temperature is fixed and the field H is varied. It is only necessary to make the following substitutions:

$$\eta_H\to\eta_H'=[H-H_{c2}(\theta,T)]/H_{c2}(\theta,T),$$
$$T_c(H)\to T,\quad \eta_H'\ll 1.$$

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