Nonequilibrium high-frequency discharge in wave fields

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We investigate the self-consistent distribution of an electromagnetic of an electromagnetic field and plasma in a time-independent diffusionless discharge with 'cold' molecules and 'hot' electrons. The condition that the problem is time independent is taken to be that the electron ionization and attachment frequencies are equal and this determines the amplitude of the electric field E in the region occupied by the discharge: $|E| = E_b'$ where E_b is the so-called breakdown field for the given type of gas, pressure, and field frequency. The determination of the structure of the discharge and the wave field producing it is thus reduced to finding the plasma density distribution $n(\mathbf{r})$ for which the solution of the Maxwell equations with given extraneous sources has the property $|E|=\text{const}=E_b$ for $n>0$ and $|E|<E_b$ outside this region. The solution is constructed on the basis of the scalar Helmholtz equation for the electric field. The resulting set of equations for the field phase and density is solved for three simple (symmetric) discharge configurations produced by plane, cylindrical, and converging spherical waves.

Despite the recently increased interest in the high-frequency discharge[1-4], its electrodynamic structure has so far been investigated mainly under thermodynamic equilibrium or quasiequilibrium. At the same time, the nonequilibrium discharge containing hot electrons ($T_e \sim 10^4$ °K) and cold heavy particles ($T_m \sim 300$°K) can be produced in a relatively broad range of parameters (at least as the first stage of the postbreakdown state), but has not been extensively investigated. Some structural calculations for such discharges have been reported in the geometric-optics approximation for the special case of a weakly absorbing and nonrefracting plasma.[5,6]

In this paper we consider some simple selfconsistent distributions of the wave field and plasma in a time-independent nonequilibrium discharge excited under diffusionless conditions when the leading mechanism responsible for electron losses is attachment to neutral molecules.[7] This discharge situation is characteristic for electronegative gases when the diffusion length for attachment $L_a = (D/\nu_a)^{1/2}$ is small in comparison with the characteristic linear size L of the discharge region (ν_a is the attachment frequency and D is the diffusion coefficient for electrons). For example, for air in the case of ambipolar diffusion the inequality $L_a \ll L$ is satisfied in this particular pressure range, i.e., p(Torr) $\gg$ 1/2L(cm).

The condition that the discharge is time-independent is that the ionization frequency (by electron impact) ν_i is equal to the attachment frequency ν_a.[1)] The difference $\nu_i - \nu_a$ may be looked upon as a known function of the amplitude of the electric field E (see, for example,[5]), which passes through zero when the amplitude is equal to E_b, the so-called breakdown value. The field E_b is a constant for a given type of gas, given pressure, and given field frequency ω, so that the equation $\nu_i = \nu_a$ in fact determines the amplitude of the electric field under time-independent conditions $|E| = \text{const} = E_b$. Therefore, the determination of the structure of the high-frequency discharge in this case reduces to the solution of a particular electrodynamic problem which can be stated as follows: given the sources of radiation, it is required to find the distribution $N(\mathbf{r})$ of the electron concentration in space [i.e., the distribution of the complex permittivity $\epsilon(\mathbf{r})$] for which

$$|E|=\text{const}=E_b \text{ for } N>0, \quad |E|<E_b \text{ for } N=0.$$

We shall consider some of the simplest solutions of this problem [2)] for the case where the electric field amplitude is described by the scalar Helmholtz equation with variable wave number:

$$\Delta E+k^2\varepsilon(\mathbf{r})E=0, \tag{1}$$

where $k = \omega/c$ is the wave number in vacuum, $\epsilon = 1 - n - in\delta$, n is the electron concentration referred to the critical point ($n = N/N_c$), $\delta = \nu/\omega$, and ν is the effective electron collision frequency.

If we write the complex field amplitude in the form $E = |E|e^{i\varphi}$ we find from (1) that the phase and concentration in the discharge region ($|E|$ = const) are the solutions of the following equations:

$$(\nabla\varphi)^2=k^2(1-n), \tag{2a}$$

$$\Delta\varphi=\delta k^2 n. \tag{2b}$$

These equations can be readily rewritten in the form of a single equation for the phase:

$$\Delta\varphi+\delta(\nabla\varphi)^2=\delta k^2. \tag{2c}$$

As can be seen, the equation which the phase satisfies in the geometric-optics approximation becomes exact in the discharge region. It follows from (2a) that the electron concentration in the discharge cannot exceed the critical value: $n \leq 1$ [this is, of course, valid only for these particular discharge conditions within the single-scalar description provided by (1)]. Outside the discharge region the fields are described by (1) with $\epsilon = 1$, and should be related to the internal field by the condition of continuity for the tangential components on the boundary, the position of which is not, of course, known in advance and is determined in the final analysis [together with the structure of n(r)] by the distribution and strength of the external sources.

We shall now construct the solution of (2) for plane layered, cylindrically symmetric, and spherically symmetric distributions of n and φ, thus obtaining the answer to the question whether the initial equation (1) can be looked upon as exact only for TE waves in plane layered structures and waves with electric field $E = E_z$ parallel to the z axis of the cylindrical set of coordinates. In so far as spherical waves and cylindrical waves with azimuthal electric field $E = E_\varphi$ are concerned, the use of (1) in these cases signifies transformation to the approximate scalar model which appears to be adequate in some regions of space for the vector problems which we are considering when the wavelength is short.

For the above three problems (plane, cylindrical, and spherical) we can eliminate the phase from (2) and obtain a single nonlinear equation for the concentration:

$$\frac{1}{r^\alpha}\frac{d}{dr}[r^\alpha(1-n)^{1/2}]=\delta kn. \qquad (3)$$

In this expression, r is the distance to the plane, axis, or center, and the coefficient α is respectively equal to 0, 1, 2; the choice of the positive sign in front of the root corresponds to the propagation of the wave in the direction of decreasing r.

Solving the resulting equation for $0 \leq n \leq 1$ for each of the three cases we obtain:

a) for the plane parallel discharge ($\alpha = 0$, $r \to x$)

$$n=\mathrm{ch}^{-2}\delta kx, \qquad (4a)$$

where x is the Cartesian coordinate,

b) for a discharge in a converging symmetric cylindrical wave ($\alpha = 1$)

$$n=1-I_1^2(\delta kr)/I_0^2(\delta kr), \qquad (4b)$$

where I_0, I_1 are the modified Bessel functions, and

c) for a discharge in a converging symmetric spherical wave ($\alpha = 2$)

$$n=1-(\mathrm{cth}\,\delta kr-1/\delta kr)^2. \qquad (4c)$$

In all three cases the concentration decreases monotonically from the critical value ($n = 1$) at $r = 0$ down to zero for $r = \infty$ with the same characteristic linear fall parameter $L = 1/k\delta = c/\nu$. When $r \ll L$ we have

$$n=1-[\delta kr/(\alpha+1)]^2$$

and when $r \gg L$

$$n=\alpha/\delta kr,\quad \alpha=1,\ 2;\quad n=4e^{-2\delta kr},\quad \alpha=0.$$

It is readily verified that the characteristic scale for the discharge satisfies the above condition that the diffusion lengths are small (for air), $2pL \gg 1$. In point of fact, since in air $\nu \approx (3\text{–}5)\times 10^9 p$,[7] we find from the expression $L = c/\nu$ that $2pL \approx 12\text{–}20$.

The above solutions can be generalized to the case of oblique incidence of waves on a surface of equal values of n. Thus, for plane waves with phase given by $\varphi = \psi(x) + ky\sin\theta$ (x, y are the cartesian coordinates and θ is the angle of incidence), the solution (4a) becomes

$$n(x)=\cos^2\theta/\mathrm{ch}^2(\delta kx\cos\theta), \qquad (5)$$

i.e., the maximum relative concentration of electrons falls to $n(0) = \cos^2\theta$ and the characteristic scale of the discharge $L = 1/k\delta\cos\theta$ increases.

We now have the important problem as to whether the above distributions can be matched to a region which is not occupied by the discharge over a finite distance. It is clear that, if there is no diffusion, this matching can be achieved by cutting off the above structures at any $r = r_0$ [$n(r) \equiv 0$ for $r > r_0$] and joining the fields on the boundary. Since inside the discharge ($r < r_0$) we have $|E| = E_b = \text{const}$, and the phase is given by (2a), the requirement that the field E and its derivative with respect to r are continuous at $r = r_0$ leads to the following boundary conditions for the modulus and phase in the external region:

$$|E|=E_b,\quad d|E|/dr=0,\quad d\varphi/dr=k(1-n(r_0))^{1/2}. \qquad (6)$$

When $r > r_0$ the field is a superposition of incident (converging) and reflected (diverging) waves, the resultant amplitude of which is an oscillating function of r but does not exceed the breakdown value E_b anywhere, including the maxima. For spherical and cylindrical waves the absolute maximum of the amplitude E_b is reached on the boundary, and the subsequent maxima (for $r > r_0$) are smaller. For plane waves, the maxima are equal ($|E|_m = E_b$) and are repeated with a period of one-half of the wavelength.

The boundary conditions (6) enable us to relate the coordinate of the boundary (r_0) and the amplitude or intensity of the incident wave, and to determine the fraction of the power Q absorbed by the discharge. In particular, when $kr_0 \gg 1$ and if the incident wave is written in the form

$$E=Ar^{-\alpha/2}e^{i(\omega t+kr)},$$

we have

$$\frac{2A}{r_0^{\alpha/2}[1+(1-n(r_0))^{1/2}]}=E_b,\quad Q=\frac{4(1-n(r_0))^{1/2}}{[1+(1-n(r_0))^{1/2}]^2}. \qquad (7)$$

As the incident-wave amplitude increases, the region occupied by the discharge expands and the concentration $n(r_0)$ on the boundary decreases while the fraction of absorbed power approaches 100%. We note that time-independent plane-layered solutions ($\alpha = 0$) exist only when plane waves are incident on the discharge on both sides and their amplitudes E_0 satisfy the conditions $E_b \geq E_0 \geq E_b/2$.

We have thus considered the simplest possible discharge configurations in the field of converging electromagnetic waves whose sources are located in the extraneous region (where the field is less than the breakdown value). The symmetric structures which we have found can probably be regarded as models of a 'cold' diffusionless discharge excited in the wave trough of a symmetric mode of a cavity resonator, or in the focal region of a highly convergent wave beam generated by a short focal length lens or mirror.

The time spent by the discharge in the above nonequilibrium state ($T_e \gg T_m$) is characterized by the time τ necessary for transferring the energy from electrons to neutral particles. Without going into the details of the very complicated processes involved in the redistribution of energy between the various degrees of freedom of a gas, we shall confine our attention to the approximate estimate of τ, regarded as the relaxation time for excitation in a cold gas containing hot electrons. The approximate formula is

$$\tau=1/\sigma N v_e \sim 10^9/N_c.$$

The excitation cross section σ and the mean electron velocity v_e can be assigned the values $\sigma \sim 10^{-17}$ cm^2 and $v_e \sim 10^8$ cm/sec (for $|E| \approx E_b$), which are typical for discharge temperatures. The electron concentration N is replaced by the maximum critical value for the given discharge $N_c = m(\omega^2 + \nu^2)/4\pi e^2$. For frequencies $\omega \sim 2\times 10^{10}\text{–}6\times 10^{10}$ sec^{-1} (and $\nu \ll \omega$) we have $N_e \sim 10^{11}\text{–}10^{12}$ cm^{-3} and $\tau \sim 10^{-2}\text{–}10^{-3}$ sec. The cooling of the gas by thermal conduction and free convection is characterized by long times for the above discharge scale $L = c/\nu$. Hence the heating of the gas in these examples can be neglected only for microwave pulse lengths less than the above figures of 10^{-2}–10^{-3} sec.

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[1)] We shall not consider the important problem of discharge stability because it is essential first to investigate the possible time-independent states.

[2)] The problem which we have formulated can be classified as a strongly-nonlinear one with a given level of nonlinear restriction on the amplitude ($|E| \leqslant E_b$).

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