The influence of condensate motion on the thermoelectric effects in superconductors

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It is shown that in the presence of an undamped current in a thin ring consisting of two superconductors the thermoelectric current depends essentially on the magnitude of the undamped current. In this case the change in the thermoelectric current is proportional to the phonon relaxation time, whereas the ordinary thermoelectric current is determined by the total relaxation time.

The problem of the thermoelectric phenomena in superconductors has been discussed in superconductivity theory, beginning with Meissner's work [1], but it was not until recently that significant progress was made in this field [2]. The aim of the present paper is to investigate the thermoelectric phenomena in a circuit in which an undamped current flows.

It was shown in [2] that a thermoelectric current can flow in a closed circuit consisting of two different superconductors in which a temperature gradient exists, and that the current will produce in the ring a magnetic field proportional to the temperature gradient. Let us carry out a simple derivation of this result. Let us consider a cylinder consisting of two superconductors (of thickness d) with a temperature gradient. The distributions of the magnetic field and the currents inside the superconductors are determined by the London equation and the equation for the current [2]

$$\mathbf{J}=-\eta\nabla T+eN_s\mathbf{v}_s,\qquad \operatorname{rot}\mathbf{H}=4\pi\mathbf{J}/c; \tag{1}$$

$$\mathbf{v}_s=\frac{1}{2m}\left(\nabla\chi-\frac{2e}{c}\mathbf{A}\right),\qquad \mathbf{H}=\operatorname{rot}\mathbf{A}, \tag{2}$$

where χ is the phase of the wave function of the condensate, N_S is the density of the superconducting electrons, η is the thermoelectric coefficient computed in [2]. The magnetic-field distribution in the k-th superconductor has, for the boundary conditions $H(d/2) = H_e$ and $H(-d/2) = H_i$, the form

$$H^{(k)}(x)=\frac{1}{2}(H_e-H_i)\operatorname{sh}\frac{x}{\delta_k}\Big/\operatorname{sh}\frac{d}{2\delta_k}+\frac{1}{2}(H_e+H_i)\operatorname{ch}\frac{x}{\delta_k}\Big/\operatorname{ch}\frac{d}{2\delta_k}. \tag{3}$$

The current distribution is found from (3) and (2), and has the form

$$J_y^{(k)}=-\frac{c}{4\pi}\frac{\partial H^{(k)}(x)}{\partial x}=-\frac{c}{8\pi\delta_k}\left[(H_e-H_i)\operatorname{ch}\frac{x}{\delta_k}\Big/\operatorname{sh}\frac{d}{2\delta_k}+(H_e+H_i)\operatorname{sh}\frac{x}{\delta_k}\Big/\operatorname{ch}\frac{d}{2\delta_k}\right]. \tag{4}$$

If the field outside the cylinder is equal to zero, i.e., if $H_e = 0$, then calculating the circulation of the current about the inner part of the cylinder, using (4) and (1), and remembering that $\oint \mathbf{A}\cdot d\mathbf{l} = \Phi_i$ is the flux inside the cylinder, we obtain for $d/R \ll 1$ and $\delta/R \ll 1$ (R is the radius of the cylinder and the ratio d/δ is arbitrary) the value

$$\Phi_i=(n+\theta/2\pi)\Phi_0; \tag{5}$$

$\Phi_0 = \pi c/e$ is the magnetic-flux quantum ($\hbar = 1$), while θ is the difference between the thermoelectric angles [2]:

$$\theta=\theta_1-\theta_2=8\pi ec^{-2}[\delta_1^2\eta_1-\delta_2^2\eta_2](T_2-T_1). \tag{6}$$

In deriving the expression (5), we used the condition

$$\oint\nabla\chi\,dl=2\pi n.$$

Let us now consider the same superconducting cylinder located in an external magnetic field whose penetration depth is large compared to the thickness of the cylinder. Then in the absence of a temperature gradient there will flow in the ring a uniform—over the thickness of the cylinder—persistent current:

$$J_y=-c(H_e-H_i)/4\pi d, \tag{7}$$

which corresponds to the superfluid velocity

$$v_{sy}=-\frac{c}{4\pi eN\,d}(H_e-H_i). \tag{8}$$

It is evident that additional terms may appear in the expression for the thermoelectric current when current flows in the ring, and the expression for the current will have the form

$$\mathbf{J}=-\eta\nabla T+\alpha\mathbf{v}_s(\mathbf{v}_s\nabla T)+\beta v_s^2\nabla T+eN_s\mathbf{v}_s. \tag{9}$$

As will be shown below, the dominant term, which depends on $\mathbf{v}_S$, is the term proportional to $\mathbf{v}_S(\mathbf{v}_S\cdot\nabla T)$, since $\alpha/\beta\sim\tau_{ph}/\tau\gg 1$, where τ_{ph} is the phonon relaxation time and τ is the total relaxation time in the normal metal. We shall assume that the inequality $\tau_{ph}/\tau\gg 1$ is always satisfied. A calculation similar to the one carried out above for the derivation of the relation (5) yields for the change in the magnetic flux inside the cylinder upon the inclusion of a temperature gradient the value

$$\delta\Phi_i=\Phi_0\theta/2\pi, \tag{10}$$

where $\theta=\theta_1-\theta_2$, which is computed with allowance for the condensate motion, will have the form

$$\theta_k=8\pi ec^{-2}\delta_k^2[\eta_k-\alpha_k v_{sk}^2](T_2-T_1), \tag{11}$$

where v_{sk} is determined by the relation (8) in the absence of a temperature gradient, i.e.,

$$v_{sk}\approx-cH_e/4\pi eN_{sk}d, \tag{12}$$

since $H_i \ll H_e$ when $\delta^2/Rd \ll 1$.

Thus, the thermoelectromotive power of a superconductor changes upon the application of a magnetic field. Let us note at once that the change in the thermoelectromotive power upon the application of a magnetic field can be larger than the thermoelectromotive power in the absence of the field, since $\alpha\sim\tau_{ph}$, while $\eta\sim\tau$, the total relaxation time.

Let us now proceed to compute the coefficient α. In the presence of a finite superfluid velocity the kinetic equation has the form [3]

$$\frac{\partial\tilde\varepsilon_p}{\partial\mathbf{p}}\frac{\partial n_p}{\partial\mathbf{r}}-\frac{\partial\tilde\varepsilon_p}{\partial\mathbf{r}}\frac{\partial n_p}{\partial\mathbf{p}}=I_{imp}\{n_p\}+I_{ph}\{n_p\}; \tag{13}$$

where

$$\tilde\varepsilon_p=(\xi_p^2+\Delta^2)^{1/2}+\mathbf{p}\mathbf{v}_s=\varepsilon_p+\mathbf{p}\mathbf{v}_s,$$
$$\mathbf{v}_s=\frac{\mathbf{p}_s}{m}=\frac{1}{2m}\left(\nabla\chi-\frac{2e}{c}\mathbf{A}\right),\quad \xi_p=\frac{p^2}{2m}+\frac{p_s^2}{2m}+\mu-e\varphi. \tag{14}$$

In the superconductor, the gradient of the electrochemical potential is equal to zero, i.e., $\nabla(\mu-e\varphi)=0$, while

$$I_{imp}\{n_{\mathbf{p}}\}=\frac{1}{2}\sum_{\mathbf{p}'}W_{\mathbf{pp}'}[n_{\mathbf{p}'}-n_{\mathbf{p}}]\left(1+\frac{\xi_{\mathbf{p}}\xi_{\mathbf{p}'}-\Delta^2}{\varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}'}}\right)\delta(\tilde{\varepsilon}_{\mathbf{p}}-\tilde{\varepsilon}_{\mathbf{p}'}),$$

$$\begin{aligned}I_{ph}\{n_{\mathbf{p}}\}=\pi\sum_{\mathbf{q}}|C_{\mathbf{q}}|^2\Big\{\{(1-n_{\mathbf{p}})n_{\mathbf{p}-\mathbf{q}}[(1+N_{\mathbf{q}})\delta(\tilde{\varepsilon}_{\mathbf{p}}-\tilde{\varepsilon}_{\mathbf{p}-\mathbf{q}}+\omega_{\mathbf{q}})\\ +N_{\mathbf{q}}\delta(\tilde{\varepsilon}_{\mathbf{p}}-\tilde{\varepsilon}_{\mathbf{p}-\mathbf{q}}-\omega_{\mathbf{q}})]-n_{\mathbf{p}}(1-n_{\mathbf{p}-\mathbf{q}})[N_{\mathbf{q}}\delta(\tilde{\varepsilon}_{\mathbf{p}}-\tilde{\varepsilon}_{\mathbf{p}-\mathbf{q}}+\omega_{\mathbf{q}})\\ +(1+N_{\mathbf{q}})\delta(\tilde{\varepsilon}_{\mathbf{p}}-\tilde{\varepsilon}_{\mathbf{p}-\mathbf{q}}-\omega_{\mathbf{q}})]\}\left(1+\frac{\xi_{\mathbf{p}}\xi_{\mathbf{p}-\mathbf{q}}-\Delta^2}{\varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}-\mathbf{q}}}\right)+\left(1-\frac{\xi_{\mathbf{p}}\xi_{\mathbf{p}-\mathbf{q}}-\Delta^2}{\varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}-\mathbf{q}}}\right)\\ \times\{(1-n_{\mathbf{p}})(1-n_{\mathbf{q}-\mathbf{p}})[(1+N_{\mathbf{q}})\delta(\tilde{\varepsilon}_{\mathbf{p}}+\tilde{\varepsilon}_{\mathbf{q}-\mathbf{p}}+\omega_{\mathbf{q}})+N_{\mathbf{q}}\delta(\tilde{\varepsilon}_{\mathbf{p}}+\tilde{\varepsilon}_{\mathbf{q}-\mathbf{p}}-\omega_{\mathbf{q}})]\\ -n_{\mathbf{p}}n_{\mathbf{q}-\mathbf{p}}[N_{\mathbf{q}}\delta(\tilde{\varepsilon}_{\mathbf{p}}+\tilde{\varepsilon}_{\mathbf{p}-\mathbf{q}}+\omega_{\mathbf{q}})+(1+N_{\mathbf{q}})\delta(\tilde{\varepsilon}_{\mathbf{p}}+\tilde{\varepsilon}_{\mathbf{q}-\mathbf{p}}-\omega_{\mathbf{q}})]\}\Big\},\end{aligned} \quad (15), (16)$$

where $|C_{\mathbf{q}}|^2 = \pi\omega_q/2Tm^2 l_a$ (k = 1), l_a is the mean free path in the normal metal when $T \gg \Theta$ (Θ is the Debye temperature). In the absence of a temperature gradient Eq. (13) has as its solution the equilibrium Fermi function of the energy $\tilde{\epsilon}_{\mathbf{p}}$:

$$n_{\mathbf{p}}^{(0)}=[\exp(\tilde{\varepsilon}_{\mathbf{p}}/T)+1]^{-1}. \quad (17)$$

It is evident from (15) that the electron-impurity collision integral computed with any $\tilde{\epsilon}_{\mathbf{p}}$-dependent distribution function vanishes, and therefore any perturbation of the distribution function that depends only on $\tilde{\epsilon}_{\mathbf{p}}$ can relax only on the phonons, in complete analogy with the case of the normal metal, where any function of the energy relaxes to the equilibrium function during the phonon relaxation time. However, in contrast to the normal metal, where such a function does not make any contribution to the current, in the superconductor the appearance of an $\tilde{\epsilon}_{\mathbf{p}}$-dependent correction to the distribution function leads to the appearance of a current whose strength is proportional to the phonon-induced-relaxation time. The effect being studied is connected precisely with this circumstance.

In the presence of a temperature gradient the kinetic equation assumes the form

$$-\left(\mathbf{v}_{\mathbf{p}}\frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}}+\mathbf{v}_s\right)\nabla T\frac{\tilde{\varepsilon}_{\mathbf{p}}}{T}\frac{\partial n_{\mathbf{p}}^{(0)}}{\partial\tilde{\varepsilon}_{\mathbf{p}}}=I_{imp}\{n_{\mathbf{p}}^{(1)}\}+I_{ph}\{n_{\mathbf{p}}^{(1)}\}, \quad (18)$$

where $\mathbf{v}_{\mathbf{p}} = \partial\xi_{\mathbf{p}}/\partial\mathbf{p}$ and $n_{\mathbf{p}}^{(1)}$ is the nonequilibrium correction to the distribution function. It can be seen from (18) that the left-hand side of the equation has two terms: one term contains the function $\mathbf{v}_{\mathbf{p}}\xi_{\mathbf{p}}/\epsilon_{\mathbf{p}}$, which changes its sign on crossing the Fermi surface and is responsible for the thermoelectromotive force in the presence of a stationary condensate, while the other term contains $\mathbf{v}_S$, the ordered excitation velocity connected with the fact that the quasiparticle spectrum $\epsilon_{\mathbf{p}}$ is determined in a reference system moving together with the condensate. Therefore, in the laboratory system the quasiparticles have the ordered velocity $\mathbf{v}_S$.

It is convenient, for the solution of Eq. (18), to represent the distribution function in the form

$$n_{\mathbf{p}}^{(1)}=\varphi_{\mathbf{p}}+\operatorname{sign}\xi_{\mathbf{p}}\Phi_{\mathbf{p}} \quad (19)$$

with the functions $\varphi_{\mathbf{p}}$ and $\Phi_{\mathbf{p}}$ depending only on $|\xi_{\mathbf{p}}|$. Then the impurity collision integral assumes the form

$$\begin{aligned}I_{imp}\{n_{\mathbf{p}}^{(1)}\}=\frac{1}{2}\sum_{\mathbf{p}'}W_{\mathbf{p}-\mathbf{p}'}[\varphi_{\mathbf{p}'}-\varphi_{\mathbf{p}}]\left(1-\frac{\Delta^2}{\varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}'}}\right)\delta(\tilde{\varepsilon}_{\mathbf{p}}-\tilde{\varepsilon}_{\mathbf{p}'})\\ +\frac{1}{2}\frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}}\sum_{\mathbf{p}'}W_{\mathbf{p}-\mathbf{p}'}\left[\frac{|\xi_{\mathbf{p}'}|}{\varepsilon_{\mathbf{p}'}}\Phi_{\mathbf{p}'}-\Phi_{\mathbf{p}}\frac{\varepsilon_{\mathbf{p}}}{|\xi_{\mathbf{p}}|}\left(1-\frac{\Delta^2}{\varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}'}}\right)\right]\delta(\tilde{\varepsilon}_{\mathbf{p}}-\tilde{\varepsilon}_{\mathbf{p}'}).\end{aligned} \quad (20)$$

Because of its linearity in $n_{\mathbf{p}}^{(1)}$, the phonon operator also splits up, in complete analogy with (20), into even and odd functions of $\xi_{\mathbf{p}}$.

Solving Eq. (18) with allowance for (19) and (20), we find that in the lowest order in $p_F v_S/\tilde{\epsilon}_{\mathbf{p}} \sim p_F v_S/T_c \ll 1$ and under the condition that $\tau_{imp} \ll \tau_{ph}$ the quantity $\Phi_{\mathbf{p}}$ has the form (see [2]):

$$\Phi_{\mathbf{p}}=\tau_{imp}\mathbf{v}_{\mathbf{p}}\nabla T\frac{\varepsilon_{\mathbf{p}}}{T}\frac{\partial n_{\mathbf{p}}^{(0)}}{\partial\varepsilon_{\mathbf{p}}}, \quad (21)$$

where τ_{imp} is the time of relaxation on the impurities in the normal metal.

Let us now turn to the equation for $\varphi_{\mathbf{p}}$:

$$-\mathbf{v}_s\nabla T\frac{\tilde{\varepsilon}_{\mathbf{p}}}{T}\frac{\partial n_{\mathbf{p}}^{(0)}}{\partial\tilde{\varepsilon}_{\mathbf{p}}}=\frac{1}{2}\sum_{\mathbf{p}'}W_{\mathbf{p}-\mathbf{p}'}[\varphi_{\mathbf{p}'}-\varphi_{\mathbf{p}}]\left(1-\frac{\Delta^2}{\varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}'}}\right)\delta(\tilde{\varepsilon}_{\mathbf{p}}-\tilde{\varepsilon}_{\mathbf{p}'})+I_{ph}\{\varphi_{\mathbf{p}}\}. \quad (22)$$

Let us, by adding and subtracting terms proportional to $\Delta^2/\tilde{\epsilon}_{\mathbf{p}}\tilde{\epsilon}_{\mathbf{p}'}$, separate out from the phonon operator the part depending only on $\tilde{\epsilon}_{\mathbf{p}}$. As a result, we shall have, after making the substitutions

$$\varphi_{\mathbf{p}}=-T\mathbf{v}_s\nabla T\frac{\partial n_{\mathbf{p}}^{(0)}}{\partial\tilde{\varepsilon}_{\mathbf{p}}}f_{\mathbf{p}}; \quad (23)$$

$$I_{ph}\{f_{\mathbf{p}}\}=I_{ph}^{(0)}\{f_{\mathbf{p}}\}+I_{ph}^{(1)}\{f_{\mathbf{p}}\}, \quad (24)$$

the expression

$$\begin{aligned}I_{ph}^{(0)}\{f_{\mathbf{p}}\}=\pi\mathbf{v}_s\nabla T\sum_{\mathbf{p}'}|C_{\mathbf{p}-\mathbf{p}'}|^2\Big\{\left(1-\frac{\Delta^2}{\tilde{\varepsilon}_{\mathbf{p}}\tilde{\varepsilon}_{\mathbf{p}'}}\right)n_{\mathbf{p}}^{(0)}(1-n_{\mathbf{p}'}^{(0)})(f_{\mathbf{p}'}-f_{\mathbf{p}})\\ \times[N_{\mathbf{p}-\mathbf{p}'}\delta(\tilde{\varepsilon}_{\mathbf{p}}-\tilde{\varepsilon}_{\mathbf{p}'}+\omega_{\mathbf{p}-\mathbf{p}'})+(1+N_{\mathbf{p}-\mathbf{p}'})\delta(\tilde{\varepsilon}_{\mathbf{p}}-\tilde{\varepsilon}_{\mathbf{p}'}-\omega_{\mathbf{p}-\mathbf{p}'})]\\ -\left(1+\frac{\Delta^2}{\tilde{\varepsilon}_{\mathbf{p}}\tilde{\varepsilon}_{\mathbf{p}'}}\right)n_{\mathbf{p}}^{(0)}n_{-\mathbf{p}'}^{(0)}(f_{-\mathbf{p}'}+f_{\mathbf{p}})[N_{\mathbf{p}-\mathbf{p}'}\delta(\tilde{\varepsilon}_{\mathbf{p}}+\tilde{\varepsilon}_{-\mathbf{p}'}+\omega_{\mathbf{p}-\mathbf{p}'})\\ +(1+N_{\mathbf{p}-\mathbf{p}'})\delta(\tilde{\varepsilon}_{\mathbf{p}}+\tilde{\varepsilon}_{-\mathbf{p}'}-\omega_{\mathbf{p}-\mathbf{p}'})]\Big\}.\end{aligned} \quad (25)$$

The operator $I_{ph}^{(1)}\{f_{\mathbf{p}}\}$ is obtained from (25) by replacing the coherence factors in the first and second terms by

$$\pm\frac{\Delta^2}{\tilde{\varepsilon}_{\mathbf{p}}\tilde{\varepsilon}_{\mathbf{p}'}}\left(1-\frac{\tilde{\varepsilon}_{\mathbf{p}}\tilde{\varepsilon}_{\mathbf{p}'}}{\varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}'}}\right).$$

The distinctive feature of $I_{ph}^{(1)}\{f_{\mathbf{p}}\}$ is its explicit dependence on the direction of the quasiparticle momentum.

It is clear that we can now solve Eq. (22) with allowance for (24) and (25) by an iterative procedure, for which purpose we represent $f_{\mathbf{p}}$ in the form

$$f_{\mathbf{p}}=f_0(\tilde{\varepsilon}_{\mathbf{p}})+f_{\mathbf{p}}^{(1)}. \quad (26)$$

Let us then assume that $f_{\mathbf{p}}^{(1)} \ll f_0(\tilde{\epsilon}_{\mathbf{p}})$. Then, if $f_0(\tilde{\epsilon}_{\mathbf{p}})$ satisfies the equation (remembering that $I_{imp}\{f_0(\tilde{\epsilon}_{\mathbf{p}})\} = 0$)

$$-\frac{\tilde{\varepsilon}_{\mathbf{p}}}{T}\frac{\partial n_{\mathbf{p}}^{(0)}}{\partial\tilde{\varepsilon}_{\mathbf{p}}}=I_{ph}^{(0)}\{f_0(\tilde{\varepsilon}_{\mathbf{p}})\}, \quad (27)$$

we have for $f_{\mathbf{p}}^{(1)}$ the equation

$$I_{imp}\{f_{\mathbf{p}}^{(1)}\}+I_{ph}^{(1)}\{f_{\mathbf{p}}^{(0)}\}+I_{ph}^{(0)}\{f_{\mathbf{p}}^{(1)}\}=0, \quad (28)$$

whence

$$f_{\mathbf{p}}^{(1)}\sim(\tau_{imp}p_F v_s/\tau_{ph}\tilde{\varepsilon}_{\mathbf{p}})f_0(\tilde{\varepsilon}_{\mathbf{p}}),$$

which justifies the assertion that $f_{\mathbf{p}}^{(1)}$ is small.

Let us multiply (27) by $\delta(\epsilon-\tilde{\epsilon}_{\mathbf{p}})$ and sum over all $\mathbf{p}$. We then obtain

$$-\frac{\varepsilon}{T}\frac{\partial n^{(0)}}{\partial\varepsilon}=\bar{I}_{ph}^{(0)}\{f(\varepsilon)\}, \quad (29)$$

where

$$\bar{I}_{ph}^{(0)}\{f(\varepsilon)\}=\sum_{\mathbf{p}}\delta(\varepsilon-\tilde{\varepsilon}_{\mathbf{p}})I_{ph}\{f_0(\tilde{\varepsilon}_{\mathbf{p}})\}\Big/\sum_{\mathbf{p}}\delta(\varepsilon-\tilde{\varepsilon}_{\mathbf{p}}).$$

The right-hand side of Eq. (29) depends on v_S^2 only through the effective transition probability. In the superconductor, as in the normal metal, scattering by the phonons is highly inelastic, and each collision is effective in respect of the transfer of energy (but not of momentum). This means that the effect of the phonon operator can be replaced by an effective relaxation time $\tau_{ph} \sim \Theta^2/T^3$, and the function φ_p' (23), has, up to terms of the order of v_s^2, the form

$$\varphi_p=\tau_{ph}(\mathbf{v}_s\nabla T)\left[\frac{\varepsilon_p}{T}\frac{\partial n_p^{(0)}}{\partial\varepsilon_p}+(\mathbf{p}\mathbf{v}_s)\frac{\partial}{\partial\varepsilon_p}\left(\frac{\varepsilon_p}{T}\frac{\partial n_p^{(0)}}{\partial\varepsilon_p}\right)\right], \tag{30}$$

where τ_{ph} does not depend on v_s.

The total current

$$\mathbf{J}=2e\sum_p \mathbf{v}_p n_p+e\mathbf{v}_s N; \tag{31}$$

setting $n_p = n_p^{(0)} + \varphi_p + \Phi_p \operatorname{sign}\xi_p$, and expanding it in a power series in $p_F v_s/T \ll 1$, we obtain

$$\mathbf{J}=-\eta\nabla T+\alpha_1\mathbf{v}_s(\mathbf{v}_s\nabla T)+e\mathbf{v}_s N_s=\mathbf{J}_T+\mathbf{J}_v'+e\mathbf{v}_s N_s \tag{32}$$

where

$$\frac{N_s}{N}=1+2\int_0^\infty d\xi\frac{\partial n^{(0)}}{\partial\varepsilon}, \tag{33}$$

$$\mathbf{J}_T=-\eta\nabla T=2e\sum_p \mathbf{v}_p(\mathbf{v}_p\nabla T)\tau_{imp}\frac{\varepsilon_p}{T}\frac{\partial n_p^{(0)}}{\partial\varepsilon_p}\operatorname{sign}\xi_p, \tag{34}$$

$$\mathbf{J}_v'=2e\sum_p \tau_{ph}(\mathbf{v}_s\nabla T)(\mathbf{p}\mathbf{v}_s)\mathbf{v}_p\frac{\partial}{\partial\varepsilon_p}\left(\frac{\varepsilon_p}{T}\frac{\partial n_p^{(0)}}{\partial\varepsilon_p}\right). \tag{35}$$

The expression (35) differs from the other terms containing v_s^2, in that it contains the phonon-induced-relaxation time and is an even function of ξ_p, which yields a nonvanishing result even in the zeroth approximation in T/μ. Let us, however, note at once that part of φ_p is an even function of **p** and ξ_p, and therefore the existence of the term $\mathbf{v}_s\cdot\nabla T$ will clearly change the energy gap entering into the expression for N_s. This correction turns out to be of the same order of magnitude as (35), and we shall consider it below. Taking the correction to the energy gap (and, consequently, to N_s) into account, we obtain for $\mathbf{J}_v = \mathbf{J}_v' + e\mathbf{v}_s N_s$ the expression

$$\mathbf{J}_v=2e\sum_p(\mathbf{p}\mathbf{v}_s)\mathbf{v}_p\frac{\partial^2 n_p^{(0)}}{\partial\varepsilon_p\partial\Delta}\delta\Delta+2e\sum_p(\mathbf{p}\mathbf{v}_s)v_p\tau_{ph}(\mathbf{v}_s\nabla T)\frac{\partial}{\partial\varepsilon_p}\left(\frac{\varepsilon_p}{T}\frac{\partial n_p^{(0)}}{\partial\varepsilon_p}\right). \tag{36}$$

Since

$$2\sum_p(\mathbf{p}\mathbf{v}_s)\mathbf{v}_p\frac{\partial n_p^{(0)}}{\partial\varepsilon_p}=\mathbf{v}_s(N_s-N),$$

while N_s depends only on Δ/T, we obtain after simple transformations the expression

$$\mathbf{J}_v=eN_s\mathbf{v}_s\left[\frac{\delta\Delta}{\Delta}+\frac{\tau_{ph}(\mathbf{v}_s\nabla T)}{T}\right]\partial\ln N_s\left(\frac{\Delta}{T}\right)\Big/\partial\ln\left(\frac{\Delta}{T}\right). \tag{37}$$

Thus, it is necessary to compute the correction (to the energy gap) proportional to $\mathbf{v}_s\cdot\nabla T$.

It follows from (37) that the $\mathbf{v}_s$-dependent contribution to the current for $\Delta \to 0$ vanishes, since $\delta\Delta$ tends to zero together with Δ as we approach the transition point.

The change in the energy gap is easily found from the self-consistency equation[3]:

$$1=\frac{\lambda}{2}\sum_p\frac{1-n_p-n_{-p}}{(\xi_p^2+\Delta^2)^{1/2}}. \tag{38}$$

Substituting $n_p = n_p^{(0)} + \varphi_p$, we find after simple transformations that (Δ_0 is the halfwidth of the energy gap at $T = 0$ and $v_s = 0$)

$$\ln\frac{\Delta_0}{\Delta}+\left(1-\frac{N_s}{N}\right)\frac{\tau_{ph}\mathbf{v}_s\nabla T}{T}=2\int_\Delta^\infty\frac{n(\varepsilon)}{(\varepsilon^2-\Delta^2)^{1/2}}d\varepsilon. \tag{39}$$

Expanding (39) in a power series in $\delta\Delta/\Delta \ll 1$, we obtain the change in the gap:

$$\frac{\delta\Delta}{\Delta}=\frac{N-N_s}{N_s}\frac{\tau_{ph}\mathbf{v}_s\nabla T}{T}\ll 1. \tag{40}$$

Notice that the sign of the change in the gap depends on the direction of $\mathbf{v}_s$ relative to the vector ∇T. The expression for the current with allowance for (40) assumes the form

$$\mathbf{J}_v=eN\mathbf{v}_s\frac{\tau_{ph}(\mathbf{v}_s\nabla T)}{T}\partial\ln N_s\left(\frac{\Delta}{T}\right)\Big/\partial\ln\left(\frac{\Delta}{T}\right), \tag{41}$$

i.e., the coefficient α is equal to

$$\alpha=\frac{eN\tau_{ph}}{T}\partial\ln N_s\left(\frac{\Delta}{T}\right)\Big/\partial\ln\left(\frac{\Delta}{T}\right). \tag{42}$$

The ratio of the change in the thermoelectric current for $\mathbf{v}_s \neq 0$ to the thermoelectric current for $v_s = 0$ is of the order of $(p_F v_s/T)^2\tau_{ph}/\tau_{imp}$, and can become larger than unity when $(p_F v_s/T)^2 \ll 1$ if $\tau_{ph}/\tau_{imp} \gg 1$.

We have assumed in all the preceding calculations that $\mathbf{v}_s$ does not depend on the coordinates, i.e., we have neglected the curling of the trajectories of the excitations in the magnetic field. To estimate the influence of this effect, let us write the kinetic equation for the function Φ_p with allowance for the spatial dependence of $\mathbf{v}_s$:

$$\begin{aligned}-\frac{|\xi_p|}{\varepsilon_p}(\mathbf{v}_p\nabla T)\frac{\varepsilon_p}{T}\frac{\partial n_p^{(0)}}{\partial\tilde\varepsilon_p}-\frac{|\xi_p|}{\varepsilon_p}\left[\mathbf{v}_p\frac{\partial\varphi_p}{\partial\mathbf{r}}-\frac{\partial}{\partial\mathbf{r}}\left(\frac{p_s^2}{2m}\right)\frac{\partial\varphi_p}{\partial\mathbf{p}}\right]\\+\left[\mathbf{v}_s\frac{\partial\Phi_p}{\partial\mathbf{r}}-\frac{\partial(\mathbf{p}\mathbf{v}_s)}{\partial\mathbf{r}}\frac{\partial\Phi_p}{\partial\mathbf{p}}\right]=I_{imp}\{\Phi_p\}.\end{aligned} \tag{43}$$

Let us allow for the terms containing the derivatives of $\mathbf{v}_s$ with the aid of perturbation theory. Then in the lowest order in $p_F v_s/T \ll 1$ it is sufficient to substitute for φ_p only the **p**-even part. Taking into account the fact that the derivatives are computed at constant $\tilde\varepsilon_p$, we obtain for the correction to the function Φ_p the equation

$$-\frac{|\xi_p|}{\varepsilon_p}\mathbf{v}_p\frac{\partial(\mathbf{v}_s\nabla T)}{\partial\mathbf{r}}\tau_{ph}\frac{\varepsilon_p}{T}\frac{\partial n_p^{(0)}}{\partial\varepsilon_p}=-\frac{\Phi_p^{(1)}}{\tau_{imp}}\frac{|\xi_p|}{\varepsilon_p}, \tag{44}$$

from which we find the first-order correction to be equal to

$$\Phi_p^{(1)}=\tau_{imp}\mathbf{v}_p\frac{\varepsilon_p}{T}\frac{\partial n_p^{(0)}}{\partial\varepsilon_p}\tau_{ph}\frac{\partial(\mathbf{v}_s\nabla T)}{\partial\mathbf{r}}, \tag{45}$$

which determines the current flowing in the direction perpendicular to ∇T (the Nernst-Ettingshausen effect). The correction to the thermoelectric current is clearly of second order in the parameter $\tau_{ph}\partial v_{si}/\partial x_k \sim \omega_c\tau_{ph}$. The ratio of the change in the thermoelectric current due to the Nernst-Ettingshausen effect to the change $\mathbf{J}_v$ in the current due to the condensate motion and determined by the expression (41) is

$$\frac{\delta J}{J_v}\sim\frac{\tau_{imp}}{\tau_{ph}}\left(\frac{T}{\mu}\right)^2\left(\frac{l_{ph}}{d}\right)^2. \tag{46}$$

We consider this ratio to be small[1)].

This effect will be observable only in films of thickness d satisfying the condition $\xi \ll d \lesssim \delta$, where ξ is the coherence length. The first condition must be imposed if the diffuse scattering by the walls is to be prevented from leading to a drastic change in the spectrum of the system. At the same time, since the scattering by the walls is elastic, it cannot lead to additional relaxation of the function φ_p, and the change in the thermoelectromotive force due to the condensate motion will still be proportional to the phonon relaxation time when the scattering by the walls is taken into account.

In conclusion, let us express the parameter $p_F v_s / T$ in terms of the magnetic field, taking into account the fact that the condition $\xi \ll d \lesssim \delta$ can be satisfied only by superconductors of the second kind. Using the coupling (see [4])

$$H_{c1} = \frac{\Phi_0}{4\pi\delta^2} \ln\frac{\delta}{\xi},$$

we obtain

$$\frac{p_F v_s}{T} = \frac{\pi}{8} \frac{\Delta}{T} \frac{H}{H_{c1}} \frac{\xi}{d} \ln\frac{\delta}{\xi} \ll 1.$$

A suitable object for such experiments is pure niobium, which is a superconductor of the second kind and in which the mean free path at low temperatures exceeds the coherence length by a factor of 300 and all the conditions are easily met. At the same time, since the considered effect is insensitive to elastic scattering mechanisms, it may be inferred that the effect will also occur when $l \ll \xi$, i.e., in alloys, for which the kinetic equation (13) is invalid.

Thus, the change in the thermoelectric effect due to the motion of the condensate can be observed in superconductors of the second kind at fields $H < H_{c1}$. This effect enables us to measure such important characteristics of a superconductor as the relaxation—in terms of energy—time of the excitations.

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1) It is obvious that the condition (46) is important only when $l_{ph} < \min(d, \delta)$. If, on the other hand, $l_{ph} > \min(d, \delta)$, then the curling of the trajectories can be neglected if $v_F/\omega_c > \min(\delta, d)$ (l_{ph} is the mean free path for relaxation in terms of energy).

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23