Nonlinear penetration of a plasma by an electromagnetic wave and nondecay mechanisms of its energy dissipation

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A nonlinear solution is otbained which describes the penetration of an electromagnetic wave at the oblique incidence into an inhomogeneous plasma, in the case of sufficiently large amplitudes, where allowance for the wave pressure is important. It is demonstrated that penetration of the wave beyond a critical point $\omega_p(z)=\omega$ is accompanied by its transformation into plasma oscillations; the corresponding transformation coefficient is found. Two nonlinear mechanisms of electromagnetic energy dissipation are investigated for the case of normal incidence. The mechanisms appear in a plasma confined by the wave pressure and involve excitation of plasma oscillations in the vicinity of the point $\omega_p=2\omega$ and absorption of the wave energy by particles in the skin layer.

1. INTRODUCTION

The problem of the absorption mechanisms of electromagnetic radiation in an inhomogeneous plasma acquires considerable significance in connection with the problem of initiation of thermonuclear reactions in a D—T target with the aid of a powerful laser.[1] The electromagnetic wave, propagating in the direction of increasing density, is reflected from a critical point $\omega \approx \omega_p(z)$. Dissipation of the energy of the wave is connected basically with parametric instability of the electromagnetic radiation, which leads to excitation of ion-acoustic and Langmuir oscillations in the plasma.[2] Along with this, additional absorption should occur on oblique incidence of the electromagnetic wave, due to transformation of a part of its energy into Langmuir oscillations in the vicinity of the point of plasma resonance $\epsilon(\omega, z) = 0$.[3]

Such a transformation is usually insignificant due to subbarrier damping of the field from the reflection point $\epsilon = \sin^2\theta$ up to the transformation point $\epsilon = 0$; however, for small angles $\theta \sim (c/\omega L)^{1/3}$ (L is the density gradient), for which such attenuation is absent, the transformation coefficient calculated from linear theory (the ratio of the energy flux in the plasma oscillations to the energy flux in the incident wave) turns out to be comparable with unity, $R_{max} \approx 0.4$.[4] Here the longitudinal electric field in the vicinity of the transformation point $z_0 \sim (Lv_T^2\omega^2)^{1/3}$ increases to a very large quantity

$$E_z \sim H_0 \left(\frac{L\omega}{v_T}\right)^{1/4} \left(\frac{c}{2\pi\omega L}\right)^{1/2} \tag{1.1}$$

(H_0 is the magnetic field in the incident wave, v_T the thermal velocity) and even at small amplitudes of the electromagnetic wave

$$H_0^2/16\pi n_0 T \gg 2\pi v_T^2/c\omega L \tag{1.2}$$

(a condition definitely satisfied in all experiments on the heating of plasma by laser radiation), account of the wave pressure becomes significant in the transformation region1) ($E_z^2/16\pi n_0 T \sim \epsilon$).

The problem of transformation of the wave into plasma oscillations then becomes essentially nonlinear, since the distribution of the particles of the plasma n(z) in the vicinity of the point of plasma resonance is determined by the wave. The wave displaces the particles and the region of resonance is shifted into the depth of the plasma. The nonlinear solution which describes the penetration of the wave in oblique incidence into the region $\epsilon_L < 0$ is obtained in Sec. 2 (ϵ_L is the linear dielectric constant, which is determined by the wave-independent distribution of the density of particles). In this solution, the field at the front of the wave increases to a value at which its pressure is sufficient for displacement of the plasma, $H_m^2 \sim |\epsilon_L|$. Penetration of the wave is accompanied by excitation of a longitudinal field E_z with a transformation coefficient determined by the following formula at small angles $\theta \lesssim (c/\omega L)^{1/3}$:

$$R \approx \frac{16\pi n_0 T|\varepsilon_L|}{H_0^2} \frac{v_T}{c} \left(\frac{m_i}{m_e}\right)^{1/2} \sin^2\theta \tag{1.3}$$

The wave penetrates into the plasma to values $|\epsilon_L|$ at which the transformation coefficient $R \sim 1$. Upon an increase in the amplitude of the incident wave to the value at which

$$H_0^2/16\pi n_0 T \gg \sin^2\theta, \tag{1.4}$$

account of the wave pressure is essential in the vicinity of the reflection point $\epsilon = \sin^2\theta$, and the wave results in nonlinear "transparency" of the barrier after this point. As a result, penetration of the wave takes place for all angles θ which satisfy the condition (1.4). This case is considered in Sec. 3.

At amplitudes of the incident wave

$$H_0^2/16\pi n_0 T \sim 1 \tag{1.5}$$

the pressure in the electromagnetic wave is sufficient to constrain the plasma. A solution has been found[5] which describes the equilibrium between the plasma and a standing electromagnetic wave for strictly normal incidence. For oblique incidence, transformation into plasma oscillations takes place for a self-consistent density gradient determined by the wave, and nonlinear penetration of the wave into the plasma $\epsilon_N < 0$ (ϵ_N is the dielectric constant of the plasma for normal incidence). The corresponding solution is obtained in Sec. 4 of this paper.

Finally, in Sec. 5 we consider some additional mechanisms of dissipation of the electromagnetic energy in a plasma which is confined by the wave pressure—mechanisms which, in contrast to the usual transformations, are preserved even for normal incidence. One such mechanism is connected with the fact that, by virtue of the high-frequency pressure $ev_y H_x/c$ acting on the electrons in the longitudinal direction, along with a time-independent term which guarantees confinement of the plasma, there is a term which varies with a frequency 2ω. In the vicinity of the point $\omega_p(z) = 2\omega$, this force leads to the excitation of longitudinal oscillations;[8] however, as is shown in the present work, the corresponding transformation coefficient contains the small quantity $(v_T/c)^2$:

$$R_{2\omega}=\pi\frac{v_T^2}{c^2}\frac{\omega}{\omega_{p0}}\ln^{1/2}\frac{\omega_{p0}^2}{4\omega^2} \tag{1.6}$$

(ω_{p0} is the Langmuir frequency, calculated at maximum equilibrium plasma density; it is assumed that $\omega \ll \omega_{p0}$).

Another mechanism of dissipation is connected with the collisionless damping of the wave—the transfer of its energy to particles which interact with the profile of the field $f(z)\cos\omega t$. For $v_T/c \ll \omega/\omega_{p0}$, when the time of flight of the particles through the skin layer is much greater than the period of the high-frequency field, the irreversibility in such an energy transfer is due in the main to the presence of a group of particles with sufficiently high velocities, passing through the barrier created by the wave. The number of such particles $\sim\exp(-\omega_{p0}^2/\omega^2)$, and the coefficient of energy dissipation of the wave due to this mechanism also contains the small quantity $\exp(-\omega_{p0}^2/\omega^2)$:

$$R_d=2\frac{\omega_{p0}^2}{\omega^2}\exp\left(-\frac{\omega_{p0}^2}{\omega^2}\right)\left(\frac{T_e}{2\pi m_e c^2}\right)^{1/2}\left(1+\frac{\omega_{p0}^2}{4\omega^2}\right). \tag{1.7}$$

2. NONLINEAR PENETRATION AT SMALL AMPLITUDES

The set of equations which describes the propagation of a wave in an inhomogeneous ($\epsilon = \epsilon(z)$) plasma at an angle θ to the direction of the inhomogeneity is written in the following form (see, for example,[3]):

$$\frac{d^2H_x}{dz^2}-\frac{1}{\varepsilon}\frac{d\varepsilon}{dz}\frac{dH_x}{dz}+\frac{\omega^2}{c^2}\varepsilon H_x=\frac{\omega^2}{c^2}\varepsilon E_z\sin\theta, \tag{2.1}$$

$$\frac{T_e}{m_e\omega^2}\frac{d^2E_z}{dz^2}+\varepsilon E_z=H_x\sin\theta, \tag{2.2}$$

$$E_y=\frac{c}{\omega\varepsilon}\frac{dH_x}{dz}. \tag{2.3}$$

In obtaining this set of equations, it has been assumed that a reflection point is always produced in a wave propagating in the direction of increasing density, with a reflection coefficient sufficiently close to unity so that one can assume the wave in the plasma to be approximately a standing one:[2)]

$$E_y=E_y(z)\cos\Phi,\ H_x=H_x(z)\sin\Phi,\ E_z=E_z(z)\sin\Phi,\quad \Phi=\omega c^{-1}y\sin\theta-\omega t. \tag{2.4}$$

The force of the high-frequency pressure acting on electrons in the wave leads to polarization of the plasma and to the generation of a static electric field $E_0(z)$. In the steady state, the quasineutral perturbation of particle density due to the wave $\delta n(z)$ and the electric field $E_0(z)$ are determined from the condition of equilibrium on the forces acting on the particles of the plasma and are given by

$$\delta n=-\frac{n_0e^2}{4m_e\omega^2}\frac{E_y^2+E_z^2}{T_e+T_i},\qquad E_0(z)=\frac{T_i}{en_0}\frac{d\delta n}{dz} \tag{2.5}$$

(T_e, T_i are the temperatures of the electrons and ions, respectively).

Using Eqs. (2.2), (2.3) to relate E_z and E_y with H_x (in the first of these equations, it is assumed that the gradient of the field E_z is not too large and that one can neglect the term $\sim T_e$, which describes the transfer of plasma oscillations), we obtain the following equation (with the help of (2.5)) for the dielectric constant of the plasma in the presence of the wave:

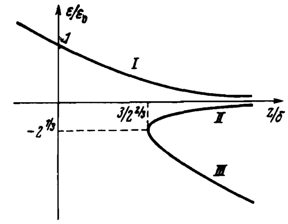

FIG. 1

$$\varepsilon=\varepsilon_L(z)+\frac{\sin^2\theta}{16\pi n_0T\varepsilon^2}\times\left[H_x^2+\frac{c^2}{\omega^2\sin^2\theta}\left(\frac{dH_x}{dz}\right)^2\right], \tag{2.6}$$

where $T = T_e + T_i$; the linear part of the dielectric constant ϵ_L will be approximated below by $\epsilon_L = -z/L$. Equation (2.6) was obtained under the condition that $|\epsilon_L| \ll 1$, and this condition determines the region of applicability of the results of this and the following sections.

In this section, we consider the case of sufficiently small field amplitudes of the incident wave, when the condition

$$\sin^4\theta\gg\frac{H_x^2(0)}{16\pi n_0T}\gg\frac{v_T^2}{\omega^2L^2\sin^2\theta}. \tag{2.7}$$

is satisfied for the magnetic field intensity in the vicinity of the plasma resonance point. In this case, at a sufficiently large distance from the plasma resonance point,

$$-z>\delta=L\sin^2\theta\left[\frac{H_x^2(0)}{16\pi n_0T\sin^4\theta}\right]^{1/3},\qquad \delta\ll L\sin^2\theta$$

the dielectric constant of the plasma is identical with the linear constant, and in agreement with the results of linear theory[3] the magnetic field of the wave is finite for small z ($H_x = H_x(0)$), and its derivative vanishes ($H_x' \sim z\ln z$).

Near the plasma resonance point $|z| \sim \delta$, account of the wave pressure is necessary and we have the cubic equation (2.6) for ϵ. The result of graphical solution of this equation is shown in Fig. 1, where $\epsilon_0 = [H_x^2(0)\sin^2\theta/16\pi n_0T]^{1/3}$, and segments I, II, III are the roots of the cubic equation (2.6). Only the root I, for which $\epsilon > 0$, has physical meaning. It is essential that intersection of the roots of Eq. (2.6) does not occur in the considered case of oblique incidence $\sin^2\theta \gg (v_T/\omega L)^{2/3}$ as well as the jumps in ϵ and the plasma density associated with such intersection. (In the problem of propagation of an electromagnetic wave in an inhomogeneous plasma, according to Ginzburg,[3] allowance for thermal motion is equivalent to allowance for collisions upon the substitution $(v_T/\omega L)^{2/3} \to \nu/\omega$.)[3)]

For $|z| \sim \delta$, the dielectric constant of the plasma $\epsilon \sim \epsilon_0$; in view of the narrowness of this region $|\delta \ll c/\omega\sin\theta|$, the magnetic field of the wave in it remains approximately constant, $H_x \approx H_x(0)$. The most interesting solutions arise in the region of large z ($|z| \gg \delta$) in which we have the following asymptotic formula for ϵ from (2.6):

$$\varepsilon(z)=\frac{\sin\theta}{(16\pi n_0T|\varepsilon_L|)^{1/2}}\left[H_x^2(z)+\frac{c^2}{\omega^2\sin^2\theta}\left(\frac{dH_x}{dz}\right)^2\right]^{1/2}. \tag{2.8}$$

The stationary distribution $H_x(z)$ of the magnetic field in the wave is determined from the equation

$$H_x''-\frac{\varepsilon'}{\varepsilon}H_x'+\frac{\omega^2}{c^2}(\varepsilon-\sin^2\theta)H_x=0, \tag{2.9}$$

which is obtained from (1.1)—(1.3) under the condition

$$\left|\frac{1}{E_z}\frac{dE_z}{dz}\right| \ll \left(\frac{m_e\omega^2}{T_e}\varepsilon\right)^{1/2}.$$

Equations (2.8), (2.9) together describe the electromagnetic wave sufficiently far beyond the linear transformation point ($z \gg \delta$). In dimensionless variables,

$$\zeta=\frac{\omega z}{c}\sin\theta,\quad h=\frac{H_x}{\sin\theta\,(16\pi n_0 T|\varepsilon_L|)^{1/2}} \qquad (2.10)$$

the nonlinear dielectric constant of the plasma is equal to

$$\varepsilon=\sin^2\theta\left[h^2+\left(h'+\frac{h}{2\zeta}\right)^2\right]^{1/2}, \qquad (2.11)$$

and the equation for $h(\zeta)$ takes on a universal form (independent of θ and ϵ_L):

$$h^2h''-2hh'^2-h\left[h^2-\left(h^2+\left(h'+\frac{h}{2\zeta}\right)^2\right)^{3/2}\right]+\frac{h'+h/2\zeta}{2\zeta}\left[\left(h'+\frac{h}{2\zeta}\right)^2-2h^2\right]+\frac{h^2h'}{2\zeta}=0 \qquad (2.12)$$

with the boundary conditions

$$h_0=\left[\frac{H_x^2(0)}{16\pi n_0 T\sin^4\theta}\right]^{1/2}\left[\frac{\omega L\sin^3\theta}{c\zeta_0}\right]^{1/2},\quad h_0'\approx-\frac{h_0}{2\zeta_0}$$

at

$$\zeta=\zeta_0\approx\frac{\omega L\sin^3\theta}{c}\left(\frac{H_x^2(0)}{16\pi n_0 T\sin^4\theta}\right)^{1/3}.$$

An analytic solution of the equation can be obtained in the limiting case of large ζ. In this case, omitting the small quantities ($\lesssim 1/\zeta$) in Eq. (2.12) and introducing the function $w(h) = (h^2 + h'^2)^{1/2}$, we obtain an equation with separable variables

$$h\frac{dw}{dh}=w(2-w), \qquad (2.13)$$

the solution of which is

$$\frac{w}{2-w}=Ch^2,\quad \text{i.e.,}\quad h'^2=\frac{h^2P(h)}{(1+Ch^2)^2}, \qquad (2.14)$$

$P(h) = 4C^2h^2 - (1 + Ch^2)^2$, C is a constant which determines h as an oscillating function of ζ varying in the limits $h_- < h < h_+$; $h_\pm$ are the roots of the polynomial $P(h)$, $h_\pm = 1 \pm (1 - 1/C)^{1/2}$.

Account of the small terms $\sim 1/\zeta$ in Eq. (2.12) leads to slow drawing together of both turning points $h_\pm$ toward unity. In this case, in the solution (2.14), C becomes a slow function of ζ, determined from the equation

$$\frac{dC}{d\zeta}=-\frac{1}{4\zeta}\int_{h_-}^{h_+}\frac{dh\,h'(1+Ch^2)^2(h'^2+3h(h'^2+h^2)^{1/2}-h^2)}{h^4(h^2+h'^2)^{1/2}}\left(\int_{h_-}^{h_+}\frac{dh}{h'}\right)^{-1}. \qquad (2.15)$$

Setting $h_\pm = 1 + \mu(\zeta)$, $\mu \ll 1$, $h^2 = 1 + \alpha$, we have from (2.15)

$$\frac{d\mu^2}{d\zeta}=-\frac{1}{2\zeta}\int_{-2\mu}^{2\mu}(4\mu^2-\alpha^2)^{1/2}d\alpha\Big/\int_{-2\mu}^{2\mu}\frac{d\alpha}{(4\mu^2-\alpha^2)^{1/2}}\approx-\frac{\mu^2}{\zeta}$$

and, consequently, the quantity ζ falls off $\sim 1/\zeta^{1/2}$ for large ζ.

A solution of Eq. (2.12) which is valid for all $\zeta \geq \zeta_0$ can be obtained only with the aid of numerical methods. Figure 2 shows the results of numerical integration of Eq. (2.12) for $\zeta_0 = 0.1$ and various values of $h_0 \ll 1$. It is seen that the asymptotic solution for large ζ is practically independent of the boundary conditions and corresponds to slowly damped oscillations of $h(\zeta)$ about the mean value $\bar{h} = 1$. For such ζ, setting $h = 1 + \varphi(\zeta)$ in Eq. (2.12), we have the following inhomogeneous Bessel equation for φ:

$$\varphi''+\frac{\varphi'}{\zeta}+\varphi=\frac{1}{8\zeta^2}. \qquad (2.16)$$

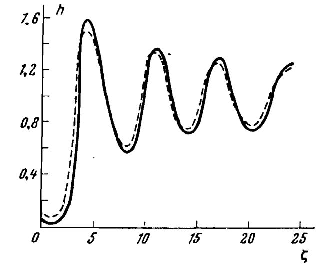

FIG. 2

The solution of this equation

$$\varphi(\zeta)=J_0(\zeta)\left[A+\frac{\pi}{16}\int_\zeta^\infty\frac{N_0(\zeta)}{\zeta}d\zeta\right]+N_0(\zeta)\left[B-\frac{\pi}{16}\int_\zeta^\infty\frac{J_0(\zeta)}{\zeta}d\zeta\right] \qquad (2.17)$$

(A and B are constants) for $\zeta \gtrsim 10$ is identical with a high degree of accuracy to the numerical solution. Here the nonlinear dielectric constant of the plasma is always positive and oscillates about the value $\epsilon = \sin^2\theta$:

$$\varepsilon=\sin^2\theta\left(1+A\left(\frac{2}{\pi\zeta}\right)^{1/2}\cos\left(\zeta-\frac{\pi}{4}\right)+B\left(\frac{2}{\pi\zeta}\right)^{1/2}\sin\left(\zeta-\frac{\pi}{4}\right)\right). \qquad (2.18)$$

The magnetic and transverse electric fields in the wave are equal to

$$H_x=\sin\theta\,(16\pi n_0 T|\varepsilon_L|)^{1/2}\left(1+A\left(\frac{2}{\pi\zeta}\right)^{1/2}\cos\left(\zeta-\frac{\pi}{4}\right)+B\left(\frac{2}{\pi\zeta}\right)^{1/2}\sin\left(\zeta-\frac{\pi}{4}\right)\right),$$
$$E_y=-\left(\frac{32n_0Tc}{\omega L\sin\theta}\right)^{1/2}\left(A\sin\left(\zeta-\frac{\pi}{4}\right)-B\cos\left(\zeta-\frac{\pi}{4}\right)\right). \qquad (2.19)$$

Penetration of the wave into the region $\zeta > 0$ is accompanied by excitation of a large longitudinal electric field

$$E_z=\frac{H_x\sin\theta}{\varepsilon}=(16\pi n_0T|\varepsilon_L|)^{1/2}\left(1-\frac{1}{\pi\zeta}\left(A\sin\left(\zeta-\frac{\pi}{4}\right)-B\cos\left(\zeta-\frac{\pi}{4}\right)\right)^2\right). \qquad (2.20)$$

Equations (2.19), (2.20) describe a stationary distribution of the components of the field of the electromagnetic wave sufficiently far behind its front. It is essential that in the motion of the wave into the plasma, the amplitudes of the magnetic and longitudinal electric fields grow $\sim(|\epsilon_L|)^{1/2}$, so that the wave pressure turns out to be sufficient for displacement of the plasma. The wave thus plays the role of a "piston" which displaces the particles and penetrates sufficiently far into the plasma. Behind the front of the wave, the density of the plasma is less than critical: $\omega_p(z) < \omega$; in front of the wave front the density is the same as in the undisturbed plasma, $n = n_L(z)$, and significantly exceeds the critical value. The plasma displaced by the wave remains quasineutral, so that a sufficiently large polarization field $E_0(z)$ is produced in the transition layer, leading to displacement of the ions.

Assuming that the velocity with which the plasma is displaced is much less than the thermal velocity of the electrons, we obtain from the condition of equality of the forces acting on the electrons

$$eE_0+\frac{T_e}{n_0}\frac{\partial\Delta n}{\partial z}-\frac{e^2}{4m_e\omega^2}\frac{\partial}{\partial z}(E_y^2+E_z^2)=0$$

and from the equations of motion and continuity for the ions

$$\frac{\partial v}{\partial t}=\frac{eE_0}{m_i}-\frac{T_i}{n_0m_i}\frac{\partial\Delta n}{\partial z},\quad \frac{\partial\Delta n}{\partial t}+n_0\frac{\partial v}{\partial z}=0$$

the following equation for the quasineutral perturbation of the density in the transition layer:

$$\frac{\partial^2 \Delta n}{\partial t^2} - c_s^2 \frac{\partial^2 \Delta n}{\partial z^2} = \frac{1}{16\pi m_i} \frac{\partial^2}{\partial z^2}(E_y^2 + E_z^2), \quad c_s = \left(\frac{T_e}{m_i}\right)^{1/2}. \quad (2.21)$$

Then, inasmuch as $E_z \gg E_y$ in the transition layer, and the velocity of displacement of the layer is, as we shall see below, greater than the velocity of sound, we have the following estimate for Δn:

$$\Delta n \sim \frac{(\Delta t)^2}{16\pi m_i} \frac{\partial^2 E_z}{\partial z^2}. \quad (2.22)$$

In the transition layer, the magnetic field of the wave falls off exponentially:

$$H_x(z+\Delta z) \approx H_x(z) \exp(-\omega \Delta z \sin\theta / c), \quad \Delta z > 0.$$

$H_x(z)$ is the field behind the front, determined by Eq. (2.19), and the plasma density undergoes a transition through the critical value $\omega_p(z) = \omega$, so that the longitudinal electric field E_z increases significantly. For $\Delta t < 1/\omega\epsilon$, when the stationary distribution of the longitudinal field has not yet been established, the field increases with time according to the linear law

$$E_z \sim \frac{H_x \omega \Delta t}{2} \sin\theta. \quad (2.23)$$

Substituting into (2.22) the field E_z from (2.23), the dimension of the transition layer $\Delta z \sim c/\omega \sin\theta$, and $\Delta n \sim n_0|\epsilon_L|$, we obtain the following estimates for the time of passage of the wave through the layer:

$$\Delta t \sim \frac{1}{\omega} \sin^{-3/2}\theta \left(\frac{m_i c^2}{T}\right)^{1/4}$$

and for the velocity of motion of the layer:

$$u \sim \frac{\Delta z}{\Delta t} \sim (\sin\theta\, c_s c)^{1/2}. \quad (2.24)$$

The longitudinal field in the transition layer, in accord with (2.23), is equal to

$$E_z \sim \frac{H_x}{\sin^{1/2}\theta} \left(\frac{m_i c^2}{T}\right)^{1/4},$$

and for the coefficient of transformation to plasma oscillations (R), defined as the ratio of the energy flux in these oscillations

$$S_l = v_g \frac{E_z^2}{8\pi} \sim \frac{v_T^2}{\Delta z \omega} \frac{H_x^2(z)}{8\pi \sin\theta} \left(\frac{m_i c^2}{T}\right)^{1/2}$$

to the energy flux in the incident wave $S_0 = cH_0^2/8\pi$, we have the relation (1.3).

3. PENETRATION OF THE WAVE FOR $H_2^0 \gg 16\pi n_0 T \times \sin^2\theta$

For such amplitudes of the incident wave, as will be shown below, the dielectric constant $\epsilon \gg \sin^2\theta$. We then have from (2.1)–(2.3)

$$H_x = -\frac{c}{\omega} E_y', \quad E_z = -\frac{c}{\omega} \frac{\sin\theta}{\varepsilon} E_y',$$
$$E_y'' + \frac{\omega^2}{c^2} \varepsilon E_y = 0. \quad (3.1)$$

The equation for the dielectric constant in this case is written in the form

$$\varepsilon = \varepsilon_L(z) + \frac{1}{16\pi n_0 T}\left(E_y^2 + \frac{c^2}{\omega^2} \frac{\sin^2\theta}{\varepsilon^2} E_y'^2\right). \quad (3.2)$$

For $\theta \ll 1$ in the region $\epsilon_L > 0$ we can neglect the last term in the equation for ϵ. For E_y we then have the equation of a nonlinear oscillator; the first integral of this equation is

$$E_y'^2 = V - U(E_y^2). \quad (3.3)$$

The amplitude in the standing electromagnetic wave

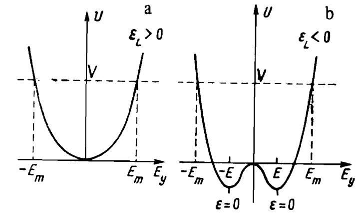

FIG. 3

oscillates within the limits $-E_m < E_y < E_m$. The quantity

$$U(E_y^2) = \frac{\omega^2}{c^2} E_y^2 \left(\varepsilon_L + \frac{E_y^2}{32\pi n_0 T}\right)$$

plays the role of the potential energy of the oscillator, and

$$V = \frac{\omega^2}{c^2} E_m^2 \left(\varepsilon_L + \frac{E_m^2}{32\pi n_0 T}\right)$$

that of the total energy. A graph of the function $U(E_y)$ is shown in Fig. 3a.

By analogy with the WKB theory for a linear oscillator, account of the slow (in comparison with the wavelength of the oscillations) dependence of ϵ_L on z leads to the result that the amplitude of the oscillations also becomes a function of z. In this case, we have the following equation for V(z) from the condition of compatibility of the equation for E_y (3.1) and the first integral of (3.3):

$$\frac{dV}{dz} \approx \frac{\omega^2}{c^2} \overline{E_y^2} \frac{d\varepsilon_L}{dz},$$

where we have replaced $E_y^2(z)$ by the approximation $\overline{E_y^2}$, inasmuch as we are interested only in the slow change of the amplitude of the basic solution with z.

By making use of the definition of V, we finally obtain an equation for the amplitude of the oscillations:

$$\left(\varepsilon_L + \frac{E_m^2}{16\pi n_0 T}\right) \frac{dE_m^2}{d\varepsilon_L} = \overline{E_y^2} - E_m^2. \quad (3.4)$$

The well-known result $E_m^2 \sim 1/\epsilon_L^{1/2}$ then follows for the linear oscillator with $E_m^2/16\pi n_0 T \ll \epsilon_L$, $\overline{E_y^2} = \frac{1}{2} E_m^2$.

In the general case, computing $\overline{E_y}$, we obtain the following equation from (3.4):

$$\frac{dE_m^2}{d\varepsilon_L} = -32\pi n_0 T \left[1 - \frac{E(\varkappa)}{K(\varkappa)}\right] \quad (3.5)$$

($\kappa = E_m\theta(32\pi n_0 T\epsilon_L + 2E_m^2)^{1/2}$, $K(\kappa)$, $E(\kappa)$ are complete elliptic integrals of the first and second kinds), which describes the finite growth of the oscillation amplitude E_m as $\epsilon_L \to 0$.

The solution has interesting features if $\epsilon_L < 0$. A graph of the function $U(E_y)$ for this case is shown in Fig. 3b. For $E_y = E^* = \pm(16\pi n_0 T|\epsilon_L|)^{1/2}$, we have $\epsilon \to 0$ and in Eq. (3.2) for ϵ account of the last term is necessary. Equation (3.2) then becomes cubic in ϵ; the graphical solution of this equation is shown in Fig. 1. In the case considered, however,

$$\varepsilon_0 = \left(\frac{c^2}{\omega^2} \sin^2\theta \frac{E_*'^2}{16\pi n_0 T}\right)^{1/3},$$

and the role of the parameter z/δ is played by the quantity

$$\lambda = \frac{|\varepsilon_L|}{\varepsilon_0}\left(1 - \frac{E_y^2}{E_*^2}\right).$$

The field E_y remains an oscillating function of z, but on the graph of this function we can distinguish three regions (see Fig. 4):

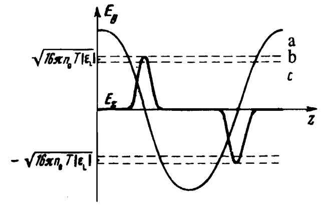

FIG. 4

a) $\lambda \ll -1$, i.e., $|E_y| > E_*$. Then the last term in the formula for ϵ (3.2) is negligibly small and for E_y we have the equation of a nonlinear oscillator (3.3);

b) $|\lambda| \lesssim 1$, $\epsilon \sim \epsilon_0$. In this region we have the following approximate relation for E_y:

$$E_y = E_* + E_*'\delta z - E_*\omega^2\varepsilon_0(\delta z)^2/2c^2 + \ldots \tag{3.6}$$

The condition of applicability of the given solution is

$$|\delta z| \leqslant \left[\frac{c}{\omega}\frac{\sin\theta}{|\varepsilon_L|}\right]^{1/3}\left|\frac{E_*}{E_*'}\right|^{1/3} \sim \frac{c}{\omega\sqrt{|\varepsilon_L|}}\left[\frac{\sin^2\theta}{|\varepsilon_L|}\left(\frac{E_m^2}{E_*^2}-1\right)\right]^{1/6} \ll \frac{c}{\omega|\varepsilon_L|^{1/2}}.$$

To obtain this estimate we used a formula for E_*' that follows from (3.3):

$$\left|\frac{E_*'}{E_*}\right| \approx \frac{\omega}{c}\left(\frac{|\varepsilon_L|}{2}\right)^{1/2}\left(\frac{E_m^2}{E_*^2}-1\right)$$

and also the fact that the parameter

$$\frac{|\varepsilon_L|^{1/2}}{\sin\theta}\frac{E_m}{E_*} \gg 1$$

for the incident-wave field amplitudes considered in this section, $H_0 \gg \sin\theta\,(16\pi n_0 T)^{1/2}$.

With the help of the estimate for δz, it is not difficult to show that the last two terms in Eq. (3.6) are small and consequently the field E_y in this solution remains approximately constant: $E_y \approx E^*$. The solution (3.6) describes the passage through zero of the parameter λ and is thus transitional, linking the regions of negative and positive values of λ;

c) $\lambda \gg 1$; i.e., $|E_y| < E_*$. For such λ we have the following asymptotic formula for ϵ from (3.2):

$$\varepsilon = \frac{c}{\omega}\sin\theta\frac{|E_y'|}{(E_*^2-E_y^2)^{1/2}}. \tag{3.7}$$

In this region, the dielectric constant of the plasma is small ($\epsilon \lesssim \epsilon_0$), but the minimum value of ϵ (at $E_y = 0$) still remains much larger than $\sin^2\theta$:

$$\varepsilon_{min} = \frac{c}{\omega}\sin\theta\left|\frac{E_*'}{E_*}\right| \simeq \sin\theta\left(\frac{|\varepsilon_L|}{2}\right)^{1/2}\left(\frac{E_m^2}{E_*^2}-1\right) \gg \sin^2\theta.$$

The equation for E_y in the case considered reduces to the form

$$E_y'' = -\frac{\omega^2}{c^2}\varepsilon E_y - \sin^2\theta(E_y'/\varepsilon)' = 0 \tag{3.8}$$

and, consequently,

$$E_y = E_* + E_*'(z-z_*). \tag{3.9}$$

Due to the smallness of ϵ a large longitudinal component of the electric field

$$E_z = -(E_*^2-E_y^2)^{1/2}\operatorname{sign}E_y \tag{3.10}$$

arises in the wave. A graph of the function $E_z(z)$ is also given in Fig. 4.

The change in the amplitude of the oscillations of E_y in motion of the wave into the plasma, $E_m(\epsilon_L)$, is determined from Eq. (3.4). Introducing the function

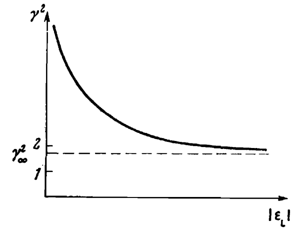

FIG. 5

$\gamma(\epsilon_L) = E_m^2/16\pi n_0 T|\epsilon_L|$ into this equation, we write it in the form

$$\frac{d\gamma}{d\varepsilon_L}\varepsilon_L = \frac{\gamma - \overline{E_y^2}/16\pi n_0 T|\varepsilon_L|}{\gamma-1} - \gamma = \Phi(\gamma). \tag{3.11}$$

The solution of this equation is shown in Fig. 5. With increase in $|\epsilon_L|$ the function γ tends to the asymptotic value γ_∞, which is determined from the equation $\Phi(\gamma_\infty) = 0$, $\gamma_\infty \approx 1.6$. For sufficiently large $|\epsilon_L|$, the amplitude of E_y increases $\sim|\epsilon_L|^{1/2}$, $E_m \approx (16\pi n_0 T|\epsilon_L|\gamma_\infty)^{1/2}$, like the amplitude of the longitudinal electric field $E_{zm} \approx (16\pi n_0 T|\epsilon_L|)^{1/2}$.

Penetrating into the plasma, the wave displaces the particles, and in the steady solution established behind the wavefront the dielectric constant of the plasma is positive and oscillates within the limits $\epsilon_{max} \approx 0.6|\epsilon_L|$, $\epsilon_{min} \approx 0.4\sin\theta\,|\epsilon_L|^{1/2}$. In contrast with the solution obtained in Sec. 2, when a barrier developed behind the reflection point $\epsilon_L = \sin^2\theta$ and a penetration took place only for $\theta \lesssim (c/\omega L)^{1/3}$, in the case considered here this barrier is lacking and the wave penetrates into the plasma over a wider range of angles satisfying the condition

$$\sin\theta \leqslant H_0/(16\pi n_0 T)^{1/2}.$$

4. NONLINEAR PENETRATION INTO A PLASMA CONFINED BY THE WAVE PRESSURE

The subsequent treatment pertains exclusively to the case of high-amplitude fields in the electromagnetic wave, $H_0^2/16\pi n_0 T \sim 1$, when its pressure is sufficient to confine the plasma. In this section, we limit ourselves to the hydrodynamic approximation. For the field amplitudes considered and a self-consistent distribution of the plasma particles, we have the following equation in the wave instead of (2.5):

$$n_e = n_0\exp\left[-\frac{E_y^2+E_z^2}{16\pi n_0 T}\frac{\omega_{p0}^2}{\omega^2}\right]. \tag{4.1}$$

The solution describing the confinement of the plasma by the wave pressure was obtained previously as $\theta \to 0$, $E_z \to 0$ for this case.[5] We now present this solution.

Determining the dielectric constant ϵ of the plasma with the help of (4.1) and substituting it in (3.1), we obtain the following equation for the transverse component of the electric field:

$$E_y'' + \frac{\omega^2}{c^2}\left[1 - \frac{\omega_{p0}^2}{\omega^2}\exp\left(-\frac{E_y^2}{16\pi n_0 T}\frac{\omega_{p0}^2}{\omega^2}\right)\right]E_y = 0. \tag{4.2}$$

The first integral of this equation is

$$(E_y')^2 = 16\pi n_0 T\frac{\omega^2}{c^2}\left[1 - \exp\left(-\frac{E_y^2}{16\pi n_0 T}\frac{\omega_{p0}^2}{\omega^2}\right) - \frac{E_y^2}{16\pi n_0 T}\right]. \tag{4.3}$$

For $\omega_{p0} \gg \omega$, Eq. (4.3) has a solution in which the plasma occupies the region $z \geq 0$ and is confined by the wave pressure. At the point $z = 0$, the field E_y is at maximum: $E_{ym} = (16\pi n_0 T)^{1/2}$ and the particle density is exponentially small: $n \approx n_0 \exp(-\omega_{p0}^2/\omega^2)$; this density corresponds to particles on the tail of the Maxwell distribution that pass through the barrier created by the wave. In the region of large z, the field does not penetrate: $E_y \sim \exp(-\omega_{p0} z/c)$ and the density of the plasma in this region is close to maximum: $n \approx n_0$.

In the present solution, the dielectric constant ϵ passes through zero as

$$E_y \to E_* = (16\pi n_0 T)^{1/2} \frac{\omega}{\omega_{p0}} \ln^{1/2} \frac{\omega_{p0}^2}{\omega^2},$$
$$E_y' \to E_*' = (16\pi n_0 T)^{1/2} \frac{\omega}{c},$$

and if $\theta \neq 0$, a singularity of the longitudinal electric field E_z arises in the vicinity of this point, in accord with (3.1). The singularity is removed if we take into account the contribution of E_z to the wave pressure. Then we have the following formula for ϵ in the vicinity of the point $\epsilon_N = 0$:

$$\varepsilon = \varepsilon_N + \frac{\omega_{p0}^2}{\omega^2} \frac{\sin^2\theta}{\varepsilon^2},$$
$$\varepsilon_N = 1 - \frac{\omega_{p0}^2}{\omega^2} \exp\left\{ -\frac{E_y^2}{16\pi n_0 T} \frac{\omega_{p0}^2}{\omega^2} \right\}, \tag{4.4}$$

ϵ_N is the dielectric constant of the plasma for normal incidence of the wave; in obtaining Eq. (4.4), we have assumed that $\omega_{p0} \sin\theta/\omega \ll 1$. In this case, the solution which describes the nonlinear penetration of the wave into the plasma for oblique incidence is analogous, to a significant degree, to that obtained in the previous section.

For $\epsilon_N \gg \epsilon_0 = (\omega_{p0} \sin\theta/\omega)^{2/3}$, the field E_y is obtained from Eq. (4.3). For $|\epsilon_N| \sim \epsilon_0$, we have the following approximate relation for E_y:

$$E_y = E_* + E_*' \delta z + \ldots,$$
$$|\delta z| = |z - z_*| \lesssim \frac{c}{2\omega_{p0}} \left(\frac{\omega_{p0} \sin\theta}{\omega} \right)^{1/3} \ln^{-1/2} \frac{\omega_{p0}^2}{\omega^2}. \tag{4.5}$$

It is easy to see that the variation of E_y is negligibly small in this solution:

$$\frac{\Delta E_y}{E_*} \sim \left(\frac{\omega_{p0} \sin\theta}{\omega} \right)^{1/3} \ll 1.$$

The solution (4.5) is an intermediate one, describing the passage of the parameter ϵ_N through zero. For $-\epsilon_N \gg \epsilon_0$, the dielectric constant ϵ remains sufficiently small and is determined by the formula

$$\varepsilon = \sin\theta \frac{c\omega_{p0}}{\omega^2} E_y' \left(16\pi n_0 T \ln \frac{\omega_{p0}^2}{\omega^2} - E_y^2 \frac{\omega_{p0}^2}{\omega^2} \right)^{-1/2}, \tag{4.6}$$

and substituting this in Eq. (3.8), we again obtain $E_y'' = 0$. The solution of this equation (3.9) is applicable for $-E_* < E_y < E_*$ in the interval

$$z - z_* \leqslant \frac{2c}{\omega_{p0}} \ln^{1/2} \frac{\omega_{p0}^2}{\omega^2}.$$

In this interval, the wave has a large longitudinal electric-field component determined by Eq. (3.10). Thus, the graphs which describe the changes of E_y and E_z in the wave are identical with those shown in Fig. 4; in the considered case, however, we should make the substitution

$$(16\pi n_0 T |\varepsilon_L|)^{1/2} \to (16\pi n_0 T)^{1/2} \frac{\omega}{\omega_{p0}} \ln^{1/2} \frac{\omega_{p0}^2}{\omega^2}.$$

Passage of the wave through the region with large longitudinal field E_z is accompanied by excitation of plasma oscillations. The transformation coefficient R can be calculated in much the same way as in Sec. 2, and turns out to be very large:

$$R \sim \frac{v_T}{c} \left(\frac{m_i}{m_e} \right)^{1/2} \sin\theta. \tag{4.7}$$

5. ABSORPTION OF ELECTROMAGNETIC ENERGY FOR NORMAL INCIDENCE OF THE WAVE

In this case the absorption is due to two mechanisms: excitation of plasma oscillations in the vicinity of the point $\omega_p(z) = 2\omega$, and collisionless damping of the wave due to transfer of its energy to the particles of the plasma. We shall consider these mechanisms of dissipation for a standing electromagnetic wave of large amplitude, $E_0^2 \sim 16\pi n_0 T$, which confines the plasma. We shall assume that within the period of the high-frequency field, the particles of the plasma are displaced by a distance that is significantly less than the depth of the skin layer: $v_T/c \ll \omega/\omega_{p0}$ (the normal skin effect). Under these conditions, the energy dissipation of the wave is small and it can be assumed in first approximation that the distribution of the field amplitude E_y is determined by the integral (4.3). The first of the dissipation mechanisms indicated above is connected with the fact that the force in the electromagnetic field acting on the electrons of the plasma in the direction of the density gradient $ev_y H_x/c$, along with the time-independent component, which guarantees the confinement of the plasma, has a component $\sim \cos 2\omega t$ which leads to resonance excitation of plasma oscillations at $\omega_p(z) = 2\omega$. Attention was first called to the existence of such a dissipation mechanism by Caruso.[8] We shall give his systematic consideration below.

Collisionless damping of the wave in interaction with particles of the plasma is due to two causes: the transfer of energy of the field to a small group of resonant particles with velocities $v_z \sim c\omega/\omega_{p0}$ and adiabatic interaction of thermal particles of the plasma with the profile of the field $f(z) \cos\omega t$. In the latter case the irreversibility in the transfer of energy from the field to the particles is connected with the presence of a group of particles with sufficiently large velocities to pass through the barrier created by the wave. The number of emerging particles and the corresponding absorption coefficient $\sim \exp(-\omega_{p0}^2/\omega^2)$, but given the condition $v_T \omega_{p0}^2/c\omega^2 \ll 1$, such a mechanism of transfer of energy to the particles is fundamental and we limit ourselves to its study.

We perform our analysis in the kinetic approximation. The solution of the kinetic equation will be sought by the method of characteristics. Inasmuch as the electromagnetic wave is, strictly speaking, not a standing wave in the presence of dissipation, then the electric and magnetic fields in this wave can be written in the form

$$E_y(t, z) = E_1(z) \cos\omega t + E_2(z) \sin\omega t, \quad E_2 \ll E_1,$$
$$H_x(t, z) = -\frac{c}{\omega} \left(\frac{dE_1}{dz} \sin\omega t - \frac{dE_2}{dz} \cos\omega t \right). \tag{5.1}$$

The equation of motion of the electrons in the direction of the electric field of the wave

$$m_e \frac{dv_y}{dt} = -eE_y - \frac{e}{c} v_z H_x.$$

can be simply integrated:

$$v_y = v_{y0} - \frac{e}{m\omega} (E_1(z) \sin\omega t - E_2(z) \cos\omega t), \tag{5.2}$$

and for motion of electrons in the direction of the gradient, we then have the equation

$$m_e\frac{dv_z}{dt}=\frac{e}{c}v_yH_x+e\frac{d\varphi_0}{dz}-eE_z(z)\cos 2\omega t=$$
$$=e\frac{d\varphi_0}{dz}+\frac{ev_{y0}}{\omega}\left[\frac{dE_1}{dz}\sin\omega t-\frac{dE_z}{dz}\cos\omega t\right]- \tag{5.3}$$
$$-\frac{e^2}{2m_e\omega^2}\left[\frac{dE_1^2}{dz}\sin^2\omega t+\frac{dE_2^2}{dz}\cos^2\omega t-\frac{dE_1E_2}{dz}\sin 2\omega t\right]-eE_z(z)\cos 2\omega t,$$

where we have taken into account the polarization field $-d\varphi_0/dz$ and the field of the plasma oscillations, which varies with frequency 2ω.

The integral for the energy of longitudinal motion follows from (5.3):

$$\frac{m_ev_z^2}{2}-e\varphi_0(z)+\frac{e^2}{4m_e\omega^2}(E_1^2+E_2^2)+$$
$$+\frac{ev_{y0}v_z}{\omega^2}\left[\frac{dE_2}{dz}\sin\omega t+\frac{dE_1}{dz}\cos\omega t\right]+\frac{e^2v_z}{8m_e\omega^3}\left[\left(\frac{dE_2^2}{dz}-\frac{dE_1^2}{dz}\right)\sin 2\omega t\right.$$
$$\left.+2\frac{dE_1E_2}{dz}\cos 2\omega t\right]+\frac{ev_z}{2\omega}\left[E_z\sin 2\omega t+\frac{v_z}{2\omega}\frac{dE_z}{dz}\cos 2\omega t-\right. \tag{5.4}$$
$$\left.-\frac{v_z^2}{4\omega^2}\frac{d^2E_z}{dz^2}\sin 2\omega t\right]=\frac{m_ev_{z0}^2}{2},$$

to obtain which we have made use of the fact that for any function $\psi(z)$ which changes over distances $\Delta z \gg v_z/\omega$, we have, approximately,

$$\int dt\psi(z)\cos\omega t=\frac{\psi(z)}{\omega}\sin\omega t+\frac{v_z}{\omega^2}\frac{d\psi}{dz}\cos\omega t+\ldots.$$

In (5.4), the term $\sim d^2E_z/dz^2$ remains; this is necessary in order that account be taken in the equation for E_z of the transport of the plasma oscillations (see (5.11) below). Assuming that the distribution function for particles which move from the region of large z is identical with the Maxwellian f_M, we have, for arbitrary z,

$$f(t,z,v_y,v_z)=f_M(v_{y0}^2+v_{z0}^2)=n_0\frac{m_e}{2\pi T_e}\exp\left\{-\frac{m_ev_z^2}{2T_e}\right.$$
$$\left.-\frac{m_e}{2T_e}\left[v_y+\frac{e}{m_e\omega}(E_1\sin\omega t-E_2\cos\omega t)\right]^2+\frac{e\varphi_0}{T_e}-\frac{e^2E_1^2}{4m_e\omega^2T_e}\right\}$$
$$\cdot\left[1-\frac{ev_zv_y}{T_e\omega^2}\frac{dE_1}{dz}\cos\omega t-\frac{e^2v_z}{8m_e\omega^3}\frac{1}{T_e}\frac{dE_1^2}{dz}\sin 2\omega t\right. \tag{5.5}$$
$$\left.-\frac{ev_z}{2\omega T_e}\left(E_z\sin 2\omega t+\frac{v_z}{2\omega}\frac{dE_z}{dz}\cos 2\omega t-\frac{v_z^2}{4\omega^2}\frac{d^2E_z}{dz^2}\sin 2\omega t\right)\right]$$

where we have neglected the small quantities $\sim E_2^2$, $v_zE_2/\omega\Delta z$.

For the distribution function (5.5), the longitudinal velocity of the particles v_z changes within the limits $-\infty < v_z < v_{z\,max}$,

$$v_{z\,max}=\left[\frac{2e}{m_e}(\varphi_0-\varphi_0(0))+\frac{e^2(E_1^2(0)-E_1^2)}{2m_e^2\omega^2}\right]^{1/2},$$

$v_{z\,max}$ is the velocity at the point z of a particle which is reflected from the barrier of the electromagnetic field at the point z = 0. For $v_z > v_{z\,max}$, the distribution function vanishes.

Determining the density of particles with the help of (5.5), we get

$$n_e=n_0\exp\left\{\frac{e\varphi_0}{T_e}-\frac{e^2E_1^2}{4m_e\omega^2T_e}\right\}\frac{1+\Phi(w)}{2},$$
$$w=\frac{e^2(E_1^2(0)-E_1^2)}{4m_e\omega^2T_e}+\frac{e(\varphi_0-\varphi_0(0))}{T_e}, \tag{5.6}$$

$\Phi(w)$ is the probability integral. The potential $\varphi_0(z)$ is found from the condition of quasineutrality and from the condition of equilibrium of the forces acting on the ions:

$$e\varphi_0\left(1+\frac{T_i}{T_e}\right)=T_i\left(\frac{e^2E_1^2}{4m_e\omega^2T_e}-\ln\frac{1+\Phi(w)}{2}\right).$$

The last term in this formula differs from zero only for $E_1^2 \approx E_1^2(0)$, when $e^2E_1^2/4m_e\omega^2T_e \approx \omega_{p0}^2/\omega^2 \gg 1$. We therefore have, finally,

$$e\varphi_0(z)=\frac{e^2E_1^2(z)}{4m_e\omega^2}\frac{T_i}{T}, \quad T=T+T \tag{5.7}$$

The field amplitudes $E_y - E_1(z)$ and $E_2(z)$ are determined from the equations

$$\frac{d^2E_1}{dz^2}+\frac{\omega^2}{c^2}E_1=-\frac{4\omega^2e}{c^2}\int_0^{2\pi/\omega}dt\sin\omega t\int dv_y\,dv_z\,v_yf(t,z,v_y,v_z),$$
$$\frac{d^2E_2}{dz^2}+\frac{\omega^2}{c^2}E_2=\frac{4\omega^2e}{c^2}\int_0^{2\pi/\omega}dt\cos\omega t\int dv_y\,dv_z\,v_yf(t,z,v_y,v_z). \tag{5.8}$$

Substituting f from (5.5), and completing the integration of t, v_y, v_z, we write these equations in the form

$$\frac{d^2E_1}{dz^2}+\left(\frac{\omega^2}{c^2}-\frac{\omega_{p0}^2}{c^2}\exp\left(-\frac{e^2E_1^2}{4m_e\omega^2T}\right)\frac{1+\Phi(w)}{2}\right)E_1=0,$$
$$\frac{d^2E_2}{dz^2}+\left(\frac{\omega^2}{c^2}-\frac{\omega_{p0}^2}{c^2}\exp\left\{-\frac{e^2E_1^2}{4m_e\omega^2T}\right\}\frac{1+\Phi(w)}{2}\right)E_2 \tag{5.9}$$
$$=\frac{\omega_{p0}^2}{c^2}\left(\frac{T_e}{2\pi m_e\omega^2}\right)^{1/2}\frac{dE_1}{dz}\exp\left(-\frac{e^2E_1^2(0)}{4m_e\omega^2T}\right)\left(1+\frac{e^2E_1^2(z)}{8m_e\omega^2T_e}\right).$$

The first of these equations is practically the same as (4.2) (a difference occurs only in the last term for $E_1^2(z) \approx E_1^2(0)$, when this term is exponentially small), and the second equation determines the amplitude E_2, the generation of which is connected with dissipation of electromagnetic energy in the plasma.

The corresponding absorption coefficient R_d can be defined as the ratio of the electromagnetic energy flux flowing into the plasma across the boundary z = 0:

$$S=-\frac{c}{4\pi}\overline{E_yH_x}(z=0)=\frac{c^2}{8\pi\omega}\left(E_1(0)\frac{dE_2}{dz}(0)-E_2(0)\frac{dE_1}{dz}(0)\right)$$

(the bar denotes averaging over the period of the high-frequency field) to the energy flux S_0 in the incident wave. From (5.9), we have

$$S=\frac{\omega_{p0}^2}{16\pi\omega^2}\left(\frac{T_e}{2\pi m_e}\right)^{1/2}\exp\left(-\frac{\omega_{p0}^2}{\omega^2}\right)E_1^2(0)\left(1+\frac{e^2E_1^2(0)}{16m_e\omega^2T_e}\right). \tag{5.10}$$

Separating the wave incident on the plasma into components of the electromagnetic field at z < 0, we obtain $S_0 = cE_1^2(0)/32\pi$, and the coefficient of absorption of the electromagnetic energy by the plasma particles, R_d, is determined by Eq. (1.7).

We now consider the absorption of electromagnetic energy due to excitation of plasma oscillations. The field of these oscillations is $E_z\cos 2\omega t$, where $E_z(z)$ is determined from the equation

$$E_z=-4e\int_{-\pi/2\omega}^{\pi/2\omega}dt\sin 2\omega t\int dv_y\,dv_z\,v_zf(t,z,v_y,v_z),$$

which can be rewritten in the following form with the help of Eq. (5.5):

$$\frac{3}{16}\frac{\omega_{p0}^2}{\omega^4}\frac{T_e}{m_e}\frac{d^2E_z}{dz^2}+\left(1-\frac{\omega_{p0}^2}{4\omega^2}\right)E_z=-\frac{e\omega_p^2(z)}{16m_e\omega^4}\frac{dE_1^2}{dz}. \tag{5.11}$$

In the vicinity of the point $2\omega = \omega_p(z)$, substituting and converting to dimensionless variables:

$$E_y=(16\pi n_0T)^{1/2}\frac{\omega}{\omega_{p0}}\left(\ln^{1/2}\frac{\omega_{p0}^2}{4\omega^2}-\frac{\omega_{p0}(z-z_0)}{c}\right)$$

(the smallness of E_z in comparison with E_y makes possible neglect of the E_z contribution to the wave pressure), we get the following equation from (5.11):

$$E_z=\mathscr{E}(16\pi n_0T)^{1/2}\frac{\omega}{\omega_{p0}}\ln^{1/6}\frac{\omega_{p0}^2}{4\omega^2}\left(\frac{v_T\omega_{p0}}{3c\omega}\right)^{1/3}$$
$$z-z_0=\frac{\zeta}{2}\left(\frac{3v_T^2c}{\omega_{p0}^2\omega}\right)^{1/3}\left(\ln\frac{\omega_{p0}^2}{4\omega^2}\right)^{-1/6}$$

$$\frac{d^2\mathscr{E}}{d\zeta^2}+\zeta\mathscr{E}=-1.$$

With the help of the solution of this equation

$$\mathscr{E}=-\int_c^\infty \sin\left(t\zeta-\frac{t^3}{3}\right)dt$$

it is not difficult to find the asymptotic form of the longitudinal field E_z for $\zeta \gg 1$:

$$\mathscr{E}(\zeta)\cos 2\omega t=-\frac{1}{4}\frac{\pi^{1/2}}{\zeta^{1/4}}\left\{\exp\left[i\left(\frac{2}{3}\zeta^{3/2}-\frac{\pi}{4}-2\omega t\right)\right]+\text{c.c.}\right. \\ \left.+\exp\left[i\left(\frac{2}{3}\zeta^{3/2}-\frac{\pi}{4}+2\omega t\right)\right]+\text{c.c.}\right\}. \tag{5.12}$$

In this formula, the first two terms correspond to a plasma wave propagating in the direction of the inhomogeneity, and the next two to the reflected plasma wave. The energy flux in the plasma wave, $E_z = E(z)\cos(\alpha(z)-\omega t)$ is equal to

$$S_l=\frac{3}{8\pi}\frac{T_e}{m_e\omega_p}E^2(z)\frac{d\alpha}{dz}. \tag{5.13}$$

Then, with the help of (5.12), we get

$$S_l=\frac{\pi}{2}n_0T\frac{\omega}{\omega_{p0}}\frac{v_T^2}{c}\ln^{1/2}\frac{\omega_{p0}^2}{4\omega^2}, \tag{5.14}$$

which corresponds to the coefficient of transformation to plasma oscillations $R_{2\omega}$, determined by Eq. (1.6).

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1) The force of the high-frequency pressure has been taken into considerations previously [5-7], but in these papers the case considered was that of normal incidence of the wave, when linear transformation into plasma oscillations is lacking.

2) Actually, along with the transformations considered in this paper, beginning with sufficiently small threshold amplitudes of the field (see [2]), dissipation of a part of the electromagnetic energy takes place due to decay processes. The approach used by us, in which the various dissipation mechanisms are considered independently, is approximate to a significant degree.

3) The jump in the density arises when the roots of Eq. (2.6) approach so closely that account of the imaginary part of ϵ becomes important. In this case, discontinuous solutions occur for ϵ and E_z with hysteretic dependence on the amplitude of the incident wave. [9,10]

1 Proceedings (Trudy) of the 7th International Conference on Quantum Electronics, Montreal, May, 1972.
2 A. A. Galeev, L. Laval', T. O'Neil, M. Rozenblyut and R. Z. Sagdeev, Zh. Eksp. Teor. Fiz. **65**, 973 (1973) [Sov. Phys.-JETP **38**, 482 (1974)].
3 V. L. Ginzburg, Rasprostranenie elektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in a Plasma) Fizmatgiz, 1960.
4 A. D. Piliya, Zh. Tekh. Fiz. **36**, 818 (1966) [Sov. Phys.-Tech. Phys. **11**, 606 (1966)].
5 R. Z. Sagdeev, in: Fizika plazmy i problemy UTR (Physics of Plasma and Problems of Controlled Thermonuclear Reactions), Akad. M. A. Leontovich (ed.), Izd. Akad. Nauk SSSR **3**, 346 (1958).
6 T. F. Volkov, ibid. **3**, 336 (1958).
7 N. S. Erokhin and S. S. Moiseev, in: Voprosy teorii plazmy (Problems of Plasma Theory), Acad. M. A. Leontovich (ed.), Atomizdat, p. 146, 1973.
8 A. Caruso, Plasma Phys. **10**, 963 (1968).
9 V. B. Gil'denburg, Zh. Eksp. Teor. Fiz. **46**, 2156 (1964) [Sov. Phys.-JETP **19**, 1456 (1964)].
10 V. B. Gil'denburg, I. G. Kondrat'ev and A. A. Markov, Izv. Vuzov, Radiofizika **12**, 655 (1969).

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