Possible generation of a magnetic field by Langmuir oscillations

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A new mechanism is established for the generation of a magnetic field in which the energy of Langmuir oscillations is transformed into magnetic-field energy. It is based on the fact that the oscillations due to the presence of the magnetic field (a pseudovector) are gyrotropic (odd). The plasma is assumed to be collisional, and the nonlinear current and its reaction on the field are calculated. An approximate criterion (threshold) is found for the instability, and the rate of increase in the field is estimated. The latter depends on the initial field becuase the generation equations are nonlinear. This effect is discussed for cosmic and laboratory plasmas.

The interaction between hydrodynamic turbulence and the regular component of the magnetic field is now well known. In particular, gyrotropic turbulence (noninvariant under reflection, odd) can act as a field generator.[1] In hydrodynamics, gyrotropy arises as a result of the presence of the Coriolis force; in plasma, gyrotropic properties are exhibited by the magnetic field itself at frequencies in excess of the ion cyclotron frequency. It is therefore natural to seek the reaction of the hf oscillations in a weak magnetic field on the field. In hydrodynamics, the field is generated by helical motions, and either the right- or left-handed helix should predominate. It would appear that helicons can excite the field in this way. It is readily verified, however, that the exact nonlinear equation for the magnetic field, from which the helicons are obtained in the linear approximation, conserves the energy of the field. In this paper, we shall consider the reaction of Langmuir oscillations on the magnetic field.

1. FORMULATION OF THE PROBLEM. ODD LANGMUIR OSCILLATIONS

Suppose the plasma lies in a self-consistent magnetic field, i.e., this field is excited not by external sources but by internal currents. The plasma is collisional, so that the electron component of the current obeys Ohm's law:*

$$\mathbf{j}=\sigma\mathbf{E}-\frac{\sigma}{nec}[\mathbf{jH}].$$

In this expression, σ represents the electron conductivity due to collisions of electrons with ions and neutral particles. We shall neglect the motion of ions, which is possible if, for example, the neutral-particle density is high, or the ions form a crystal lattice (solid). This assumption does not appear to be fundamental (this will be clear from the example discussed in Sec. 2). The dynamic equation then has the form (displacement currents can be neglected):

$$\frac{\partial\mathbf{H}}{\partial t}=-\frac{c^2}{4\pi\sigma}\operatorname{rot}\operatorname{rot}\mathbf{H}-\frac{c}{4\pi ne}\operatorname{rot}[\operatorname{rot}\mathbf{H},\mathbf{H}]. \quad (1)$$

The fact that the displacement currents are neglected means that we are considering quasistationary electromagnetic fields. This, in turn, means that the rate of decay of the field must be small in comparison with the collision frequency. We shall consider the reaction of low-frequency oscillations on this quasistationary magnetic field. We note that, in contrast to the present work, the author of[2] considers the effect of Langmuir oscillations on low-frequency electromagnetic waves in which $H \gg E$.

Suppose that Langmuir oscillations are excited in the plasma, so that we must assume that $\omega_p > \nu$, where ω_p is the plasma frequency and ν is the collision frequency between electrons and other particles. If we neglect the plasma pressure, the equations for the electron oscillations in the Fourier representation take the form (the wavelength is assumed to be negligible in comparison with the size of inhomogeneities in **H**):

$$-i\omega\mathbf{v}=\frac{e}{m}i\varphi\mathbf{k}+[\mathbf{v}\boldsymbol{\omega}_e],\qquad -k^2\varphi=4\pi ne, \quad (2)$$

$$\omega n=n_0(\mathbf{kv}),\quad \boldsymbol{\omega}_e=e\mathbf{H}/mc.$$

We shall suppose henceforth that $\omega_e < \omega_p$, and this will be the sense in which the magnetic field will be regarded as weak. It is readily verified that

$$\mathbf{v}=\left\{\omega\mathbf{k}-\frac{(\boldsymbol{\omega}_e\mathbf{k})\boldsymbol{\omega}_e}{\omega}+i[\mathbf{k}\boldsymbol{\omega}_e]\right\}\psi, \quad (3)$$

$$\psi=-\frac{e}{m}\frac{\varphi}{\omega^2-\omega_e^2}\approx-\frac{e}{m}\frac{\varphi}{\omega_p^2-(\mathbf{k}\boldsymbol{\omega}_e)^2/k^2}.$$

When collisions are taken into account in (2), the result is that ω in (3) must be replaced by $\omega-\nu\mathrm{I}$, which is unimportant for our ensuing analysis since $\omega \gg \nu$. When phase correlation is lost, i.e., in the case of a random process, we can introduce the spectral tensor

$$\langle v_\alpha(\mathbf{k},\omega)v_\beta^*(\mathbf{k}',\omega')\rangle=\delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega')\Phi(\mathbf{k},\omega)\left\{\omega^2k_\alpha k_\beta-\omega_jk_j[\omega_\alpha k_\beta+\omega_\beta k_\alpha]+\frac{(\omega_jk_j)^2}{\omega^2}\omega_\alpha\omega_\beta+\omega\omega_fk_ji[\varepsilon_{\alpha jf}k_\beta-\varepsilon_{\beta jf}k_\alpha]+\frac{\omega_ak_a}{\omega}ik_j\omega_f[\varepsilon_{\beta jf}\omega_\alpha-\varepsilon_{\alpha jf}\omega_\beta]+\varepsilon_{\alpha jf}\varepsilon_{\beta cd}k_jk_c\omega_f\omega_d\right\}. \quad (4)$$

The angular brackets in this expression represent averaging over a wave ensemble. The quantity ω_α in (4) represents the components of the vector $\boldsymbol{\omega}_e$, and $\Phi(\mathbf{k},\omega)=\Phi(-\mathbf{k},-\omega)$, $\Phi(\mathbf{k},\omega)>0$.

The gyrotropy of the oscillations is measured by $\langle\mathbf{v}\cdot\operatorname{curl}\mathbf{v}\rangle$: if we have invariance under reflection, then $\langle\mathbf{v}\cdot\operatorname{curl}\mathbf{v}\rangle=0$. However, it is readily seen that for (4)

$$\langle\mathbf{v}\operatorname{rot}\mathbf{v}\rangle=2\int[k^2\omega_e^2-(\mathbf{k}\boldsymbol{\omega}_e)^2]\frac{(\mathbf{k}\boldsymbol{\omega}_e)}{\omega}\Phi(\mathbf{k},\omega)\,d\omega d\mathbf{k}. \quad (5)$$

Of course, in the isotropic case, when $\Phi(\mathbf{k},\omega)=\Phi(|\mathbf{k}|,\omega)$, the expression given by (5) will be equal to zero. If, on the other hand, the phase velocity of the waves has a special direction, then $\langle\mathbf{v}\cdot\operatorname{curl}\mathbf{v}\rangle\neq 0$. Suppose, for example, that the phase velocity of all the waves lies along $\mathbf{k}_0$, in which case

$$\Phi(\mathbf{k},\omega)=\frac{\Phi_1}{2}[\delta(\mathbf{k}+\mathbf{k}_0)\delta(\omega+\omega_p)+\delta(\mathbf{k}-\mathbf{k}_0)\delta(\omega-\omega_p)] \quad (6)$$

(in the dispersion relation for ω we have neglected corrections due to the presence of **H**) and

$$\langle\mathbf{v}\operatorname{rot}\mathbf{v}\rangle=2\Phi_1[k_0^2\omega_e^2-(\mathbf{k}_0\boldsymbol{\omega}_e)^2](\mathbf{k}_0\boldsymbol{\omega}_e)/\omega_p. \quad (7)$$

We note that $\langle \mathbf{v}\cdot \mathrm{curl}\,\mathbf{v}\rangle$ is a pseudoscalar, since $\mathbf{k}_0$ is a vector and $\boldsymbol{\omega}_e$ a pseudovector, as indeed should be the case. We shall see below that it is precisely pseudoscalars such as those given by (5) and (7) that are important for field generation. Finally, it is clear from (3) that, strictly speaking, the oscillations are nonpotential. It is important to know when the nonpotential component of the electric field E_b, which unavoidably appears, can be neglected. We shall suppose that $ck \gg \omega_p$, in which case we have the approximate result

$$E_b \approx \omega_p^2 \omega_e |\mathbf{k}\varphi| / c^2 k^2 \omega_p.$$

The quantity ω_e/ω_p is regarded as small but, nevertheless, we do take into account second-order terms in this parameter in (3). The expression for E_b contains the additional small parameter ω_p/ck. This is why E_b need not be taken into account, whilst the nonpotential component of the electron velocity is included. Thus, the wave vector k lies within the range $\omega_p/c \ll k \ll \omega_p/v_T$, where v_T is the thermal velocity of electrons and the upper limit is associated with Landau damping.

2. NONLINEAR GENERATION

We must now calculate the average current due to the presence of Langmuir oscillations. We shall use the averaged form of the Maxwell equation:

$$\mathrm{rot}\,\mathbf{H} = \frac{4\pi}{c}\left(\sigma\mathbf{E} - \frac{\sigma}{4\pi n e}[\mathrm{rot}\,\mathbf{H}, \mathbf{H}] + \langle n_1 e \mathbf{v}_1\rangle + n_0 e \langle \mathbf{v}_2\rangle\right) + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \quad (8)$$
$$\mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}$$

(We recall that $e < 0$). The expression in parentheses on the right-hand side of (8) represents the resultant current; the first two terms are the usual ohmic current (including the Hall current). Next, it is clear that averaging over a period greater than the time of the oscillations will ensure that the linear part of the microcurrents will not contribute to the oscillations themselves. The nonlinear current is therefore introduced into (8). If we now write the density and velocity in a form which includes the quadratic correction,

$$n = n_0 + n_1 + n_2, \quad \mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2,$$

it will be clear that, after averaging over the oscillations, i.e., over a time interval greater than the reciprocal of the frequency, the terms $\langle n_1 e \mathbf{v}_1\rangle$ and $\langle n_0 e \mathbf{v}_2\rangle$ will remain in the quadratic approximation, and are included in (8). The displacement current can again be neglected, and when $\langle n_1 e \mathbf{v}_1\rangle + n_0 e\langle \mathbf{v}_2\rangle = 0$, we come back to (1). The quantity $\langle \mathbf{v}_2\rangle$ represents the nonlinear effect of the oscillations on the slow motion of the medium (this is the analog of the Miller force). Let us begin by evaluating the quantity $\langle n_1 e \mathbf{v}_1\rangle$. To do this, we shall express n_1 and $\mathbf{v}_1$ in terms of φ, using (2) and (3). The greatest contribution will be given by the first term in (3). However, the resulting vector is of no interest because, owing to the assumed homogeneity $[\delta(\mathbf{k} - \mathbf{k}')$ in (4)], this vector will be independent of the coordinates. Such a nonlinear current will lead to the appearance of a surface charge on the boundary of the plasma, and the resulting electric field will give rise to the reverse current which will compensate the former current. Allowance for the inhomogeneity in the fluctuations is evidently of no interest because these inhomogeneities are transported with the group velocity (which appears when the thermal spread is taken into account). In the presence of a stationary source, for example, two-stream instability, the fluctuations should be more or less homogeneous. The third term on the right-hand side of (3) is equated to zero because it is an integral of an odd function of $\mathbf{k}$. The second term, even though it is odd in $\mathbf{k}$, does not contain ω either and, therefore, it provides the contribution

$$\langle n_1 e \mathbf{v}_1\rangle = -n_0 e \boldsymbol{\omega}_e \int \frac{(\boldsymbol{\omega}_e \mathbf{k})}{\omega} k^2 \Phi(\mathbf{k}, \omega)\, d\omega\, d\mathbf{k}. \quad (9)$$

The significance of the function $\Phi(\mathbf{k}, \omega)$ is clear from (4). The expression given by (9) resembles (5), as expected. The essential point is that, here again, we see the importance of the existence of a special direction for the phase velocity. This situation is, in fact, realized during the excitation of Langmuir oscillations: the phase velocity is largely parallel to the beam. To estimate the quantity given by (9), we shall use (6) for the sake of simplicity, and will assume that the energy of the Langmuir oscillations is localized near $k \sim k_d$ ($k_d = 2\pi\omega_p/v_T$). We then have

$$\langle n_1 e \mathbf{v}_1\rangle = -(\boldsymbol{\varkappa}\mathbf{H})\mathbf{H}\frac{e\langle(\nabla\varphi)^2\rangle}{8\pi v_T c^2 n_0 m^2} = -(\boldsymbol{\varkappa}\mathbf{H})\mathbf{H}\frac{e v_T \beta}{2c^2 m}, \quad (10)$$

where $\langle(\nabla\varphi)^2\rangle/8\pi$ is the mean square energy density of the oscillations, κ is the unit vector in the direction of the beam, and $\beta = \langle(\nabla\varphi)^2\rangle/4\pi n_0 m v_T^2$.

We must now evaluate $\langle \mathbf{v}_2\rangle$. The problem reduces to the evaluation of the correction to Ohm's law due to the nonlinear force $\mathbf{F} = \langle(\mathbf{v}\cdot\nabla)\mathbf{v}\rangle$.

Let us now consider the equation for the second-order correction

$$\frac{\partial \mathbf{v}_2}{\partial t} + (\mathbf{v}_1\nabla)\mathbf{v}_1 = \frac{e}{m}\nabla\varphi_2 + [\mathbf{v}_2\boldsymbol{\omega}_e] - \nu\mathbf{v}_2 + \frac{e}{m}\mathbf{E}.$$

In this expression $\mathbf{E}$ is the induced regular magnetic field (we have neglected thermal corrections; they did not contribute to the induced component $\mathbf{v}_2$). It is clear that $\partial\mathbf{v}_2/\partial t$ will vanish in the course of averaging, because of the quasistationary nature of Ohm's law; the quantity $\langle\nabla\varphi_2\rangle$ will not provide a contribution either. The quantity $(\mathbf{v}\cdot\nabla)\mathbf{v}$ is usually assumed to be small in the derivation of this law but, in the present case, this is not obvious because this expression includes the derivative with respect to the pulsation velocity, and the pulsation scales are small (i.e., the derivative is large). We shall use (4) and (6) to calculate $\mathbf{F}$:

$$\mathbf{F} = -\frac{\beta v_T^2[\mathbf{k}_0\boldsymbol{\omega}_e]}{\omega_p}\left(1 - \frac{(\boldsymbol{\omega}_e\mathbf{k}_0)^2}{\omega_p^2 k_0^2}\right).$$

Next, using the equation for the second-order correction, we have $\langle\mathbf{v}_2\rangle = -\mathbf{F}/\nu$.

Taking the curl of the first equation in (8), we obtain the following equation for $\mathbf{H}$ instead of (1):

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{c^2}{4\pi\sigma}\mathrm{rot}\,\mathrm{rot}\,\mathbf{H} - \frac{c}{4\pi n e}\mathrm{rot}[\mathrm{rot}\,\mathbf{H}, \mathbf{H}] - \mathrm{rot}\,\alpha\mathbf{H} + \mathrm{rot}\,a[\boldsymbol{\varkappa}\mathbf{H}], \quad (11)$$
$$\alpha = \frac{(\boldsymbol{\varkappa}\boldsymbol{\omega}_e)}{2\sigma}v_T\beta, \quad a = 2\pi\beta v_T\left(1 - \frac{(\boldsymbol{\omega}_e\mathbf{k})^2}{\omega_p^2 k^2}\right).$$

It is clear from general ideas that this form of (11) is possible if α is a pseudoscalar; it is precisely for this reason that the property of gyrotropy was demanded at the beginning of this paper. Equation (11) is, in fact, the generation equation and is similar to the equation for $\mathbf{H}$ in the case of odd hydrodynamic turbulence (see [3]). The difference between them is that α itself is a function of $\mathbf{H}$ and, therefore, the equation is nonlinear and the growth in the field is not exponential (as in the usual theory of the turbulent dynamo). The other difference is that (11) contains additionally the second and fourth terms. However, these terms do not complicate the situation because they do not introduce anything basically new: the second term (Hall current) conserves the energy, whilst the fourth describes simple field transport in the direction of $\boldsymbol{\kappa}$ if we neglect ω_e^2 in comparison with ω_p^2. It is important to note that the first term on the right of (3) will also contribute to the expression for $\langle n_1 e\mathbf{v}_1\rangle$ when the correction for ω due to $\mathbf{H}$ is taken into account. The nonlinear current will then no longer depend on the coordinate (because $\mathbf{H}$ is not uniform). It is possible, however, to show that when this contribution is taken into account the general form of (11) will not change. Since the exact value of the coefficients will be unimportant in the ensuing analysis (we shall be interested only in the general character of the solution), we shall ignore this correction.

We shall now show the existence of growing solutions of (11). We note that the linear theory of stability is of no interest in this context because we shall be concerned with the particular growth of the field for which its final state will not be very different from the initial state.

The correct formulation of the problem of field generation is characterized by the following features: 1) the field must be zero at infinity, since vacuum or superconducting boundary conditions must be introduced for a bounded system (physically, this means that the field is not supported by external forces), and 2) we must consider the asymptotic behavior of the field for $t \to \infty$, since there are trivial mechanisms which may lead to a temporary amplification of the field.

We shall seek solutions of this kind. It will be convenient to introduce cylindrical coordinates, and we shall assume that $\partial/\partial z = 0$ and $\partial/\partial\varphi = 0$, in which case $H_r = 0$. Suppose that $\boldsymbol{\kappa}$ lies along the z axis. Equation (11) will then assume the form

$$\frac{\partial H_\varphi}{\partial t}=\frac{\partial \gamma H_z^2}{\partial r}+\nu_m\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}rH_\varphi, \qquad (12)$$
$$\frac{\partial H_z}{\partial t}=-\frac{1}{r}\frac{\partial}{\partial r}\gamma rH_\varphi H_z+\nu_m\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial H_z}{\partial r},$$

where $\nu_m = c^2/4\pi\sigma$, $\gamma = \alpha/H_z$, and γ is independent of the magnetic field.

It is interesting to note the advantages exhibited by this example. The "uninteresting" second and fourth terms are automatically equated to zero. All that remains is the first term which describes the usual ohmic damping of the field and, as expected, the third term, which competes with it and gives rise to the dynamo. Moreover, the fact that curl(curl $\mathbf{H} \times \mathbf{H}$) is zero means that the electromagnetic force acting on the ions is potential and can be compensated by ion pressure. This justifies the fact that we have neglected the motion of ions in this example.

Next, for the sake of mathematical simplification, we shall consider the following artificial situation. We shall be interested only in the fundamental aspects of the problem, i.e., whether the equation given by (11) will, in general, result in generation. Suppose that γ is not zero and is constant in the region $r_0 \le r \le r_1$, $r_1 - r_0 \ll r$, i.e., the oscillations are excited only for these values of r. In that case, $1/r \ll \partial/\partial r$. When $r = R > r_1$, we must introduce boundary conditions of the form described above. We shall refer to this region as the generation region. It is clear that the field may grow when the second terms on the right-hand side of (12), which describe the attenuation of the field, can be neglected, i.e., when γ is sufficiently large. Suppose that this is indeed the case. The solution in the generation region can then be sought in the form $H_\varphi = f_\varphi(r)/(1-bt)$, $H_z = f_z(r)/(1-bt)$ and we shall consider the solution for $t \ge 0$ where $t < 1/b$, since an infinite value is of no physical significance. Equation (11) will, in fact, cease to be valid well before the condition $\omega_e \ll \omega_p$ is violated. From the resulting system

$$bf_\varphi=\gamma(f_z^2)', \quad bf_z=\gamma(rf_\varphi f_z)'/r$$

It is clear that if f_z^0, f_φ^0 is its solution with $b < 0$, then f_z^0, $-f_\varphi^0$ will give the solution of the system with $b > 0$, i.e., a growing solution for $t \ge 0$.

Thus, we need only show the existence of the solution. Neglecting $1/r$ in comparison with $\partial/\partial r$, we can solve our set of equations in terms of quadratures. Substituting for f_φ from the first equation into the second, and reducing the order of the resulting second-order equation, we obtain

$$f_z'=(C-f_z^4b^2/4\gamma^2)^{1/2}/f_z^2$$

where C is the constant of integration. The solution of the last equation can be expressed in terms of quadratures. We can satisfy all the assumptions made above if, in addition, we suppose that $C \gg f_z^4 b^2/4\gamma^2$. In that case,

$$f_z=3^{1/3}C^{1/6}(r-r_0+\delta)^{1/3}, \quad f_\varphi=-3^{-1/3}C^{1/3}2\gamma(r-r_0+\delta)^{-1/3},$$

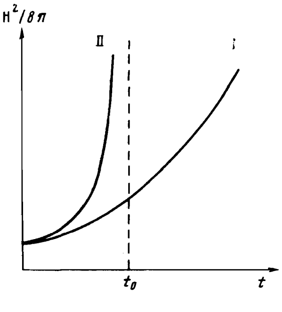
where δ is a new constant (let us suppose that $0 < \delta \ll r_0$).

In fact, the above solution has no singularities, and the assumption that $1/r \ll \partial/\partial r$ is satisfied under the adopted restrictions on δ and $r - r_0$. The requirement that $C \gg f_z^4 b^2/4\gamma^2$ is a restriction on C and, finally, the fact that we have neglected dissipation amounts to a restriction on ν_m. Only the dissipative terms will remain outside the generation region. The dynamo will, as a whole, be described by the diffusion equation with a source having an increasing strength in the generation region. The field outside this region will therefore decay in space over the skin-layer depth.

Thus, a growing solution characterized by the above two generation conditions does, in fact, exist.

3. DISCUSSION

The nonlinearity of the generation equation means that the growth of the field acquires new features as compared with the usual theory of the dynamo. Thus, firstly, the field becomes infinite in a finite time t_0. In the figure, curve I shows the growth in the magnetic energy in the usual theory with the growth rate $1/t_0$, whereas curve II shows the growth in the present problem. Thus, if the initial field is weak, then, in the usual theory, saturation will set in in a time of the order of a



few t_0. On the other hand, in the present problem, if we know that the field does increase, we may conclude that it will reach a predetermined value in the time t_0. Secondly, the rate of growth itself depends on the initial value of the field. This quantity will be estimated below.

2. Consider the saturation field. From a formal standpoint, violation of the condition $\omega_e < \omega_p$ will modify much of the foregoing analysis. Thus, the approximate equation in (3) will cease to be valid, and $\psi \approx e\varphi/m\omega_e^2$ for $\omega_e > \omega_p$. Moreover, the quantity ω_e^2 in the expression for α will then be present in the denominator, so that this important α-term will no longer depend on the field energy but only on the ratio H/H. It is clear that the growth of the field will terminate. The estimated steady field obtained by equating the α-term to the dissipative term yields a field such that $\omega_p \ll \omega_e$. However, it is hardly likely that a field of this order can be realized since, in that case, the assumed uniformity of the pulsations becomes physically incorrect. In point of fact, the stationary equation for the number n_k of plasmons in the "τ approximation" has the form

$$-\frac{\partial\omega}{\partial r_i}\frac{\partial n_k}{\partial k_i}+\frac{\partial\omega}{\partial k_i}\frac{\partial n_k}{\partial r_i}=-\frac{n_k-n_k^0}{\tau}, \qquad (13)$$

where τ is the relaxation time and n_k^0 is the stationary number of plasmons in the absence of the inhomogeneity; $\tau \approx \omega_p^{-1}$ when $k \approx k_d$ and two-stream instability is present. So long as $\omega_e < \omega_p$, the left-hand side of (13) is small in comparison with n_k/τ. Consequently, $n_k \approx n_k^0$, i.e., the pulsations are homogeneous. If, on the other hand, $\omega_e \gg \omega_p$, the conclusion is no longer valid. Therefore, the saturation field is determined by the quantity $\omega_e \lesssim \omega_p$.

3. If we take the situation opposite to that illustrated in the above example, the electromagnetic force can no longer be balanced by the pressure, the ions can no longer be regarded as fixed, and this leads to the appearance of the term $\mathrm{curl}(\mathbf{v}\times\mathbf{H})$ in (11), where $\mathbf{v}$ is the velocity of the ions (as in the usual induction equation). At the same time, the magnetic field energy will be transformed into the kinetic energy of the ions, so that the energy density of the latter will be $\approx H^2/8\pi$. The situation when a large fraction of the magnetic field energy is irreversibly transformed into kinetic energy, and thus the dynamo process is violated, is very artificial.

4. Finally, let us estimate the characteristic magnitudes of the rate of increase in the field and of the instability threshold. The condition that generation will predominate over diffusion reduces to $\alpha > \nu_m/L$, where L is the characteristic field scale [this follows from (11)], i.e.,

$$2\pi L\omega_e v_T\beta > c^2. \qquad (14)$$

When $\beta = 0.1$ and $v_T = 10^9$ cm/sec (as in thermonuclear plasma), it is necessary that $HL > 1.5\times10^5$ (L is in centimeters and H in gauss). Thus, when H = 1500 gauss, the size of the system should not be less than 1 m. Under astrophysical conditions, when $v_T = 10^8$ cm/sec and $\beta = 0.01$, we have $HL > 1.5\times10^7$. When H = 150 gauss, we have $L > 10^5$ cm. In these examples, H is interpreted as the initial field.

We now estimate the characteristic time t_0 when (14) is satisfied. From (11), it is clear that $t_0 = L/\alpha$, i.e.,

$$t_0 = 2\sigma L/\omega_e v_T\beta. \qquad (15)$$

For the first example $\sigma = 10^{16}-10^{17}$ sec^{-1} and $t_0 = 1.3-13$ sec, and for the second example $\sigma = 10^{13}$ sec^{-1} and $t_0 = 1.3\times10^3$ sec. In both cases, the rate of growth is very high.

It is clear from (15) that the situation improves as the electrical conductivity decreases, so that with the classical value of σ, the estimate given by (15) must be regarded as the worst; when collisions of electrons with waves are more frequent than with other particles, and σ decreases, i.e., assumes the turbulent value, the estimate given by (15) will improve. The instability threshold given by (14) is independent of σ.

Finally, we note the following point. In the usual theory of the dynamo, the growing solution is equivalent to linear instability, i.e., from the existence of the solution we necessarily conclude its realization in nature, since the necessary initial fluctuations are always present (at least, thermodynamic fluctuations). Here, on the other hand, we have an instability threshold (14) and, in each case, we must check whether the necessary initial fields of sufficient strength are present.

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*[jH] ≡ j × H.

[1] M. Shteenbek and F. Krauze, Magnitnaya gidrodinamika **3**, 19 (1967).
[2] V. N. Tsytovich, Dokl. Akad. Nauk SSSR **181**, 60 (1968) [Sov. Phys.-Dokl. **13**, 672 (1969)].
[3] S. I. Vaĭnshteĭn and Ya. B. Zel'dovich, Usp. Fiz. Nauk **106**, 431 (1972) [Sov. Phys.-Uspekhi **15**, 159 (1972)].

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