

# Investigation of the surface conductivity of tin in a magnetic field by a high-frequency technique

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The dependence of the real  $R$  of the surface impedance on the magnetic field strength  $H$  (0-70 kOe), frequency (3.6-1400 kHz), and temperature (1.8-4.2 °K) has been investigated for tin single crystals. It is found that for a strong magnetic field  $H$  perpendicular to the sample surface, the behavior of  $R$  can be described by the normal skin-effect theory. The  $R(H, T)$  behavior is qualitatively different for a field parallel to the sample surface. This difference, which depends exclusively on the field orientation with respect to the sample surface, can be explained by the static skin-effect theory. In a parallel field, the scattering of the electrons by the sample surface results in an increase in the static conductivity in a thin surface layer with a thickness of the order of the Larmor radius. An analysis of the experimental results indicates that the surface scattering is mainly diffuse. However, it is necessary to assume that a small fraction (0.01) is due to specular reflection, which yields a significant contribution to the surface layer conductivity at the lowest temperatures.

## 1. INTRODUCTION

It has been shown in a number of theoretical papers that the interaction of conduction electrons with the surface in metals with equal numbers of electrons and holes in a strong magnetic field leads to the appearance of a current density that is inhomogeneous over the depth of the sample—the "static skin effect."<sup>[1,2]</sup> In a magnetic field that is transverse to the current, under the condition that the Larmor radius  $r$  of the electron orbit is much less than the free path length  $l$ , the electrical conductivity of a surface layer of thickness  $r$  turns out to be of the order of  $\sigma_T(0)r/l$  for the case of diffuse scattering of the electrons from the surface of the metal, and  $\sigma_T(0)$  for specular reflection. At the same time, the specific electrical conductivity in the bulk of the metal is  $\sigma_T(0)(r/l)^2$  ( $\sigma_T(0)$  is the specific electrical conductivity at a temperature  $T$  and magnetic field  $H = 0$ ).

Studies of the static skin effect can be carried out both with direct current and by high-frequency (hf) methods. In both cases, the conductivity of the near-surface layer must be separated from the total conductivity of the sample. At constant current, the contribution of the surface conductivity changes with the sample thickness  $d$ . In hf methods, a similar role is played by the depth of the skin layer  $\delta < d$ .

The surface impedance of a half-space, with account of the interaction of electrons with the boundary, can be expressed in terms of the impedance of the "unbounded" metal  $Z_\infty$ , in which the surface current is not taken into account, and the surface conductivity  $S$ :<sup>[3]</sup>

$$Z^{-1} = Z_\infty^{-1} + S. \quad (1)$$

That the surface and volume conductivities are in parallel is explained by the fact that in a half-space filled with the metal, the surface current does not change the distribution of the electric field over depth if the condition  $\delta/r \gg 1$  is satisfied. It was also established in<sup>[3]</sup> that the real part of the impedance  $Z = R + iX$  does not depend on the frequency over a wide range of frequencies corresponding to the condition  $r \ll \delta \ll l$ , and is proportional to the square of the magnetic field.

In the experimental papers<sup>[4,5]</sup>, the surface impedance of single-crystal samples of tin was investigated; it was shown that the theory of the static skin effect can satisfactorily describe the results obtained. In particu-

lar, a significant difference was discovered in<sup>[6]</sup> in the imaginary parts of the impedance in cases in which the field  $H$  is parallel to the surface of the sample and when it is normal to it; this observation has been explained by the different values of the surface conductivity, which is comparable with the hf conductivity of the metal in the parallel case, and is insignificant in the normal case.

The purpose of the present investigation was to make a more detailed study of hf conductivity in a strong magnetic field. As the object of investigation, we chose tin, for which a strong quadratic increase in the transverse magnetoresistance with field corresponds to most directions of the magnetic field relative to the crystallographic axes. This produces favorable conditions for observation of the contribution of the surface conductivity to the total conductivity of the sample.

## 2. SAMPLES AND METHOD OF EXPERIMENT

Single-crystal tin samples were grown in a dismantlable quartz mold with optically polished surfaces, according to the method of Sharvin and Gantmakher,<sup>[6]</sup> and took the form of disks of diameter 18 mm and thickness 1 mm. The initial tin had a resistance ratio  $\rho(293^\circ\text{K})/\rho(4.2^\circ\text{K}) \approx 10^5$ , which corresponds to  $l(4.2^\circ\text{K}) \approx 0.07$  cm. Experimental results are reported for the direction of the magnetic field along the [100] axis of the crystal in a hf current in the (100) plane in a direction close to the [010] axis (see Table). Such conditions correspond to maximal growth of resistance with field. However, study of cases of fields normal and parallel to the surface required the use of two samples.

The measurements consisted of study of the dependence of the real part  $R$  of the surface impedance (the losses) on the magnetic field  $H$  and the temperature  $T$  in the range of frequencies 3.6-1400 kHz. For this purpose we used a simple resonance bridge.<sup>[7]</sup> The source of sinusoidal voltage was a standard-signal generator, to the output of which was connected a broadband amplifier

Sample	Orientation of the normal to the surface $n$	Direction of the vector $E$	Direction of the vector $H$
№ 1	[100]	in plane (100), $\sphericalangle(E, 010) = 27^\circ$	[100]
№ 2	in plane (100), $\sphericalangle(n, 001) = 27^\circ$	in plane (100), $\sphericalangle(E, 010) = 27^\circ$	[100]
Error	1°	2°	1°

with automatic gain control, which was accomplished by negative dc feedback. Thanks to the use of the automatic gain control, amplitude stability of the amplifier output voltage was maintained with accuracy no worse than 0.05% during the course of the experiment. The amplitude-stabilized voltage was fed to the input of the bridge, which represents a parallel tank circuit tuned to resonance, with the sample placed in the induction coil. The detected bridge voltage was applied to the input of a differential dc amplifier. A voltage proportional to the amplitude of the input signal of the bridge was maintained at the second input of the amplifier. The drift noise reduced to the input of the direct-current amplifier, amounted to  $\sim 5 \mu\text{V}$ , which was at the intrinsic noise level of the vibropack input stage of the amplifier.

The induction coil was wound with brass wire, which made it possible practically to eliminate the dependence of its resistance on the magnetic field and the temperature. For comparison of the losses  $R$  in the samples at different frequencies, the bridge was sensitivity calibrated against the superconducting transition in lead samples of the same geometry as the samples studied. The resistance ratio amounted to  $\rho(293^\circ\text{K})/\rho(4.2^\circ\text{K}) \approx 50$ ; therefore the skin effect in the lead samples could be regarded as normal. The latter was further confirmed by measurement of the penetration depth of the electromagnetic field into the metal. The loss-comparison error did not exceed 15% in the frequency range 120–1400 kHz. At a fixed frequency, the error in the reproducibility of the results amounted to 2%. The losses in the samples were measured in relative units. The accuracy of duplication of the sample orientations in the magnetic field was no worse than  $1/4$  degree.

The measurements were carried out in the tempera-

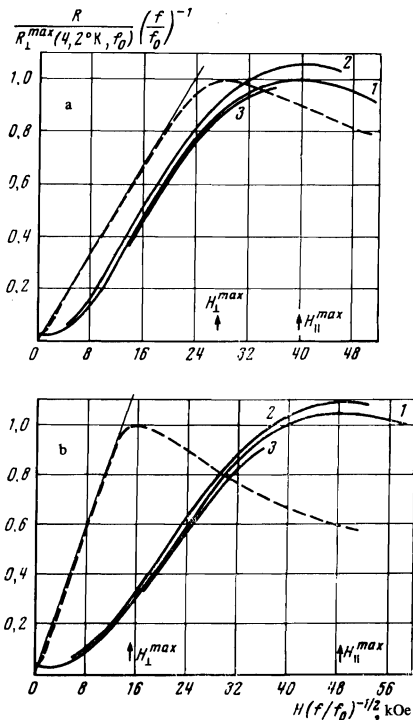


FIG. 1. Dependence of the real part of the surface impedance of tin on the magnetic field: a—at  $T = 4.2^\circ\text{K}$ ; b—at  $T = 2.1^\circ\text{K}$ . The dashed curves correspond to the experimental conditions  $H \parallel [100] \parallel n$  at a frequency  $f = 200$  kHz. The solid curves 1, 2, 3 are for  $H \perp [100] \perp n$  at the frequencies  $f = 200, 300, 600$  kHz, respectively. The frequency  $f_0 = 200$  kHz.

ture range 1.8–4.2° K. Magnetic fields up to 70 kOe were produced by a superconducting solenoid.

### 3. RESULTS

1. Figure 1 shows records of the dependence of the real part  $R$  of the surface impedance on the value of the magnetic field  $H$  at temperatures  $4.2^\circ\text{K}$  and  $1.8^\circ\text{K}$  and an electromagnetic-field frequency  $f = 200$  kHz. (An explanation of the choice of coordinates for these plots is given in par. 3). The losses in the samples in the superconducting state were taken as zero. Similar records were obtained at other frequencies in the interval 120–1400 kHz. At lower frequencies, it was difficult to obtain satisfactory accuracy of the measurements; at the high-frequency end, the interval was limited by the maximum value of the magnetic field achieved in the experiment.

Since the path lengths in the samples differed insignificantly (see below), we shall henceforth speak of losses in magnetic fields parallel and normal to the surface of the sample,  $R_{\parallel}$  and  $R_{\perp}$ , respectively.

2. Let us consider the behavior of the losses  $R_{\perp}$ . The value of  $R_{\perp}$  increases monotonically with the field, reaching a maximum value at  $H = H_{\perp}^{\text{max}}$ , and thereafter decreases monotonically with increasing field.

In fields  $H \ll H_{\perp}^{\text{max}}$ , the losses are approximately proportional to the square of the field. At a temperature  $T = 4.2^\circ\text{K}$  (Fig. 1a), in the range of fields from  $0.1H_{\perp}^{\text{max}}$  to  $H \approx 0.7H_{\perp}^{\text{max}}$ , the losses  $R_{\perp}(H)$  are proportional to the field  $H$ . If this linear dependence is extended into the low-field region, the straight line passes through the origin. As the temperature is decreased, the origin of the linear part of the  $R_{\perp}(H)$  curve is shifted in the direction of larger fields. For fixed  $H < H_{\perp}^{\text{max}}$  ( $1.8^\circ\text{K}$ ), the quantity  $R_{\perp}(H, T)$  increases monotonically with decrease in temperature, but at  $T < 2.0^\circ\text{K}$  the growth practically ceases (Fig. 2). Along with this, the value of the losses at the maximum  $R_{\perp}^{\text{max}}$  is independent of temperature within the accuracy of the experiment; this has been observed in the frequency range 120–600 kHz. In this same interval, the losses at the maximum are proportional to the frequency:  $R_{\perp}^{\text{max}} \propto f$ .

Figure 2 also represents the temperature dependence of the reciprocal of the field  $H_{\perp}^{\text{max}}$ , referred to  $H_{\perp}^{\text{max}}(4.2^\circ\text{K})$ . This dependence is identical with the ratio  $R_{\perp}(T)/R_{\perp}(4.2^\circ\text{K})$ , if the values of  $R_{\perp}$  are taken from the region where  $R_{\perp} \propto H$ . According to the measurements,

$$[H_{\perp}^{\text{max}}(T)/H_{\perp}^{\text{max}}(4.2^\circ\text{K})]^2 = [R_{\perp}(4.2^\circ\text{K})/R_{\perp}(T)]^2 = \alpha + \beta T^{5.6 \pm 0.1},$$

where  $\alpha \approx 0.28$  and  $\beta = 0.24 \times 10^{-3} \text{ deg}^{-5.6}$ . The constant

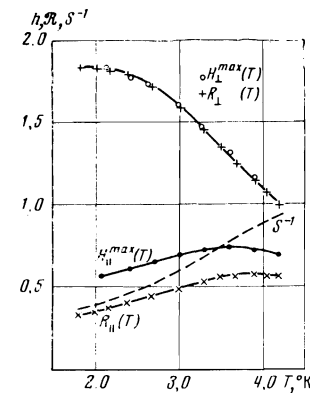


FIG. 2. Temperature dependences of the relative values of the real part of the surface impedance  $\mathcal{R} = R_{\perp \parallel}(T)/R_{\perp}(4.2^\circ\text{K})$ , of the reciprocal of the magnetic field  $h = H_{\perp}^{\text{max}}(4.2^\circ\text{K})/H_{\perp}^{\text{max}}(T)$  and of the reciprocal value of the surface conductivity  $1/S$  (in units of  $R_{\perp}(4.2^\circ\text{K})$ ; measurements were made in a field  $H = 12$  kOe) Frequency  $f = 200$  kHz.

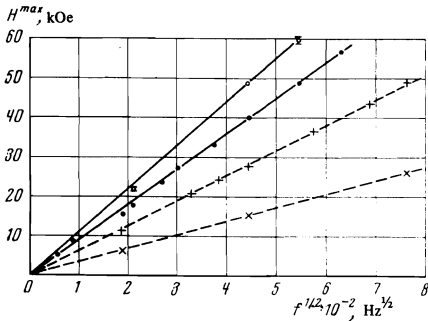


FIG. 3. Dependence of the quantity  $H_{\perp}^{\max}$  on  $f^{1/2}$ :  
 $\times - H_{\perp}^{\max}$  (2.1°K),  $\circ - H_{\parallel}^{\max}$  (2.1°K),  $+ - H_{\perp}^{\max}$  (4.2°K),  $\bullet - H_{\parallel}^{\max}$  (4.2°K).

$\alpha$  was chosen so that the experimental points would lie on a single line in log-log coordinates. Investigations carried out in the frequency range 3.6–600 kHz have shown that at a fixed temperature,  $H_{\perp}^{\max} \propto f^{1/2}$  (Fig. 3).

3. In the case in which the magnetic field is parallel to the surface of the sample, the dependence of the losses on the magnetic field also has a maximum at  $H = H_{\parallel}^{\max}$  (Fig. 1). However, in the range of fields where  $R_{\perp} \propto H$ , the quantity  $R_{\parallel}$  is approximately proportional to the square of the field. Furthermore, at fixed values of  $H$  in this region, there is a broad temperature range with an upper bound close to  $T = 4.2^{\circ}\text{K}$ , in which the losses decrease monotonically with decrease in temperature. This temperature dependence remains important down to the lowest temperature of the experiment,  $T = 1.8^{\circ}\text{K}$  (Fig. 2). The value of the losses at the maximum depends weakly on the temperature.

The value of the field  $H_{\parallel}^{\max}$  decreases on a decrease in the temperature from  $T = 4.2^{\circ}\text{K}$  to  $T = 3.7^{\circ}\text{K}$ ; however, it does so much more slowly than  $H_{\perp}^{\max}$ , and at  $T \approx 3.7^{\circ}\text{K}$ , in contrast with  $H_{\perp}^{\max}$ , it begins to increase monotonically. Figure 2 shows the temperature dependence of the ratio  $H_{\perp}^{\max}(4.2^{\circ}\text{K})/H_{\parallel}^{\max}(T)$ .

The frequency dependences of  $H_{\perp}^{\max}$  and  $H_{\parallel}^{\max}$  are the same (Fig. 3). It was observed that the losses in a magnetic field parallel to the surface of the sample do not depend on the frequency in the range of fields for which  $R_{\parallel} \propto H^2$ .

The experimental results allow us to combine the curves for the different frequencies at a fixed temperature if in place of the variables  $R$  and  $H$  we introduce the new variables  $R(f/f_0)^{-1}$  and  $H(f/f_0)^{-1/2}$ , where  $f_0$  is arbitrarily chosen to be the frequency  $f = 200$  kHz. The choice of coordinates in Fig. 1 is explained in this fashion; there, to illustrate the agreement of the curves of  $R_{\parallel}(H, f)$  in these coordinates, the two records which have the maximum difference are given (curves 2, 3).

#### 4. DISCUSSION OF RESULTS

1. We first consider the problem of the origin of the loss maximum in the magnetic field. In the work of Fisher and Kao,<sup>[8]</sup> it was shown that under the conditions of the normal skin effect for a bilateral, E-antisymmetric excitation of the electromagnetic field in a plate, the losses cannot exceed some maximum value corresponding to the condition  $\delta = 0.45d$ , where  $d$  is the thickness of the plate. Since  $R_n = 4\pi^2 c^{-2} f \delta_n$  for the normal skin effect, the losses at the maximum,  $R_n^{\max}$ , should be proportional only to  $f$  and  $d$ . Taking into account that the magnetoresistance  $\rho_T(H) = \rho_T(0)(l/r)^2$

$\propto l/r$  in a strong magnetic field, we can write for the skin-layer depth

$$\delta_n^{\max} = c^{-1} [\rho_T(H)/4\pi^2 f]^{1/2} \propto l^{1/2} f^{-1/2} r^{-1}, \quad (2')$$

whence

$$H^{\max} \propto f^{1/2} l^{-1/2}. \quad (2'')$$

It was shown by Perov<sup>[5]</sup> that the value of  $H^{\max} \propto f^{1/2} T^{5/2}$  in tin with  $H \parallel [100] \parallel n$  (this is equivalent to (2''), since  $l^{-1} \propto T^5$ ) and that the imaginary part of the surface impedance is identical to that calculated from the formulas of the normal skin effect at  $H > 0.5H^{\max}$ , a sample thickness of 1 mm and a resistance ratio  $\rho(293^{\circ}\text{K})/\rho(4.2^{\circ}\text{K}) \approx 10^5$ .

The results of our measurements of  $H_{\perp}^{\max}(f, T)$  agree with the results of<sup>[5]</sup>. Moreover, it has been established that at  $H \gtrsim 0.5H^{\max}$ ,  $R \propto HT^{5/2}$ , which also corresponds to the normal skin effect. (The case in which  $\delta/d > 1$  is not considered quantitatively.) It is natural that the magnetic-field region of existence of the normal skin effect is limited from below; in the limiting case, when  $\delta/r \ll 1$ , the skin effect is anomalous. If we assume that only the ratio  $\delta/r$  is responsible for the position, with regard to the field, of the lower boundary of the region of the normal skin effect, then we must expect a shift of the boundary to smaller fields on a temperature decrease, inasmuch as  $\delta/r$  increases in this case. It is observed, however, that this boundary shifts to larger fields (cf. the dashed curves of Fig. 1), i.e., the real losses turn out to be somewhat smaller than expected. It is evident that this can be connected with the existence of an electromagnetic wave of the type described in<sup>[9]</sup>.

Since the following relations are valid for the normal skin effect:

$$\left[ \frac{R(H, T_2, f)}{R(H, T_1, f)} \right]^2 = \left[ \frac{H^{\max}(T_1, f)}{H^{\max}(T_2, f)} \right]^2 = \frac{\rho(T_1)}{\rho(T_2)}$$

( $\rho(T)$  is the resistivity at  $H = 0$ ), then, considering the results that we have obtained, we have for the temperature dependence of the resistance:

$$\rho(T)/\rho(4.2^{\circ}\text{K}) = \alpha + \beta T^{6.8 \pm 0.4}.$$

This is in accord with the results of Zernov and Sharvin<sup>[10]</sup> on the temperature dependence of the resistance of tin, from a comparison with which we obtain an estimate of the value of the resistivity of sample No. 1:  $\rho(4.2^{\circ}\text{K}) \approx 1.4 \times 10^{-10} \Omega\text{-cm}$  and  $\rho(1.8^{\circ}\text{K}) \approx 0.4 \times 10^{-10} \Omega\text{-cm}$ . Using the known value<sup>[11]</sup> of the product  $\rho l = 1.05 \times 10^{-11} \Omega\text{-cm}^2$ ,<sup>[11]</sup> we find the free path lengths:  $l(4.2^{\circ}\text{K}) \approx 0.07$  cm and  $l \approx 0.25$  cm (these values will be used below).

2. When the magnetic field is parallel to the surface of the sample, the dependence of the losses  $R_{\parallel}$  on the field, temperature, and frequency have a character that differs in principle from  $R_{\perp}$ . In the region of magnetic fields and frequencies where  $R_{\perp}$  can be determined from the formulas of the normal skin effect,  $R_{\parallel}$  is proportional to  $H^2$  and does not depend on frequency. Such behavior is identical with that predicted for the active part of the impedance of the half-space in the case of a dominant role of the surface conductivity over the volume conductivity.<sup>[3]</sup> Moreover, inasmuch as the dependence  $R_{\perp}(H, f)$  in the coordinates  $R(f/f_0)^{-1}$  and  $H(f/f_0)^{-1/2}$  is universal in the normal skin effect for fixed temperature, then, in accord with the experiment, the ratio  $R_{\parallel}/R_{\perp}$  depends only on  $Hf^{-1/2}$  or  $\delta_n(H, f)$ . This also supports the existence of surface scattering, and can be ob-

tained from expression (1). Therefore, the experimental results will be discussed with account of the contribution of the surface scattering to the total conductivity of the sample.

3. In a magnetic field normal to the surface of the sample, the larger part of the electrons in the skin layer can collide with the surface not more than once during the time of relaxation in the volume. In this case, the active part of the impedance of the sample in a strong magnetic field is determined by the volume magneto-resistance of the metal. In a parallel field, the number of collisions of electrons with the surface is much greater than in the normal field and the surface conductivity accordingly makes a significant contribution to the impedance of the metal.

We find the relation between the surface conductivity  $S$  and the active parts of  $Z$  and  $Z_\infty$  from Eq. (1). Setting  $Z = R + iX$  and  $Z_\infty = R_\infty(1 + i)$ , we get

$$R = R_\infty \frac{1 + 2R_\infty S}{1 + 2R_\infty S + 2(R_\infty S)^2}. \quad (3)$$

Thus,  $S$  can be found by using the experimental results if we substitute  $R_{\parallel}$  and  $R_{\perp}$  for  $R$  and  $R_\infty$  in (3).

It must be noted that such a calculation is possible if the path lengths in the samples are the same. The difference in path lengths in samples No. 1 and No. 2 did not exceed 20% at  $T = 4.2^\circ\text{K}$ . (This was established from measurements of the losses in the samples for identical directions of field and current relative to the crystallographic axes and close to the angles of inclination of the field to the surface. In a strong magnetic field, where the skin effect was close to normal, the losses in the samples differed within 10%.) Therefore, for qualitative considerations, we can regard the lengths as the same. The results given below are based on a calculation of the surface conductivity from measurements at  $f = 200$  kHz in the range of fields 8–16 kOe and temperatures 1.8–4.2°K. Under these conditions, the relation of the depth of the skin layer  $\delta$  to the thickness of the sample  $d$  was such that the impedance of the sample differs little from the impedance of the half-space.

We shall assume that the character of the scattering of electrons from the boundary is diffuse, i.e.,  $S = S_{\text{diff}} \propto \sigma_T r^2 / l$  ( $S_{\text{diff}}$  does not depend on the temperature, inasmuch as  $\sigma_T / l = \text{const}$ ). Taking into account the results obtained earlier for  $\rho(4.2^\circ\text{K})$  and  $l(4.2^\circ\text{K})$ , we find that at  $T = 4.2^\circ\text{K}$ ,  $H = 16$  kOe and  $f = 200$  kHz, the product

$$R_{\perp} S_{\text{diff}} = 4\pi^2 c^{-2} f \delta_{\perp} S_{\text{diff}} \approx 0.3.$$

Using expression (3) and the experimental value of the ratio  $R_{\parallel} / R_{\perp} = 0.7$  for the same values of the field, temperature, and frequency, we obtain the result that  $R_{\perp} S_{\text{diff}}^2 \approx 0.75$ . Further, it is easy to see that the product  $R_{\perp} S_{\text{diff}}$  is proportional to  $l \delta_{\perp}^{-1}$ . Assuming  $R_{\perp} = R_\infty$ , we find that the ratio  $R / R_\infty$  is a function of  $\delta_{\perp}(H, f)$  at fixed temperature.

Thus, at a fixed temperature, the model of the static skin effect, under the assumption of diffuse scattering, satisfactorily explains the experimental results. However, the temperature dependence of the losses in the parallel field, i.e., the decrease in losses with decreasing  $T$ , does not satisfy the model of completely diffuse scattering. Actually, if the surface conductivity does not depend on the temperature, then, even in the limiting case in which  $S \gg 1/R$ , a decrease in losses with decreasing temperature is impossible.

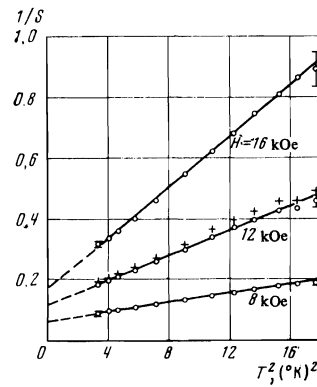


FIG. 4. Dependence of the surface resistance  $1/S$  on the square of the temperature for various  $H$ . The quantity  $1/S$  is given in units of  $R_{\perp}^{\text{max}}(4.2^\circ\text{K})$  at  $f = 200$  kHz. The crosses were plotted on the assumption that the free path of the electrons in sample No. 2 was 1.2 times smaller than in sample No. 1. The magnetic field  $H = 12$  kOe.

Figure 2 shows the result of calculation of the temperature dependence of  $1/S$ , for a fixed value of the magnetic field, on the basis of relation (3). Figure 4 illustrates the change in the surface resistivity with the square of the temperature for several values of the magnetic field. For the temperature dependence of the surface resistivity we obtained the expression

$$1/S(T, H) = 1/S_0(H) + \zeta(H) T^{2 \pm 0.5}.$$

Here  $S_0(H)$  is obtained by extrapolation of the temperature dependence  $S(H, T)$  to zero; the form of the function  $\zeta(H)$  is determined by experiment. At a fixed temperature,  $1/S$  is approximately proportional to  $H^2$ .

The value of the error is connected basically with the indeterminacy of the ratio of the path lengths in the samples. For illustration, Fig. 4 shows the calculation in the case in which the path length  $l(4.2^\circ\text{K})$  in sample No. 2 is 20% less than in sample No. 1. It is seen from the same figure that at the lowest experimental temperatures the effect of this indeterminacy on the result of calculation is small, which is explained by the dominant contribution of the surface conductivity to the active part of the impedance of the sample.

In explaining the temperature dependence of the surface conductivity, it is natural to suppose that some fraction of the electrons is scattered specularly by the surface, and that the contribution to the conductivity from the specular electrons increases with decrease in temperature. We estimate the fraction of specular electrons  $k$  at  $T = 0^\circ\text{K}$  that is necessary to explain the experimental results. The surface conductivity due to specular electrons is  $S_{\text{spec}} \propto k \sigma_T r$ ; the ratio of the conductivities  $S_{\text{spec}} / S_{\text{diff}} \propto kl/r$ . Since the estimates showed that at  $4.2^\circ\text{K}$  the surface conductivity obtained experimentally is of the order of the diffuse conductivity, we can take as the ratio  $S_{\text{spec}} / S_{\text{diff}}$  the quantity  $[S(0) - S(4.2^\circ\text{K})] \times S(4.2^\circ\text{K})$ , which is approximately equal to 4 at  $H = 16$  kOe (see Fig. 4), from which we obtain the result that the fraction of specular electrons at  $T = 0^\circ\text{K}$  is  $k \approx 10^{-2}$ .

As is well known, electrons with very small approach angles can scatter specularly from a metallic surface with<sup>[12]</sup>. We use a model according to which all the electrons are reflected specularly from the surface at angles of approach  $\varphi \leq \varphi_0$  and diffusely at  $\varphi > \varphi_0$ . If we assume the orbits of the electrons to be circles with equal radii, then the fraction of specular electrons is  $k \approx 1 - \cos \varphi_0$ . The limiting angle  $\varphi_0 \approx 0.1$  rad corresponds to the value  $k \approx 10^{-2}$ . The orbits of electrons with angles of approach less than  $\varphi_0$  congregate in a layer of thickness  $kr$  at the surface. If an electron from this

layer is scattered on an orbit at an angle greater than  $k$ , the center of the orbit will be shifted through a distance greater than  $kr$ , and the electron either leaves the surface layer or arrives at the surface at an angle greater than the limiting angle and is reflected diffusely from it. In a strong magnetic field ( $r/l \ll 1$ ) parallel to the boundary of the metal, the frequency of collisions of the electrons of the surface layer with the boundary is significantly greater than the frequency of collisions with volume phonons. Moreover, the collisions in the volume do not change the number of electrons in the surface. Therefore, in the case of diffuse scattering, the volume collisions of the electrons should not effect the surface conductivity and the temperature dependence of the surface conductivity  $S$  should indicate the presence of specular electrons.

Study of the conductivity of filamentary single crystals (whiskers) has shown that the coefficient of diffusivity  $q$  is proportional to  $T^4$ . This dependence has been explained by the fact that the diffusivity is proportional to the density of surface phonons ( $\propto T^2$ ), and a factor of the order of  $(T/\Theta_D)^2$  is added because of the ineffective scattering of the electrons by the surface phonons ( $\Theta_D$  is the Debye temperature). In a strong magnetic field, as has been noted previously, small-angle scattering should be effective and, consequently, it is natural to ascribe the observed proportionality between  $S$  and the square of the temperature solely to the temperature dependence of the density of surface excitations.

Finally, in the case of a predominant role of specular electrons in the surface conductivity, which obtains at the lowest temperatures in this experiment, the quantity  $S$  should be proportional to  $H^{-1}$  ( $S \propto r$ ). The departure from this dependence ( $S \propto H^{-2}$ ) should evidently be explained by the decrease in the probability of specular

reflection, since the number of collisions of the electrons with the surface in a time corresponding to the free path length increases with increase in the magnetic field.

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