Resonance size effect in high-frequency conductivity of thin metallic plates in the absence of a static magnetic field

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(Submitted January 10, 1974)
Zh. Eksp. Teor. Fiz. 67, 223–231 (July 1974)

The surface impedance Z and transmission coefficient $\mathcal{T}$ for E and H waves passing through a thin metallic plate are calculated for the case of specular reflection of the electrons by the metal boundary. The resonant dependence of Z and $\mathcal{T}$ at the near-resonance frequencies, $\omega\approx\omega_n=\pi n v_k/d$, $n=1,2,3,\ldots$, where v_k are the extreme values of the electron-velocity projections on the normal to the plate surface and d is the plate thickness, is analyzed. The shape of the resonance curves is sensitive to the geometry of the Fermi surface. The difference in the passage of E and H waves through the plate are discussed.

The size effect in a constant magnetic field[1] is widely used at the present to determine the diameters of the Fermi surfaces of metals. Theoretical formulas that describe the electrodynamic properties of metallic plates have as a rule been derived under the assumption that the static magnetic field is strong, so that it is impossible to go to the limit of a zero magnetic field. On the other hand, the periodic motion of the electrons between the two boundaries of the plates leads (as shown in[2]) to a unique resonance effect. Since this effect should be observed at relatively high frequencies $\omega \gg \nu$ ($\nu = 1/\tau$ is the collision frequency and τ is the electron mean free path time), it is necessary to use the equations of electrodynamics to calculate this effect (the dispersion of the dielectric constant of a metallic plate was calculated in[2]). The frequency of the periodic motion is determined by the projection v_z of the electron velocity $\mathbf{v} = d\epsilon/d\mathbf{p}$ ($\epsilon = \epsilon(\mathbf{p})$ is the energy of an electron with quasimomentum $\mathbf{p}$) on the normal to the surface of the plate, and is different for different electrons. It is therefore necessary to use a kinetic approach in the calculations. The dependence of the resonant frequency on the electron momentum, of course, smears out the effect, and after integration with respect to the momenta we are left only with the resonant terms that correspond to the extremal frequency[1)] (in analogy with cyclotron resonance for metals with an arbitrary electron dispersion law (see[3])).

We emphasize that the nature of the considered effect is not connected with quantization of the z-projection of the electron momentum, so that the effect is not as sensitive to temperature as effects due to spatial quantization[4].

The interaction of electrons with the electromagnetic field depends significantly on the configuration of the wave near the plate boundary. We have therefore considered two cases: incidence of an E or H wave on the plate and their passage through it[5]. The calculation is carried out under the following assumptions: 1) the temperature is low, so that $df_F/d\epsilon = -\delta(\epsilon - \epsilon_F)$ (f_F is the Fermi distribution function and ϵ_F is the Fermi energy); 2) the electron reflection from the film boundaries is specular; 3) Fermi-liquid effects are neglected; 4) as already mentioned, $\omega\tau \gg 1$; 5) $l \gg d$, d is the plate thickness, $l = v_F\tau$ is the mean free path; 6) $d \ll c/\omega = \lambda_0$, c is the speed of light and λ_0 is the wavelength in vacuum.

1. PASSAGE OF H WAVE THROUGH A PLATE

In this section we investigate the reaction, on the field, of an electromagnetic wave whose vector **E** lies in the plane of the plate. The z axis of the employed coordinate system is normal to the plate surface and x is directed along the vector **E**. We assume that the polarization of the electric field of the incident wave coincides with one of the principal axes of the conductivity tensor of the metal, so that the anisotropy does not intermix the different components of the electric field in the metal[2)]. If we seek the increment to the Fermi distribution function in the form

$$f_1(z,\mathbf{v},t)=-\frac{\partial f_F}{\partial \varepsilon}f(z,\mathbf{v})e^{-i\omega t},$$

then the kinetic equation for $f(z, \mathbf{v})$ takes the form

$$-i\omega^* f+v_z\frac{\partial f}{\partial z}-ev_xE_x(z)=0, \qquad (1.1)$$

where $\omega^* \equiv \omega + i/\tau$, $E_x(z, t) = E_x(z)e^{-i\omega t}$. In the case of specular reflection of the electrons from the surface, the distribution-function component that is antisymmetrical with respect to v_z

$$f_a(z,\mathbf{v})={}^1/_2[f(z,v_z,v_x)-f(z,-v_z,v_x)]$$

vanishes identically on the film boundaries.

Using (1.1), we can find a connection between $f_a(z, \mathbf{v})$ and the analogously-introduced symmetrical component $f_s(z, \mathbf{v})$:

$$f_a(z,\mathbf{v})=\frac{v_z}{i\omega^*}\frac{\partial f_s(z,\mathbf{v})}{\partial z}, \qquad (1.2)$$

and an equation for $f_s(z, \mathbf{v})$

$$(\omega^*)^2f_s+v_z^2\frac{\partial^2 f_s}{\partial z^2}-i\omega^* ev_xE_x=0, \qquad (1.3)$$

the boundary condition for which, as follows from (1.2), is

$$\left.\frac{\partial f_s}{\partial z}\right|_{z=0}=\left.\frac{\partial f_s}{\partial z}\right|_{z=d}=0.$$

Expanding $f_s(z, \mathbf{v})$, $E_x(z)$, and the x-component of the current $j_x(z)$ in a Fourier cosine series

$$j_x(z)=\frac{1}{2}j_{x0}+\sum_{n=1}^{\infty}j_{xn}\cos\frac{\pi n}{d}z,$$

$$E_x(z)=\frac{1}{2}E_{x0}+\sum_{n=1}^{\infty}E_{xn}\cos\frac{\pi n}{d}z, \qquad (1.4)$$

$$f_s(z,\mathbf{v})=\frac{1}{2}f_0^{(s)}(\mathbf{v})+\sum_{n=1}^{\infty}f_n^{(s)}(\mathbf{v})\cos\frac{\pi n}{d}z,$$

and using (1.3), we obtain

$$j_{xn}=\sigma_n(\omega)E_{xn},\quad \sigma_n(\omega)=\sigma_0 J_n(\omega),$$
$$J_n(\omega)=\left(\int\frac{ds}{v}v_x^2\right)^{-1}\int\frac{ds}{v}\frac{v_x^2}{1-(\pi n v_z/\omega^* d)^2},\tag{1.5}$$
$$\sigma_0=\frac{i\omega_0^2}{4\pi\omega^*},\quad \omega_0^2=\frac{8\pi e^2}{(2\pi\hbar)^3}\int\frac{ds}{v}v_x^2;$$

The integration is carried out over the Fermi surface. For a quadratic isotropic dispersion law $\omega_0^2 = 4\pi ne^2/m^*$ is the square of the plasma frequency of the metal (m^* is the effective mass and $n = {}^8/_3\pi p_F^3/(2\pi\hbar)^3$ is the electron density). The Fourier cosine transformation of the Maxwell equations yields an equation that connects E_{xn} with the magnetic field intensity $H_y(z)$ on the plate boundary:

$$\frac{2i\omega}{cd}[H_y(d)(-1)^n-H_y(0)]+\left[\frac{4\pi i\omega}{c^2}\sigma_n(\omega)-\left(\frac{\pi n}{d}\right)^2\right]E_{xn}=0.\tag{1.6}$$

With the aid of Eq. (1.6) and the expansion (1.4), making use of the usual boundary conditions for the electromagnetic field on the metal boundaries, we obtain the impedance $Z(0)$ and the wave transmission coefficient $\mathcal{T}$:

$$Z(0)\equiv\frac{E_x(0)}{H_y(0)}=\frac{(1+S_1)S_1-S_2^2}{1+S_1},\quad \mathcal{T}\equiv\frac{E_x(d)}{E_x^{\text{inc}}}=\frac{2S_2}{(1+S_1)(1+Z(0))},$$
$$S_1=-i\frac{\omega^*}{\omega}\frac{\delta_0^2}{d\lambda_0}-2i\frac{d}{\pi^2\lambda_0}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[1+\frac{\omega}{\omega^*}\left(\frac{d}{\pi n\delta_0}\right)^2 J_n\right]^{-1},\tag{1.7}$$
$$S_2=-i\frac{\omega^*}{\omega}\frac{\delta_0^2}{d\lambda_0}-2i\frac{d}{\pi^2\lambda_0}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\left[1+\frac{\omega}{\omega^*}\left(\frac{d}{\pi n\delta_0}\right)^2 J_n\right]^{-1},$$

where $\delta_0 \equiv c/\omega_0$ and E_x^{inc} is the electric field intensity of the wave incident from the vacuum on the metal. It will be shown below that $|S_1| \ll 1$, $|S_2| \ll 1$ at $d \ll \delta_0^2/\lambda_0$. Therefore

$$Z(0)\approx S_1,\quad \mathcal{T}\approx 2S_2.\tag{1.8}$$

At $\omega \sim v_F/d$ (see below), the impedance and the transmission coefficients are of the following order of magnitude

$$|Z(0)|=|\mathcal{T}|\sim\frac{d}{\lambda_0}\left(\frac{\delta_0}{d}\right)^2\quad\text{if}\quad \delta_0\gg d\gg\frac{\delta_0^2}{\lambda_0},\tag{1.9}$$
$$|Z(0)|\sim\frac{d}{\lambda_0}\left(\frac{\delta_0}{d}\right)^{2/3},\quad |\mathcal{T}|\sim\frac{d}{\lambda_0}\left(\frac{\delta_0}{d}\right)^{4/3}\quad\text{if}\quad d\gg\delta_0.\tag{1.10}$$

Let us return to formula (1.5) for the conductivity. The denominator in the integrand of the expression for $J_n(\omega)$ in (1.5) vanishes at $|v_z| = \omega d/\pi n$ (as $\tau \to \infty$). This condition corresponds to resonant interaction of the electrons with the electric field[3)] and specifies a "narrow strip" on the Fermi surface of the metal. The number of the resonantly interacting electrons (length and width) of the strip varies with the frequency ω and with the geometry of the Fermi surface, and in some cases quite abruptly. In particular, this occurs when the connectivity of the strip changes, namely at the saddle point (Fig. 1).

Since the spectrum of the velocities v_z in the metal is bounded, a resonant interaction of the electrons with the n-th mode of the field either appears or disappears when the frequency goes to the value $\omega_n = \pi n v_{z\,\text{extr}}/d$ (depending on the type of the extremum of v_z). It is precisely these cases which call for a special analysis, since $J_n(\omega)$ can have near the corresponding frequencies singularities analogous to those of Migdal and Kohn (see[7]).

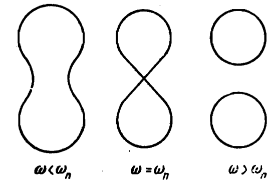

FIG. 1. Three investigated phases of the evolution of the strip $v_z = \omega d/\pi n$ on going through the "saddle" point.

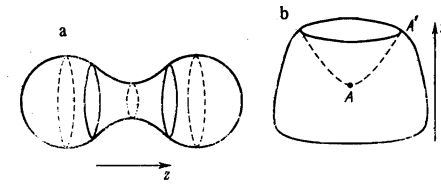

FIG. 2. Extremal strips on Fermi surfaces with "nicks" and "dents": a—between two "strips" $v_z = 0$ (dashed) there must be present a belt for which v_z is maximal; b—"dent." The maximum of v_z is reached at the point A on the belt A′. Between A′ and A there is certain to be a belt on which v_z is minimal, and this minimum can be different from zero.

1. If the x-component of the electron velocity v_x does not vanish at the point on the Fermi surface where v_z is maximal[4)], then at frequencies close to $\omega_n = n v_{z\,\max}/d$ we obtain

$$J_n(\omega)=D_1\ln(\omega^*/\omega_n-1).\tag{1.11}$$

2. On going through the "saddle" point (Fig. 1) we have

$$J_n(\omega)=-iD_2\ln(\omega^*/\omega_n-1),\tag{1.12}$$

where $\omega_n = \pi n v_{z0}/d$ (v_{z0} is the velocity v_z at the "saddle" point).

3. If the extremum of v_z is reached on a whole line, then at frequencies close to $\omega_n = \pi n v_{z\,\text{extr}}/d$, we obtain

$$J_n=\gamma D_3(\omega^*/\omega_n-1)^{-1/2}.\tag{1.13}$$

The values of the dimensionless real constants D_i are given in Appendix; throughout we have $\ln z \equiv \ln|z| = i \arg z$, with $0 \le \arg z < 2\pi$. In (1.13) we have $\gamma = 1$ if v_z has a maximum (possibly a local one) on the line and $\gamma = -i$ if v_z has a local minimum ($v_{z\,\min} \neq 0$). Extremal strips are present on Fermi surfaces with necks and with "dents" (Fig. 2). Thus, at the frequencies $\omega = \omega_n$ the "dimensionless conductivity" $J_n(\omega)$ has the singularities $J_n(\omega_n) \to \infty$ as $\tau \to \infty$, and this is reflected in the behavior of the impedance and of the transmission coefficient near the frequencies $\omega = \omega_n$. It follows from (1.7) and (1.8) that the singularities become manifest not in the dependence of the impedance and of the transmission coefficient on the frequency, but in their derivatives with respect to frequency, We have the following in particular cases:

1) a maximum of v_z at a point—"end" point:

$$\frac{dZ(0)}{d\omega}=-\frac{2i\delta_0^2}{\omega_n D_1 d\lambda_0}\left[\left(\frac{\omega^*}{\omega_n}-1\right)\ln^2\left(\frac{\omega^*}{\omega_n}-1\right)\right]^{-1},$$
$$\frac{d\mathcal{T}}{d\omega}=2(-1)^n\frac{dZ(0)}{d\omega},\quad \omega_n=\frac{\pi n}{d}v_{z\,max};\tag{1.14}$$

2) a "saddle" point:

$$\frac{dZ(0)}{d\omega}=-\frac{2\delta_0^2}{\omega_n D_2 d\lambda_0}\left[\left(\frac{\omega^*}{\omega_n}-1\right)\ln^2\left(\frac{\omega^*}{\omega_n}-1\right)\right]^{-1},$$
$$\frac{d\mathcal{T}}{d\omega}=2(-1)^n\frac{dZ(0)}{d\omega},\quad \omega_n=\frac{\pi n}{d}v_{z\,0};\tag{1.15}$$

3) an extremal strip:

$$\frac{dZ(0)}{d\omega}=-\frac{i\delta_0^2}{\gamma\omega_n D_3 d\lambda_0}\left(\frac{\omega^*}{\omega_n}-1\right)^{-1/2},\quad \frac{d\mathcal{T}}{d\omega}=2(-1)^n\frac{dZ(0)}{d\omega},\quad \omega_n=\frac{\pi n}{d}v_{z\,extr}. \tag{1.16}$$

Since the electron free path time τ is finite, the singularities of the derivatives (1.14)–(1.16) are smeared out, and to separate them from the background dependence of $dZ(0)/d\omega$ and $d\mathcal{T}/d\omega$ on the frequency, obtained by differentiating (1.10), it is necessary to satisfy certain conditions, namely: $\omega\tau \gg (d/\delta_0)^{4/3}$ for impedance singularities and $\omega\tau \gg (d/\delta_0)^{2/3}$ for singularities of the transmission coefficient in formulas (1.14) and (1.15), and respectively $\omega\tau \gg (d/\delta_0)^{8/3}$, $\omega\tau \gg (d/\delta_0)^{4/2}$ in formulas (1.16) for the impedance and the transmission coefficient. At $d \ll \delta_0$ formulas (1.15) and (1.16) are not valid; the singularities are then much weaker than in the case described here.

2. PASSAGE OF AN E WAVE THROUGH A PLATE

An E wave propagating along a waveguide contains in addition to the component E_z also a field component normal to the propagation direction. We designate it E_x. The coordinate axes are chosen the same as in the preceding section (in particular, the axes x, y, and z coincide with the principal directions of the conductivity tensor). In the E wave, all the components of the electromagnetic field depend at least on two variables; we assume them to be z and x.

The fact that the part of the electron distribution function that is antisymmetrical in v_z vanishes makes it possible to use, as in Sec. 1, a Fourier expansion for the solution of the kinetic equation. After transformations we obtain ($k = \pi m/L$, m = 1, 2, 3, ..., L is the waveguide width)

$$j_z=\sin kx\left(\sum_{n=1}^{\infty} j_{zn}\sin\frac{\pi n}{d}z\right),\quad j_x=\cos kx\left(\frac{1}{2}j_{x0}+\sum_{n=1}^{\infty} j_{xn}\cos\frac{\pi n}{d}z\right),$$
$$E_z=\sin kx\left(\sum_{n=1}^{\infty} E_{zn}\sin\frac{\pi n}{d}z\right),\quad E_x=\cos kx\left(\frac{1}{2}E_{x0}+\sum_{n=1}^{\infty} E_{xn}\cos\frac{\pi n}{d}z\right) \tag{2.1}$$

where

$$j_{zn}=\sigma_{zz}^{(n)}E_{zn}+\sigma_{zx}^{(n)}E_{xn},\quad j_{xn}=\sigma_{xx}^{(n)}E_{xn}+\sigma_{xz}^{(n)}E_{zn},$$
$$\sigma_{zz}^{(n)}=\sigma_0 I_n(\omega),\quad \sigma_{xx}^{(n)}=\sigma_0 J_n(\omega),\quad \sigma_{xz}^{(n)}=\sigma_{zx}^{(n)}=\sigma_0\frac{kd}{\pi n}G_n(\omega),$$
$$I_n(\omega)=\left(\int\frac{ds}{v}v_z^2\right)^{-1}\int\frac{ds}{v}v_z^2\frac{1}{1-(\pi n v_z/\omega^* d)^2},$$
$$G_n(\omega)=\left(\int\frac{ds}{v}v_z^2\right)^{-1}\int\frac{ds}{v}v_z^2\frac{2(\pi n v_z/\omega^* d)^2}{[1-(\pi n v_z/\omega^* d)^2]^2}. \tag{2.2}$$

Taking the Fourier transforms of Maxwell's equations in accordance with the expansions (2.1) and taking into account the boundary conditions for the electric field intensity, we get for $d \gg \delta_0^2/\lambda_0$

$$Z(0)=\frac{E_x(0)}{H_y(0)}\approx \tilde{S}_1,\quad \mathcal{T}=\frac{E_x(d)}{E_x^{\rm inc}}\approx 2\tilde{S}_2, \tag{2.3}$$

where

$$\tilde{S}_1=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n,\quad \tilde{S}_2=\frac{1}{2}a_0+\sum_{n=1}^{\infty}(-1)^n a_n,$$
$$a_0=-i\frac{\omega^*}{\omega}\frac{\delta_0^2}{d\lambda_0}, \tag{2.4}$$
$$a_n=-2i\frac{d}{(\pi n)^2\lambda_0}\left\{1+\frac{\omega}{\omega^*}\left(\frac{d}{\pi n\delta_0}\right)^2\left(J_n-\frac{k^2d^2}{\pi^2n^2}\frac{G_n^2}{I_n}\right)\right\}^{-1}$$

In a derivation of $a_n(\omega)$ we took into account the inequality $\lambda_0 \gg d$. Formulas (2.3) and (2.4) go over, as $k \to 0$, into expressions (1.7) and (1.8) for the impedance and for the transmission coefficient of the H wave.

In order of magnitude, the impedance and the transmission coefficient of the E wave coincide with the corresponding expressions for the H wave (see formulas (1.9) and (1.10)). At the frequencies $\omega = \omega_n$ ($\omega_n = \pi n v_{z\,cr}/d$, n = 1, 2, 3, ..., $v_{z\,cr}$ is the value of the velocity v_z at the singular point or on the extremal strip), the impedance and the transmission coefficient have singularities. If the H wave has a singularity at the frequency $\omega = \omega_n$, the E wave has a similar singularity (see (2.4) and (1.7)).

It was emphasized in Sec. 1 that the H wave has no singularity if the velocity component v_x vanishes at a critical ("end" or "saddle") point. The E wave has singularities also at these critical points, regardless of the vanishing of the velocity v_x.

In the case of a spherical Fermi surface (singular "end" point $v_z = v_F$, $v_x = 0$), the impedance and the transmission coefficient of the H wave have singularities at the frequencies $\omega_n = \pi n v_F/d$ (n = 1, 2, 3...), with

$$\frac{dZ(0)}{d\omega}=-\frac{8}{3}i\frac{\lambda_0}{d}\frac{1}{(k\lambda_0)^2}\left(\pi n\frac{\delta_0}{d}\right)^2\left\{(\omega^*-\omega_n)\ln^2\left(\frac{\omega^*}{\omega_n}-1\right)\right\}^{-1},\quad \frac{d\mathcal{T}}{d\omega}=2(-1)^n\frac{dZ(0)}{d\omega} \tag{2.5}$$

at

$$d>\delta_0,\quad \omega\tau\gg\frac{1}{(kd)^2},\quad kd\gg\frac{\delta_0}{d},\quad \left|\frac{\omega-\omega_n}{\omega_n}\right|\ll 1.$$

However, observation of this specific singularity of the E wave is possible under much more stringent conditions than the observation of singularities that are common to the H and E waves (see the inequalities written out above with the condition for the applicability of formulas (1.14)–(1.16)).

CONCLUSION

An examination of the passage of E and H waves through a thin metallic plate ($d \ll l$) show that the surface impedance and the transmission coefficient have resonant singularities at frequencies satisfying the condition $\omega = \pi n v_k/d$, n = 1, 2, 3, ..., where v_k are the extremal values of the projection of the electron velocity on the normal to the plate of the values of this projection at the "saddle" point. To each critical value v_k there corresponds its own series of singularities (angles 1, 2, 3, ...). The shapes of the resonant curves are sensitive to the geometry of the Fermi surface. The singularities of the E and H waves coincide, except when the velocity component v_x vanishes at the critical point (the x axis lies in the plane of the plate and is directed along the electric field), then the H wave has no singularities, whereas the E wave does have them.

The resonance singularities are most significant at a plate thickness equal to the plasma wavelength $\delta_0 = c/\omega_0$. The described resonance is the classical high-frequency size effect, which is not connected with the dimensional quantization of the electron motion.

To observe the resonant effect it is necessary to use perfect single crystal plates with a mean free path much larger than the sample thickness. Any diffuseness (even partial) of the reflection of the electrons from the surface of the plate plays the same role as a finite character of the relaxation time, and leads to additional smearing of the singularities. The entire aggregate of requirements and limitations is apparently best satisfied by semimetals and by degenerate semiconductors in which, in particular, specular reflection of the electrons from the sample boundaries is possible.

We take the opportunity to thank N. B. Brandt and I. M. Lifshitz for useful discussions.

APPENDIX

A complicated Fermi surface can have several "end" and "saddle" points on several extremal strips. Each of the singular points is characterized by its own critical value of the velocity component v_z. If the Fermi surface has symmetry properties, this can lead to equality of the critical velocities v_z, of the singular points, or else of the belts. If such an equality does take place, then the corresponding D must be summed over those end and saddle points and extremal strips for which the values of the critical velocities coincide. The constants D_i are given by

$$D_1=\left(\int\frac{ds}{v}v_x^2\right)^{-1}\frac{\pi v_x^2\sin\theta}{Kv\sqrt{R}}|v_{z\,max}|, \quad (A.1)$$

$$D_2=\left(\int\frac{ds}{v}v_x^2\right)^{-1}\frac{\pi v_x^2\sin\theta}{Kv\sqrt{|R|}}|v_{z\,0}|, \quad (A.2)$$

$$D_3=\left(\int\frac{ds}{v}v_x^2\right)^{-1}|v_{z\,extr}|\oint_{(\Gamma)}d\varphi\left[1+\left(\frac{d\theta}{d\varphi}\right)^2\right]^{1/2}\frac{v_x^2\sin\theta}{Kv\sqrt{|F|}}, \quad (A.3)$$

Here (θ,φ) are the angle variables in velocity space, defined in such a way that $v_z/v=\cos\theta$ and $K=K(\theta,\varphi)$ is the average Gaussian curvature of the Fermi surface

$$R=\left(\frac{\partial^2 v_z}{\partial\theta^2}\right)\left(\frac{\partial^2 v_z}{\partial\varphi^2}\right)-\left(\frac{\partial^2 v_z}{\partial\theta\,\partial\varphi}\right)^2,\quad F=\frac{1}{2}v_{z\,extr}\left(\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial\varphi^2}\right).$$

All the quantities in (A.1) and (A.2) are taken directly at the singular points. The integration in (A.3) is over the strip ($v_z = v_{z\,extr}$).

[1)]Or to the frequencies corresponding to "saddle" points on the Fermi surface (see below).

[2)]This means, in particular, that $j_z = 0$ (j is the current density).

[3)]According to Landau [6], the electrons that interact resonantly are those for which $k \cdot v = \omega$. Here $k_z = \pi n/d$ and $k_x = k_y = 0$. The resonance condition can be given a simple mechanical meaning: $2d/v_z = nT$, where T is the period of the oscillations of the electromagnetic field ($\omega = 2\pi/T$). When it returns to the plate boundary, the electron finds the phase of the field unchanged.

[4)]This holds true for a spherical Fermi surface, where $v_z = 0$ at $v_z = v_F$. However, even for an ellipsoid that is rotated through a certain angle relative to the z axis we have $v_x \neq 0$ at $v_z = v_{z\,max}$. Of course, the Fermi surface should then consist of not one ellipsoid, but of several of them, such that the condition $\sigma_{xz} = 0$, assumed in the derivation of formulas (1.5)–(1.7), is satisfied. We emphasize that the condition $v_x \neq 0$ at $v_z = v_{z\,max}$, which is necessary for the validity of formula (1.11), does not contradict the condition $\sigma_{xz} = 0$ if the axes x and z are reasonably chosen (normal to the plate and to the polarization of E wave).

[1] I. D. Shoenberg, Electron Structure, Experimental Results, in: Physics of Metals, I, ed. by J. M. Ziman, Cambridge Univ. Press, 1969. A. A. Abrikosov, Vvedenie v teoriyu normal'nykh metallov (Introduction to the Theory of Normal Metals), Nauka, 1972, Chap. VIII.

[2] N. B. Brandt, M. I. Kaganov, and A. S. Mikhaĭlov, ZhETF Pis. Red. 17, 150 (1973) [JETP Lett. 17, 105 (1973)].

[3] I. M. Lifshitz, M. Ya. Azbel', and M. I, Kaganov, Életronnaya teoriya metallov (Electron Theory of Metals), Nauka (1971), Part IV, Sec. 36.

[4] B. A. Tavger and V. A. Demikhovskiĭ, Usp. Fiz. Nauk 96, 61 (1968) [Sov. Phys.-Uspekhi 11, 644 (1969)].

[5] L. deBroglie, Electromagnetic Waves in Waveguides and in Cavity Resonators (Russ. transl.), IIL, 1948.

[6] L. D. Landau, Sobranie trudov (Collected Works), Vol. 2, Nauka (1969), p. 7.

[7] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Quantum Field Theoretical Methods in Statistical Physics), Fizmatgiz (1962), Chap. IV, Sec. 21 [Pergamon, 1965].

Translated by J. G. Adashko
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