Nonlinear theory of thermonuclear Alfven instability in plasma

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The nonlinear stage of the thermonuclear Alfven instability in plasma, i.e., the instability excited by high-energy ions produced in the course of thermonuclear reactions, is discussed. It is shown that the instability results in the directed motion of deuterium and tritium ions toward the center of the plasma column and that a radial electric field appears near the plasma boundary.

1. INTRODUCTION

When thermonuclear systems are operated under reactor conditions, or conditions close to them, high-energy ions from thermonuclear reactions (α particles, if the plasma consists of a deuterium-tritium mixture) appear in the confinement region and remain there. This increases the deviation of the system from thermodynamic equilibrium and, as the concentration n_α of the reaction products increases, 'thermonuclear' instabilities develop in the plasma.[1-8] These instabilities may arise when the relative concentration of the high-energy ions is very low ($n_\alpha^{cr}/n \ll 1$) and, therefore, in the analysis of plasma instability the initial distribution function can be taken to be an isotropic and monoenergetic, or almost monoenergetic, function, i.e., $f_\alpha(\mathbf{v}) \sim \delta(v-u)$, or

$$f_\alpha(\mathrm{v}) \sim \frac{1}{v}\left[\exp\left\{-\frac{m_0(v-u)^2}{2T}\right\}-\exp\left\{-\frac{m_0(v+u)^2}{2T}\right\}\right],$$

where T and m_0 are, respectively, the temperature and sum of masses of the reacting ions, and u is the root mean square velocity of the α particles [1].

It seemed correct to assume until now that the instabilities considered previously [1-8] cannot lead to appreciable macroscopic effects (see, for example, [3]). However, the rigorous analysis of the nonlinear development of thermonuclear Alfven instabilities given in the present paper has shown that plasma containing α particles exhibits turbulence with certain very specific properties. In particular, the interaction between particles and waves gives rise to a directed motion of the deuterium and tritium ions toward the center of the plasma column, whilst the α-particle flux $n_\alpha V_\alpha$ and electron flux $n_e V_e$ are directed toward the periphery of the plasma and are such that $n_\alpha V_\alpha > n_e V_e$, $\Sigma e_j n_j V_j = 0$ ($j = e, i, \alpha$). Therefore, the radial component of the electric current in the plasma vanishes because the motion of the ions and α particles takes place in opposite directions. However, near the plasma boundary, this effect cannot ensure that the current becomes zero, and this leads to the appearance of a radial electric field which may, in fact, be responsible for MHD plasma instabilities.

It is also shown in this paper that turbulent fluxes of α particles and ions are substantially greater than those due to classical diffusion. An expression is established for the size of plasma for which the number of α particles produced in fusion reactions is equal to the number of these particles which escape as a result of anomalous diffusion. When the size of the plasma column is less than the quantity given by this expression, the number of α particles will eventually approach n_α^{cr}.

We shall use the linear theory to analyze plasma stability [4]. We start by summarizing the main results. The expression for the imaginary part of the Alfven wave (γ^L) in plasma containing thermonuclear reaction products is given by

$$\gamma^L=\gamma_e^M+\gamma_\alpha^\delta=-\frac{1}{2}\left(\frac{\pi}{2}\right)^{1/2}\frac{m_e}{m_i}\frac{k_\perp^2 v_A^2}{\omega_{Hi}^2}|k_\parallel| v_e - \frac{\pi}{4}\frac{n_\alpha}{n}\frac{m_\alpha}{m_i}\frac{\omega_{H\alpha}^2}{|k_\parallel| u}\sum_l \frac{l^2}{z_l}\frac{\partial J_l^2(z_l)}{\partial z_l}. \tag{1}$$

In this expression γ_e^M is the attenuation of the wave by electrons with a Maxwellian distribution function, and γ_α^δ is the growth or attenuation of the wave by α-particles with the distribution function $f_\alpha(v) \sim \delta(v-u)$,

$$z_l=\frac{k_\perp u}{\omega_{H\alpha}}\left(1-\frac{l^2\omega_{H\alpha}^2}{k_\parallel^2 u^2}\right)^{1/2};$$

$J_l(z_l)$ is the Bessel function of order l, v_i and v_e are the thermal velocities of the ions and electrons, respectively, v_A is the Alfven velocity, and ω_{Hi} and $\omega_{H\alpha}$ are the cyclotron frequencies of the reacting ions and reaction products.

The instability in which we are interested appears in the plasma whose parameters satisfy the inequalities

$$v_* \ll v_A \ll v_e,\quad v_A<u,\quad v_*=(T_e/m_i)^{1/2}, \tag{2}$$

if the relative concentration of the reaction products n_α/n approaches the critical value n_α^{cr}/n for which $\gamma^L = 0$. It follows from (1) that

$$\frac{n_\alpha^{cr}}{n} \cong 4\cdot 10^{-6}\frac{1}{\beta}\left(\frac{v_e}{u}\right)^3\left(1+\frac{T_i}{T_e}\right), \tag{3}$$

where $\beta = 8\pi n(T_e + T_i)/H^2$. This is accompanied by the growth of oscillations with wave vectors $k_\perp \sim k_\parallel \gtrsim \omega_{H\alpha}/u$. For plasma parameters which, according to modern ideas, are necessary for the operation of a toroidal thermonuclear reactor, using a balanced deuterium-tritium mixture,[9] the critical relative concentration is

$$n_\alpha^{cr}/n \sim 10^{-3}. \tag{4}$$

2. BASIC EQUATIONS

The spectrum of oscillations which occur during the development of thermonuclear instabilities is, as a rule, quite broad. This enables us to assume that the instability will take the plasma to a weakly turbulent state. Therefore, when the effect of thermonuclear instabilities on transport processes in plasma is investigated, we shall start with the set of kinetic equations for the waves and the quasilinear equations for the particle distribution functions. In the case of Alfven instability these can be written in the following form ($\beta = \alpha$, e):

$$\frac{\partial f_\beta}{\partial t}=\pi\frac{e_\beta^2}{m_\beta^2}\sum_{\mathbf{k},l}\frac{E_{\mathbf{k}\gamma}E_{\mathbf{k}\varepsilon}^*}{\omega_{\mathbf{k}}^2}\hat{\Pi}_0 q_\varepsilon^* q_\gamma \delta(\omega_{\mathbf{k}}-k_\parallel v_\parallel - l\omega_{H\beta})\hat{\Pi}_0 f_{0\beta}+\mathrm{St}_\beta^{coll}, \tag{5}$$

$$\frac{\partial f_i}{\partial t}=-\frac{e_i^2}{m_i^2}\sum_{\mathbf{k},l}\frac{E_{\mathbf{k}\gamma}E_{\mathbf{k}s}^*}{\omega_{\mathbf{k}}^2}[\hat{\Pi}_0 q_s^*+\hat{\Pi}_{\alpha s}(\omega_{\mathbf{k}}-k_\parallel v_\parallel-l\omega_{Hi})]\gamma_{\mathbf{k}}\frac{\partial}{\partial\omega}$$

$$\times\frac{\mathrm{P}}{\omega-k_\parallel v_\parallel-l\omega_{Hi}}[q_\gamma\hat{\Pi}_0+(\omega_{\mathbf{k}}-k_\parallel v_\parallel-l\omega_{Hi})q_\alpha\hat{\Pi}_{\alpha\gamma}]f_{0i}+\mathrm{St}_i^{coll}, \tag{6}$$

$$\frac{\partial N_{\mathbf{k}}}{\partial t}=2\gamma_{\mathbf{k}}N_{\mathbf{k}}+4\pi\int\frac{d\mathbf{k}'}{8\pi^3}\int\frac{d\mathbf{k}''}{8\pi^3}|V_{\mathbf{k},\mathbf{k}',\mathbf{k}''}|^2\{N_{\mathbf{k}'}N_{\mathbf{k}''}-$$

$$-N_{\mathbf{k}}N_{\mathbf{k}'}\,\mathrm{sign}(\omega_{\mathbf{k}}\omega_{\mathbf{k}''})-N_{\mathbf{k}}N_{\mathbf{k}''}\,\mathrm{sign}(\omega_{\mathbf{k}}\omega_{\mathbf{k}'})\}\cdot \tag{7}$$

$$\times\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}'}-\omega_{\mathbf{k}''})\delta(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')+\Delta_k,$$

$$N_{\mathbf{k}}=\frac{1}{16\pi}|E_{\mathbf{k}}|^2 e_i e_j^*\frac{\partial\Lambda_{ij}}{\partial\omega}. \tag{8}$$

In these expressions

$$\hat{\Pi}_0=k_\parallel\frac{\partial}{\partial v_\parallel}+\frac{l\omega_H}{v_\perp}\frac{\partial}{\partial v_\perp}-\frac{k_x}{\omega_H}\frac{\partial}{\partial y},$$

$$\hat{\Pi}_{\alpha\gamma}=\frac{\delta_{\alpha\gamma}-\delta_{\alpha z}\delta_{\gamma z}}{v_\perp}\frac{\partial}{\partial v_\perp}+\frac{\delta_{\alpha z}\delta_{\gamma z}}{v_\parallel}\frac{\partial}{\partial v_\parallel}-\frac{\delta_{\alpha z}\delta_{\gamma x}}{v_\parallel\omega_H}\frac{\partial}{\partial y},$$

$$\mathbf{q}=\left\{\frac{lv_\perp}{\xi}J_l(\xi),\ iv_\perp J_l'(\xi),\ v_\parallel J_l(\xi)\right\},\quad J_l'(\xi)=\frac{\partial J_l}{\partial\xi},\quad \xi=\frac{k_\perp v_\perp}{\omega_H}, \tag{9}$$

$$\Lambda_{ij}(\omega,\mathbf{k})=\frac{k^2c^2}{\omega^2}\left(\frac{k_ik_j}{k^2}-\delta_{ij}\right)+\varepsilon_{ij}(\omega,\mathbf{k}),$$

$$\gamma_\beta=\frac{4\pi^2e_\beta^2}{m_\beta}\left(\frac{\partial}{\partial\omega}\omega^2\varepsilon_{\mathbf{k}}\right)^{-1}_{\omega=\omega_{\mathbf{k}}}\sum_l\int d\mathbf{v}|\mathbf{qe}|^2\delta(\omega_{\mathbf{k}}-k_\parallel v_\parallel-l\omega_{H\beta})\hat{\Pi}_0 f_{0\beta}, \tag{10}$$

$$\varepsilon_{\mathbf{k}}=e_i\varepsilon_{ij}e_j^*\cong\varepsilon_{\mathbf{k}}^t=c^2/v_A^2,$$

$\gamma_{\mathbf{k}}=\gamma_e+\gamma_\alpha$ is the quasilinear instability growth rate and $\mathbf{e}$ is the polarization vector. The term $\mathrm{St}_\alpha^{\mathrm{coll}}$ describes the scattering of the α particles by plasma electrons and also the creation of α-particles during the thermonuclear reactions, and $\mathrm{St}_e^{\mathrm{coll}}$ and $\mathrm{St}_i^{\mathrm{coll}}$ are collisional terms describing scattering during Coulomb encounters of electrons and ions respectively. Spontaneous emission of waves by plasma particles is represented by the term $\Delta_{\mathbf{k}}$.[6]

The quasilinear equations (5) and (6) are consistent with the inequalities given by (2), according to which the electrons and α particles interact with the waves mainly in a resonant fashion whilst the ions interact adiabatically. Moreover, these equations contain collisional terms which are essential for the determination of the turbulent fluxes in the case of quasilinear relaxation of the distribution functions. Allowance for spontaneous effects in quasilinear equations under the conditions prevailing in developed turbulence ($N_{\mathbf{k}}\gg N_T=T/\omega_{\mathbf{k}}$) is unnecessary.

In the kinetic equation for the waves, given by (7), we have allowed for the stimulated scattering of waves by waves and for the stimulated and spontaneous emission of waves by plasma particles. The last process is important when the system passes through the stability boundary. We note that the spontaneous scattering of waves by waves can be neglected because for $\gamma_{\mathbf{k}}\gtrsim 0$ we have $N_{\mathbf{k}}\gg N_T$. We can also neglect stimulated scattering of waves by particles because the characteristic time for the interaction in the case of stimulated scattering [10] is

$$\tau^N\cong nm_iv_A^2/\omega_{\mathbf{k}}\sum_{\mathbf{k}}\omega_{\mathbf{k}}N_{\mathbf{k}}$$

which is lower by a factor of $\beta^{-1/2}$ in comparison with the case of decay.

3. MACROSCOPIC EFFECTS DUE TO THERMONUCLEAR ALFVEN INSTABILITY

Let us now consider the quasilinear equations (5) and (6) for regular parts of the distribution functions for electrons, ions, and α particles. Integrating these equations in velocity space, we obtain the following expressions for the turbulent particle fluxes across the magnetic field:

$$n_\beta V_\beta=\frac{c}{4\pi e_\beta H}\sum_{\mathbf{k}}\frac{k_x}{\omega_{\mathbf{k}}^2}\gamma_{\mathbf{k}}^\beta|\mathbf{E}_{\mathbf{k}}|^2\frac{\partial}{\partial\omega}\omega^2\varepsilon_{\mathbf{k}}\Big|_{\omega=\omega_{\mathbf{k}}}, \tag{11}$$

$$n_iV_i=\frac{c}{4\pi e_iH}\sum_{\mathbf{k}}\frac{k_x}{\omega_{\mathbf{k}}^2}\gamma_{\mathbf{k}}|\mathbf{E}_{\mathbf{k}}|^2\frac{\partial}{\partial\omega}\omega^2\varepsilon_{\mathbf{k}}\Big|_{\omega=\omega_{\mathbf{k}}}, \tag{12}$$

where V_e, V_α, V_i are the hydrodynamic velocities of the electrons, α particles, and ions, and $\gamma_{\mathbf{k}}^\beta$ is given by (10).

It follows from (8) and (9) that the electric current across the magnetic field is zero. Moreover, it also follows that, for homogeneous plasma, since the spectrum of the excited waves is symmetric (in $\mathbf{k}$), the turbulent particle fluxes are absent. The presence of weak inhomogeneity leads to the appearance of asymmetric corrections $\delta|\mathbf{E_k}|^2$ and $\delta\gamma_{\mathbf{k}}$ because the fluxes given by (11) and (12) are then not zero. When the corrections are determined, it is sufficient to take into account the inhomogeneity in the distribution of the α particles because $\delta\gamma_{\mathbf{k}}^\alpha/\delta\gamma_{\mathbf{k}}^e\sim m_\alpha u^2/T_e\gg 1$. Using this result together with (7), we can readily show that, for steady-state oscillations, $\delta N_{\mathbf{k}}=N_{\mathbf{k}}\delta\gamma_{\mathbf{k}}^\alpha/\gamma_{\mathbf{k}}$. Equations (11) and (12) therefore assume the following form:

$$n_\alpha V_\alpha=\frac{c}{4\pi e_\alpha H}\sum_{\mathbf{k}}\frac{k_x}{\omega_{\mathbf{k}}^2}\delta\gamma_{\mathbf{k}}^\alpha\left(1+\frac{\gamma_{\mathbf{k}}^\alpha}{\gamma_{\mathbf{k}}}\right)|\mathbf{E}_{\mathbf{k}}|^2\frac{\partial}{\partial\omega}\omega^2\varepsilon_{\mathbf{k}}\Big|_{\omega=\omega_{\mathbf{k}}}, \tag{13}$$

$$n_eV_e=\frac{c}{4\pi eH}\sum_{\mathbf{k}}\frac{k_x}{\omega_{\mathbf{k}}^2}\delta\gamma_{\mathbf{k}}^\alpha\frac{\gamma_{\mathbf{k}}^e}{\gamma_{\mathbf{k}}}|\mathbf{E}_{\mathbf{k}}|^2\frac{\partial}{\partial\omega}\omega^2\varepsilon_{\mathbf{k}}\Big|_{\omega=\omega_{\mathbf{k}}}, \tag{14}$$

$$n_iV_i=-\frac{c}{2\pi e_iH}\sum_{\mathbf{k}}\frac{k_x}{\omega_{\mathbf{k}}^2}\delta\gamma_{\mathbf{k}}^\alpha|\mathbf{E}_{\mathbf{k}}|^2\frac{\partial}{\partial\omega}\omega^2\varepsilon_{\mathbf{k}}\Big|_{\omega=\omega_{\mathbf{k}}}; \tag{15}$$

$$\delta\gamma_{\mathbf{k}}^\alpha=-\frac{4\pi^2e_\alpha^2}{m_\alpha}\left(\frac{\partial}{\partial\omega}\omega^2\varepsilon_{\mathbf{k}}\right)^{-1}_{\omega=\omega_{\mathbf{k}}}\sum_l\int d\mathbf{v}\frac{k_x}{\omega_{H\alpha}}\frac{\partial}{\partial y}|\mathbf{qe}|^2\delta(\omega_{\mathbf{k}}-k_\parallel v_\parallel$$

$$-l\omega_{H\alpha})f_{0\alpha}=-\frac{\pi}{4}\frac{n_\alpha}{n}\frac{m_\alpha}{m_i}\frac{k_xk_{0\alpha}}{k_\parallel^2}\frac{\omega_{H\alpha}^3}{k_\perp^2v_Au}\sum_l l^2J_l^2(z_l),\quad k_{0\alpha}=\frac{1}{n_\alpha}\frac{dn_\alpha}{dy} \tag{16}$$

Equations (13)–(15) give the turbulent particle fluxes when the amplitudes of turbulent pulsations and the quasilinear instability growth rate are known. These quantities can be found from (5)–(7). Let us begin with (7). This equation describes the development of the time-independent oscillation spectrum in the course of the competition between two processes, namely, the emission of Alfven waves by the α particles and the stimulated scattering of these waves leading to the absorption of electromagnetic energy by plasma electrons. The most probable are the three-plasmon decay processes with the participation of slow magnetosonic waves (when $T_e\gg T_i$): $A\rightleftarrows A+S$ and $A\rightleftarrows M+S$, where A, M, S are, respectively, the Alfven wave, the fast wave, and the slow magnetosonic wave. Equation (7) therefore leads to the following estimate for the steady-state turbulent-pulsation amplitude:

$$N_{\mathbf{k}}^A\cong\frac{\gamma_{\mathbf{k}}\omega_A}{2\pi\Gamma|V_{A,A',s}|^2}, \tag{17}$$

where

$$\Gamma=\frac{1}{2\pi}|k_\parallel|k_\perp^2\cong\frac{1}{2\pi}\frac{\omega_{H\alpha}^3}{u^3},\qquad |V_{A,A',s}|^2=\frac{k_\parallel^3v_A^2}{16nm_iv_s}$$

is the phase volume and matrix eleme[illegible] for the interaction of the waves A, A′, and S.

The amplitude $N_{\mathbf{k}}^A$ is calculated on the assumption that the nonlinear term in (7) can be approximated as follows:

$$4\pi\int\frac{dk'}{8\pi^3}\int\frac{dk''}{8\pi^3}|V_{A,A',s}|^2\{N_{A'}N_s-N_AN_{A'}-N_AN_s\}$$
$$\times\delta(\omega_A-\omega_{A'}-\omega_s)\delta(\mathbf{k}_A-\mathbf{k}_{A'}-\mathbf{k}_s)\cong-\frac{4\pi\Gamma|V_{A,A',s}|^2}{\omega_A}N_AN_{A'}.$$

Since this approximation does in fact assume the maximum possible flow of energy into the attenuation region (the reverse energy flux connected with the coalescence of waves is ignored), Eq. 17 gives an underestimate for N_k^A. However, even with this value of N_k^A the quasilinear relaxation of the distribution functions for the α particles and electrons is quite appreciable. In fact, it follows from the quasilinear equations (5) and (6) that the γ_k, in (17) is equal to the linear instability growth rate when

$$\nu_e\gg\frac{c^2}{H^2}\sum_k\frac{k_\perp^2v_e^2}{\omega_{Hi}^2}\frac{k_\parallel^2}{\omega_k}|\mathbf{E}_k|^2, \tag{18}$$

$$\tau_f^{-1}\gg2\pi\frac{c^2}{H^2}\sum_k\frac{k_\perp^2}{\omega_{H\alpha}}\frac{n_\alpha^{cr}}{n}|\mathbf{E}_k|^2, \tag{19}$$

where ν_e is the electron collision frequency, $\tau_f = 4/n\langle\sigma v\rangle$, and $\langle\sigma v\rangle$ is the product of the cross section for the thermonuclear reactions and the relative velocity of the reacting ions averaged over the Maxwellian distribution. However, when the amplitude of the turbulent pulsations is given by (17) and the plasma parameters are [9]

$$T=15\,\mathrm{keV}\; n=3\cdot10^{14}\;\mathrm{cm}^{-3},\; H=40\;\mathrm{kG} \tag{20}$$

the inequalities given by (18) and (19) are not satisfied, and the opposite inequalities are valid. The development of the thermonuclear Alfven instability is therefore accompanied by the quasilinear relaxation of the distribution functions f_α and f_e, which leads to a reduction in the instability growth rate.

The quasilinear instability growth rate can be determined by perturbation theory because, in accordance with the foregoing $St_e^{coll}\ll St_e^{QL}$ and $St_\alpha^{coll}\ll St_\alpha^{QL}$ where St^{QL} is the quasilinear collisional term. (This method was first used in [11,12] to investigate the nonlinear stage of instability.) Let us write the particle distribution function in the form $f_\beta = f_\beta^{(0)}+f_\beta^{(1)}$, $f_\beta^{(1)}\ll f_\beta^{(0)}$. In the steady state, the corrections to the electron and α-particle distribution functions, which ensure that the quasilinear instability growth rate is not zero, are then determined by

$$St^{QL}\{f_\beta^{(1)}\}+St^{coll}\{f_\beta^{(0)}\}=0, \tag{21}$$

where the distribution function $f_\beta^{(0)}$ is a solution of the equation $St^{QL}\{f_\beta^{(0)}\}=0$. Integrating (21) in velocity space, we obtain

$$\sum_k\gamma_k^e|\mathbf{E}_k|^2=-\frac{1}{2}\left(\frac{\pi}{2}\right)^{1/2}\frac{m_e}{m_i}\nu_e\left(\frac{v_A}{c}\right)^2\frac{v_A}{v_e}H^2, \tag{22}$$

$$\sum_k\gamma_k^\alpha|\mathbf{E}_k|^2=2\pi^2\frac{m_\alpha nv_Au}{\tau_f}\left(\frac{v_A}{c}\right)^2. \tag{23}$$

It follows from the last two equations that, in plasma with the parameters given by (20), the quasilinear relaxation results in the attainment of the steady state with $\gamma_k^\alpha\gg\gamma_k^e$. Using this fact and the relations given by (8), (17), (22), and (23), we can readily show that the amplitude of turbulent pulsations is

$$|\mathbf{E}_k|^2\cong2\pi\frac{v_A^2}{c^2}\frac{\omega_A}{\Gamma|V_{A,A',s}|}\left(\frac{m_\alpha nv_Au}{\tau_f}\right)^{1/2}=4\pi\frac{H^2}{c^2}\frac{\beta^{1/4}v_A\rho_\alpha^4}{\tau_f}(\omega_{H\alpha}\tau_f)^{1/2}, \tag{24}$$

where $\rho_\alpha = u/\omega_{H\alpha}$ is the Larmor radius of the α particles. It is clear from (24) that the steady-state oscillation amplitude is a function of τ_f which characterizes the rate of production of the α particles.

Substituting (22)–(24) into (13)–(15), we obtain the turbulent particle fluxes in terms of the plasma parameters:

$$n_\alpha V_\alpha\cong-\frac{e_i}{e_\alpha}n_iV_i=-2\frac{\beta^{1/4}}{(\omega_{H\alpha}\tau_f)^{1/2}}\frac{u}{v_A}\rho_\alpha^2\omega_{H\alpha}\frac{\partial n_\alpha}{\partial y}, \tag{25}$$

$$n_eV_e=-\frac{\beta^{1/4}}{(\omega_{H\alpha}\tau_f)^{1/2}}v_e\tau_f\frac{v_A}{v_e}\frac{u^2}{\omega_{He}}\frac{\partial n_\alpha}{\partial y}\ll n_\alpha V_\alpha. \tag{26}$$

It is clear from the last two equations that the turbulent flux of ions is in the direction of the center of the plasma column. This flux exceeds the outward ion flux due to classical diffusion by a factor of $\omega_{He}\tau_{ei}/(\omega_{H\alpha}\tau_f)^{1/2}$, i.e., by a factor of 100–1000. Consequently, the development of the thermonuclear Alfven instability is accompanied by the pinching of the ions which in turn ensures that the radial component of the electric current is zero. It is, however, obvious that, near the plasma boundary, the current can be made zero in this way only in the presence of special ion injection. In the opposite case, we should see the appearance of an ambipolar electric field, leading to the development of centrifugal and slipping instabilities.

It is interesting to compare the turbulent α-particle flux obtained above with the flux

$$n_\alpha V_\alpha^0=\frac{1}{a}\int\frac{n^2}{4}\langle\sigma v\rangle r\,dr=\frac{\delta_1an}{2\tau_f} \tag{27}$$

for which the number of α particles created in the volume is equal to the number of these particles escaping from the system (a is the radius of the plasma column and $\delta_1<1$). This comparison shows that, when the radius of the plasma is determined by

$$\left(\frac{a_0}{\rho_\alpha}\right)^2\cong10^{-2}\beta^{1/4}(\omega_{H\alpha}\tau_f)^{1/2}\frac{v_ev_A}{u^2}, \tag{28}$$

we have $n_\alpha V_\alpha = n_\alpha V_\alpha^0$. For the above plasma parameters, $a_0\sim100$ cm. If the transverse size of the plasma is less than a_0, the concentration of the thermonuclear reaction products will eventually approach n_α^{cr}. If, on the other hand, the plasma size is greater than a_0, further accumulation of α particles will take place. Nevertheless, the diffusion of plasma will, as before, be described by (25) and (26).

We have considered the effect of thermonuclear Alfven instability on the diffusion of nonisothermal plasma and the reaction products. When $T_i\sim T_e$, a possible nonlinear mechanism governing the amplitude of the steady-state oscillations appears to be the coalescence of Alfven waves into attenuating fast magnetosonic waves. Since the probability of this nonlinear interaction is lower by a factor of $\beta^{-1/2}$ than the interaction with the participation of slow magnetosonic waves, the energy of the steady-state oscillations in this case will be greater by a factor of $\beta^{-1/4}$. Consequently, as in the case when $T_e\gg T_i$, the inequalities given by (18) and (19) will not be satisfied, and turbulent diffusion will be described by (25) and (26).

In conclusion we note that the macroscopic effects derived in this paper will probably also appear during the development of certain other thermonuclear instabilities.

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174