Thermoelectric effects in superconductors

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A theory of thermoelectric effects in isotropic superconductors is constructed. In the presence of a temperature gradient, a current of normal excitations, proportional to the temperature gradient, is produced in the superconductor. An equal and opposite superconducting current appears in the volume of the bulky superconductor. This current can be observed in a superconducting interferometer by the difference between the order-parameter phases on superconductor end surfaces having different temperatures. It a closed circuit is made up of two different superconductors and a temperature difference is produced on their contacts, an additional magnetic flux is produced and can be measured in experiment.

1. INTRODUCTION AND FORMULATION OF PROBLEM

In the presence of a temperature gradient in a closed circuit consisting of normal conductors, a thermoelectric current is produced whereas a potential difference (thermal emf) is produced at the terminals of an open circuit. The current density j produced in a normal conductor[1] by the gradient of the temperature T and by the electric field

$$E = -\nabla(U + \mu/e), \qquad (1.1)$$

is

$$j = -\eta \nabla T + \sigma E, \qquad (1.2)$$

where U is the electrostatic potential, μ is the chemical potential, e is the electron charge, and σ is the conductivity. For an open-circuited conductor (j = 0) this yields the thermoelectric field E:

$$E = \frac{\eta}{\sigma} \nabla T, \qquad (1.3)$$

where η/σ is the so-called differential thermoelectric power. In experiment one always measures the difference $\eta_1/\sigma_1 - \eta_2/\sigma_2$ (the relative thermoelectric power) for two different conductors.

As is well known, no stationary electric fields can exist in superconductors. In addition, no volume current can flow in bulky superconductors, with dimensions much larger than the penetration depth δ of the magnetic field. These properties of superconductors should cause the thermoelectric effect to occur in them in a manner different and much more peculiar than in ordinary conductors. Our purpose in this article is to study this effect. We start with a qualitative description, and then proceed to a quantitative solution of the problem.

In a superconductor, at temperatures lower than T_c, there is a superconducting condensate of Cooper pairs responsible for all the distinguishing properties of the superconductors. In addition, it contains normal excitations. They can be scattered by the phonons and by the impurity atoms, i.e., they behave in the same manner as conduction electrons in a normal metal, and differ from the latter only in the dispersion law. In particular, a temperature gradient produces a volume current of normal excitations[2]

$$j^T = -\eta \nabla T, \qquad (1.4)$$

where η denotes now the corresponding kinetic coefficient of the superconductor. There is no term proportional to $E = -\nabla(U + \mu/e)$ in this expression, since the

electrochemical potential in a superconductor in the stationary state should be constant.

The volume current in bulky superconductors, however, should be equal to zero. This means that a volume superconducting current j^S, i.e., a Cooper-pair condensate current, should appear and cancel out the normal current:

$$j^s + j^T = 0. \qquad (1.5)$$

Thus, an impression may be gained that there is no thermoelectric effect at all, since in principle there should be no electric field in the superconductor, and the total volume current is also equal to zero. This would be the case were we not able to measure separately either of the two currents that add up to zero. In fact, however, the current j^S can be directly measured. Let us explain how this can be done.

A superconductor is a system whose quantum properties become manifest on a macroscopic scale. The state of a Cooper-pair condensate is described by specifying the order parameter—by means of the wave function

$$\Delta(r) = |\Delta(r)| e^{i\varphi(r)},$$

which characterizes the entire assembly of superconducting electrons. In the stationary state, in which we are interested, it depends only on the coordinate r and does not depend on the time.

The density of the superconducting current is proportional to the gradient of the phase φ:

$$j^s = \frac{e\hbar N_s}{2m} \nabla\varphi, \qquad (1.6)$$

where m is the effective mass of the free electron and N_S is the so-called concentration of the superconducting electron[2]. At T = 0 it is equal to the total electron concentration N_0, and near the transition point T_c we have

$$N_s(T) = 2N_0(T_c - T)/T_c. \qquad (1.7)$$

Thus, a phase difference of the order parameter[3] $\Delta\varphi = \varphi_2 - \varphi_1$ is produced at the end points of the superconductor section at which a temperature difference $\Delta T = T_2 - T_1$ has been produced. This phase difference can be measured with the aid of a quantum Josephson interferometer. The method of measuring the phase difference is discussed in detail in Sec. 3 below. As a result, the superconducting current j^S, meaning also the normal-excitation current j^T, is a measurable quantity. Thus, the simplest thermoelectric phenomenon in a superconductor consists in the appearance of an irreversible flux of normal excitations j^T, proportional to

the temperature gradient. The thermoelectric coefficient η will be calculated in Sec. 2.[4] However, a direct observation of the phase difference in a superconducting interferometer is by far not the only method of investigating thermoelectric effects in an isotropic superconductor. In Sec. 4 we discuss another such method, wherein one observes the additional magnetic flux that penetrates into a section of a closed circuit consisting of two different superconductors, the junctions of which are kept at different temperatures. The value of this flux depends on the temperature difference and can be measured in experiment.

We have considered both the case of a circuit made up of bulky superconductors (of thickness much larger than the field penetration depth δ), and the opposite limiting case of a circuit made up of thin superconductors. What is the result of the study of the thermoelectric effects in superconductors? By determining the value of η, we can assess the relaxation processes in which normal excitations of the superconductor take part. In addition, once a sufficient sensitivity is reached in the method, these effects can be useful for the registration of the inhomogeneous heating of a superconducting thermoelectric circuit. Finally, measurement of η in the superconductor can be one of the methods of determining the absolute thermoelectric power. The point is that the coefficient η near T_C varies very little on going from the superconducting state to the normal state (see Sec. 2). This means that by determining the coefficient in the superconductor at a temperature somewhat lower than T_C we determine by the same token the value of this coefficient at a temperature somewhat higher than T_C.

2. CALCULATION OF THERMOELECTRIC COEFFICIENT

Our problem is to obtain an expression for the current density in a superconductor in the presence of a temperature gradient and with account taken of the motion of the condensate. To this end it would be possible to use for the kinetic equation the solution obtained in the review of Abrikosov and Khalatnikov[6] or in the book by Geĭlikman and Kresin[4]. With the aid of this solution it is easy to calculate the normal-excitation current, to which it is necessary to add the superconducting current. We, however, will proceed somewhat differently, and use the kinetic equation we have previously introduced[7] for excitations in a superconductor, where account is taken of the motion of the condensate. With the aid of such an equation we can obtain immediately the complete expression for the current, In the approximation linear in $\nabla\varphi$ and ∇T, as we shall show, the total current is indeed a sum of two terms, normal and superconducting. This method seems to us somewhat preferable from the methodological point of view, since it makes it possible to obtain in simpler fashion, when necessary, estimates for the higher-approximation corrections.

The kinetic equation for the distribution function n_p of the normal excitations, with allowance for the motion of the condensate, takes the form[7]

$$\frac{\partial \tilde{\varepsilon}_p}{\partial p}\frac{\partial n_p}{\partial r} - \frac{\partial \tilde{\varepsilon}_p}{\partial r}\frac{\partial n_p}{\partial p} + I\{n_p\} = 0, \qquad (2.1)$$

$$\tilde{\varepsilon}_p = \varepsilon_p + vp_s = (\xi_p^2 + |\Delta|^2)^{1/2} + vp_s. \qquad (2.2)$$

Here $v = p/m$, p is the quasimomentum of the electron,

$\xi_p = (p^2 + p_s^2)/2m - \mu$; $|\Delta|$ is the width of the superconducting gap; $I\{n_p\}$ is the operator for the collisions of the quasiparticles with the phonons and with the impurity atoms, and is of the usual form[8,7], but contains the energies $\tilde{\varepsilon}_p$ in the arguments of the δ functions in place of the energies ε_p. Finally, $v_s = p_s/m$ is the velocity of the superfluid motion of the condensate, the momentum p_s being connected with $\nabla\varphi$ and with the vector potential A by the gauge-invariant relation

$$p_s = \frac{\hbar}{2}\left(\nabla\varphi - \frac{2e}{\hbar c}A\right). \qquad (2.3)$$

In the derivation of (2.1), the external conditions are assumed to be stationary. In addition, we assume the superconductor to be pure enough, so that the electron mean free path in it greatly exceeds the coherence length.

We assume the temperature gradient (and the current j^T due to it) to be small enough. This means that the nonequilibrium increment of the excitation distribution function must be small in comparison with its equilibrium value. This condition is practically always satisfied in the experiments. In addition, we also assume that the velocity of the resultant superfluid motion is small enough to make unity much larger than the parameter

$$p_s v_F/|\Delta| \ll 1, \qquad (2.4)$$

and we confine ourselves to the first nonvanishing approximation in this parameter. The condition (2.4) may turn out to be relatively stringent near the transition point T_C itself.

Equation (2.1) must be supplemented with an equation for the quantity $|\Delta(r)|$. If the distribution function of the excitations were at equilibrium, then the corrections to the standard equation for $|\Delta|$ would be of second order in the small parameter (2.4), and this would be beyond the limits and accuracy of our calculation. In our case, the distribution function has a small nonequilibrium increment proportional to ∇T (see below), and the additional term in the equation for $|\Delta|$ can contain besides p_s^2 also the product $p_s \cdot \nabla T$. These terms will also be regarded as small and discarded. As a result we arrive at the conclusion that, at the accuracy assumed by us, $|\Delta(r)|$ is the equilibrium value of the gap at the temperature $T(r)$.

The expression for the current density in terms of the quasiparticle distribution function n_p, with allowance for the condensate motion, is[7]

$$j = 2e\int\frac{d^3p}{(2\pi\hbar)^3}[u_p^2 n_p + v_p^2(1 - n_{-p})](v + v_s),$$
$$\qquad (2.5)$$
$$u_p^2 = \frac{1}{2}(1 + \xi_p/\varepsilon_p); \quad v_p^2 = \frac{1}{2}(1 - \xi_p/\varepsilon_p).$$

In the linear approximation in the temperature gradient, we seek the solution of (2.1) in the form

$$n_p = n_0(\tilde{\varepsilon}_p) + n_p^{(1)},$$
$$\qquad (2.6)$$
$$n_0(\tilde{\varepsilon}_p) = [1 + \exp(\tilde{\varepsilon}_p/T(r))]^{-1},$$

where n_0 is an equilibrium function that depends on the local temperature $T(r)$. Substituting (2.6) in (2.5), we obtain in the approximation linear in the small parameter[6] proportional to p_s

$$j = j^s + j^T; \qquad (2.7)$$

$$j^s = 2e\int\frac{d^3p}{(2\pi\hbar)^3}\left\{v_s[u_p^2 n_0(\varepsilon_p) - v_p^2(1 - n_0(\varepsilon_p))] + v(vp_s)\frac{\partial n_0}{\partial \varepsilon_p}\right\}, \qquad (2.8)$$

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$$j^\tau = 2e \int \frac{d^3p}{(2\pi\hbar)^3} n_p^{(1)} \mathbf{v}. \qquad (2.9)$$

Taking into account the relation

$$N_s(T,\Delta) = N_0 \left[1 - \int_0^\infty dx\, \mathrm{ch}^{-2}\left(x^2 + \left(\frac{|\Delta|}{2T}\right)^2 \right)^{1/2} \right] \qquad (2.10)$$

we can rewrite (2.8) in the form

$$j^s = eN_s \mathbf{v}_s,$$

i.e., it is the density of the superfluid current. Expression (2.9) is the density of the normal current, and to calculate it it is necessary to solve the kinetic equation and determine the function $n_p^{(1)}$. We do so by assuming that the phonon scattering can be neglected in comparison with the impurity scattering, i.e., for a normal conductor the considered temperature region would correspond to the residual resistance.

To calculate $n_p^{(1)}$ we can put in (2.1) $p_s = 0$[7]. The solution of this equation in the case when scattering by impurities plays an important role is given in[4,6]. Substituting $n_p^{(1)}$ in the expression for the density of the normal current, we arrive at the following result for the coefficient η:

$$\eta = \eta_n G(\Delta/T), \qquad (2.11)$$

$$G(x) = \frac{6}{\pi^2} \int_x^\infty \frac{y^2 dy}{\mathrm{ch}^2(y/2)}, \qquad (2.12)$$

where η_n is the thermoelectric coefficient in the normal state. If the amplitude for scattering by impurities does not depend on the momentum transfer, then

$$\eta_n = \frac{2\pi^2}{9} \frac{eT}{m} \frac{d}{d\mathscr{E}} [\tau_n(\mathscr{E}) \nu(\mathscr{E})\mathscr{E}]_{\mathscr{E}=\mu}, \qquad (2.13)$$

$\mathscr{E} = \xi + \mu$, $\nu(\mathscr{E})$ is the density of states, and τ_n is the electron relaxation time in the normal state. We note that formula (2.11) is valid for any elastic scattering mechanism.

At $x \ll 1$ we have

$$G(x) = 1 - 2x^3/\pi^2, \qquad (2.14)$$

so that at $x = 0$ we obtain the known expression (see, e.g.[11]) for the thermoelectric power of the normal metal. At $x \gg 1$ we get

$$G(x) = 24\pi^{-2} x^2 e^{-x}, \qquad (2.15)$$

so that at $T \ll T_c$ the thermoelectric coefficient η is exponentially small.

Expression (2.14) for η differs from the formulas of Geĭlikman and Kresin[4,5]. The reason for the discrepancy, in our opinion, is the following. Our initial expression for the current density is formula (2.9), whereas they use a formula in which $\mathbf{v}$ contains the quantity $\mathbf{v}\xi/\epsilon$, which is equal to the excitation rate $\partial\epsilon_p/\partial p$. That our expression is correct is easiest to verify in the following manner. It is necessary to take as the initial expression

$$j = e \sum_\alpha \int \frac{d^3p}{(2\pi\hbar)^3} \mathbf{v} \langle a_{p\alpha}^+ a_{p\alpha} \rangle, \qquad (2.16)$$

where $a_{p\alpha}^+$ ($a_{p\alpha}$) are the creation (annihilation) operators for an electron with quasimomentum p and spin α (the angle brackets denote averaging over the non-equilibrium density matrix of the system). It is then necessary to go over, with the aid of the Bogolyubov transformations, to the operators of the creation and annihilation of the quasiparticles in the superconductor, and as a result we obtain formula (2.9).

It should also be noted that the formula given in[4,5] for η differs from ours in order of magnitude. Namely, it gives a result larger than ours by roughly speaking a factor μ/T. Naturally, as $\Delta \to 0$ it does not make it possible to obtain the limiting transition to the normal metal[8].

3. MEASUREMENT OF THERMOELECTRIC EFFECT WITH THE AID OF A SUPERCONDUCTING JOSEPHSON INTERFEROMETER

We consider the superconducting interferometer shown schematically in Fig. 1. Arms I and II of the interferometer are assumed to be made of difficult superconductors. A temperature gradient is produced in the region B of each of the arms, whereas the other sections of the interferometer are at a constant temperature, T_1 or T_2. The Josephson junctions J_{0I} and J_{0II} are also at a temperature T_2. For simplicity, assume first that the Josephson junctions J_{0I} and J_{0II} have the same critical current J_c (the calculation of the general case leads to more cumbersome calculations). Then the currents through arms I and II are connected with the phase discontinuities $\Delta\varphi_I$ and $\Delta\varphi_{II}$ across the junctions by the known relations

$$J_I = J_c \sin\Delta\varphi_I, \quad J_{II} = J_c \sin\Delta\varphi_{II}. \qquad (3.1)$$

The expression for the current density in the volume of the superconductor is written, as we have seen, in the following gauge-invariant form:

$$j = \frac{e\hbar N_s}{2m}\left(\nabla\varphi - \frac{2e}{\hbar c}\mathbf{A} \right) - \eta\nabla T. \qquad (3.2)$$

On the other hand, as is well known, the density of the volume current at a distance much larger than the penetration depth from the superconductor surface should be equal to zero. Integrating (3.2) along a closed contour in the interior of the superconductors at a distance much larger than δ from their surfaces, we obtain

$$0 = \oint \nabla\varphi\, dr - \Delta\varphi_I + \Delta\varphi_{II} - \frac{2e}{\hbar c}\oint \mathbf{A}\, dr + (\theta_I - \theta_{II}), \qquad (3.3)$$

$$\theta \equiv \frac{2m}{e\hbar} \int_{T_1}^{T_2} \frac{\eta(T)}{N_s(T)} dT. \qquad (3.4)$$

The first integral in the right-hand side of (3.3) is equal to $2\pi n$, where n is an integer, inasmuch as the change in phase of the wave function on going over the closed contour should be a multiple of 2π. The integral is $\oint \mathbf{A}\cdot dr = \Phi$, where Φ is the magnetic flux linked with the contour. As a result, we can rewrite (3.4) in the form

$$2\pi n - \Delta\varphi_I + \Delta\varphi_{II} - 2\pi\Phi/\Phi_0 + (\theta_I - \theta_{II}) = 0,$$

where $\Phi_0 = \pi\hbar c/e$ is the magnetic-flux quantum.

We introduce the quantity $\Delta\varphi$ with the aid of the relations

$$\Delta\varphi_I = \Delta\varphi + \left(\frac{\pi\Phi}{\Phi_0} - \frac{\theta_I - \theta_{II}}{2}\right), \quad \Delta\varphi_{II} = \Delta\varphi - \left(\frac{\pi\Phi}{\Phi_0} - \frac{\theta_I - \theta_{II}}{2}\right) + 2\pi n. \qquad (3.5)$$

Then the current J in the external circuit in which the interferometer is connected can be represented in the form

$$J = J_I + J_{II} = 2J_c \cos\left(\pi\Phi/\Phi_0 - \frac{\theta_I - \theta_{II}}{2}\right)\sin\Delta\varphi. \qquad (3.6)$$

The maximum superconducting stationary current that can flow through the interferometer is therefore

$$2J_c \cos\left(\pi\frac{\Phi}{\Phi_0} - \frac{\theta_I - \theta_{II}}{2}\right). \qquad (3.7)$$
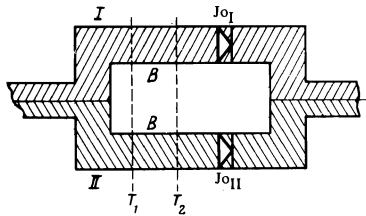
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FIG. 1

By measuring it we can directly assess the value of the thermoelectric effect in the superconductors. The magnitude of the thermoelectric effect can thus be characterized by the "thermoelectric angle" θ, which is determined by the expression (3.4).

If the critical currents J_{cI} and J_{cII} through the Josephson junctions J_{0I} and J_{0II} are different, we obtain for the maximum current through the interferometer the expression

$$\left[(J_{cI}-J_{cII})^2 + 4J_{cI}J_{cII}\cos^2\left(\pi\frac{\Phi}{\Phi_0} - \frac{\theta_I - \theta_{II}}{2}\right)\right]^{1/2}. \quad (3.7a)$$

We note that the conditions for the applicability of the initial relation (3.2), with the aid of which our result is obtained, include the requirement that the change of ∇T be small over a distance on the order of the field penetration depth δ, a condition that is practically always satisfied with large margin. It is immaterial here whether the external field penetrates into the superconductor in accordance with the London law or the Pippard law.

The thermoelectric effect in superconductors, just as in normal conductors, is a difference effect—it is proportional to the difference $\theta_I - \theta_{II}$. But whereas the ratio η/σ for the normal conductor depends little on the concentration of the impurity atoms, the coefficient η itself (meaning also the thermoelectric angle θ), being proportional to τ_n, is very sensitive to the impurity content. η can change by many orders of magnitude with changing impurity concentration. If it is desired to study the thermoelectric properties of some definite superconductor I, it must be chosen to be pure enough. The superconductor II can be dirty in this case, in order to satisfy the inequality $\theta_{II} \ll \theta_I$. The answer will then contain only the quantity θ_I. We consider henceforth this case (we omit the subscript I).

We proceed to the calculation and estimate of θ. If the relative change of both the width of the gap and of the temperature along the superconductor is small, then

$$\theta = 2m\eta\Delta T/e\hbar N_s, \quad (3.8)$$

i.e., the expected effect is proportional to $\Delta T = T_2 - T_1$, which is natural. It is assumed that $\tau_n \approx 10^{-9}$ sec, which is apparently close to the limit of modern experimental capabilities, then far from the transition temperature T_C the angle θ amounts to several degrees of angle at a temperature difference on the order of several hundredths of a degree.

Near the transition point itself, the coefficient η changes little and it can be assumed that $\eta_C = \eta(T_C)$. However, $N_S(T)$ changes strongly in this region, and we shall use expression (1.7) for its value. Substituting it in (3.4), we get

$$\theta = \frac{m\eta_c T_c}{e\hbar N_0}\ln\frac{T_c - T_1}{T_c - T_2}. \quad (3.9)$$

Near the transition point, the angle θ can amount to

several dozen degrees at a temperature difference of several hundredths of a degree. It must be borne in mind, however, that for superconductors with relatively high T_C the temperature region near T_C may not correspond to the residual resistivity. The phonon scattering may prevail in this temperature interval. In this case, formula (2.13) can no longer be used to estimate the coefficient η, and we can employ, for example, estimates of the coefficients η in a normal conductor on the basis of the experimental data at temperatures somewhat higher than T_C.

4. THERMOELECTRIC EFFECT IN A CLOSED CIRCUIT MADE UP OF DIFFERENT SUPERCONDUCTORS

We consider a closed thermoelectric circuit made up of two superconductors (Fig. 2). We investigate first the case when the transverse dimensions of the superconductors greatly exceeds the penetration depth δ. For the current densities in the superconductors I and II we have

$$j_{I,II} = -\eta_{I,II}\nabla T + \frac{e\hbar N_{sI,II}}{2m_{I,II}}\left(\nabla\varphi - \frac{2e}{\hbar c}A\right). \quad (4.1)$$

From this we get in the first and second superconductors, respectively,

$$\nabla\varphi = \frac{2e}{\hbar c}A + \frac{2m_I}{eN_{sI}\hbar}j + \frac{2m_I\eta_I}{eN_{sI}\hbar}\nabla T,$$

$$\nabla\varphi = \frac{2e}{\hbar c}A + \frac{2m_{II}}{eN_{sII}\hbar}j + \frac{2m_{II}\eta_{II}}{eN_{sII}\hbar}\nabla T. \quad (4.2)$$

Calculating the integral $\oint\nabla\varphi dr$ along the closed contour passing in the interior of the bulky superconductors, and equating it to $2\pi n$, we obtain

$$n = \Phi/\Phi_0 + (\theta_{II} - \theta_I)/2\pi, \quad (4.3)$$

whence

$$\Phi = \Phi_0[n - (\theta_{II} - \theta_I)/2\pi]. \quad (4.4)$$

Thus, the temperature difference produces an increment to the quantized magnetic flux linking with the section of the closed contour; this increment is proportional to the difference between the thermolectric angles of the two superconductors making up the circuit. If the magnetic flux in the absence of the temperature difference was equal to zero (i.e., at n = 0), then the effect reduces simply to the onset of a magnetic flux under the influence of the temperature difference.

We consider now a thermoelectric circuit made up of two thin superconductors of equal length P, the thickness of which is much less than the penetration depth δ. We assume that the current density J is constant over the cross section of the conductor[9], so that the total current is J = jS. Here S is the cross section area, which we assume to be the same in both superconductors. Calculating $\nabla\varphi$ from formula (4.2) and equating $\oint\nabla\varphi dr$ to $2\pi n$, we obtain

$$n = \frac{\Phi}{\Phi_0} + \frac{PJ}{\pi e\hbar S}\left(\frac{m_I}{N_{sI}} + \frac{m_{II}}{N_{sII}}\right) + \frac{\theta_{II} - \theta_I}{2\pi}. \quad (4.5)$$

But[10]

$$J = \Phi c/L, \quad (4.6)$$

where L is the self-induction of the circuit. From this we get

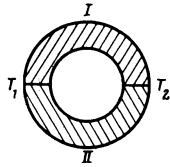$$\Phi = \Phi_0\frac{n - (\theta_{II} - \theta_I)/2\pi}{1 + (Pc^2/e^2SL)(m_I/N_{sI} + m_{II}/N_{sII})}. \quad (4.7)$$

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FIG. 2

It should be noted, incidentally, that the angles θ in the thin samples can by themselves not be as large as in bulky ones, inasmuch as the coefficient η is proportional to the mean free path of the excitations, which in the case of a thin sample is at best of the order of the diameter of its cross section. The latter, as we have agreed, is less than the penetration depth δ. Therefore the largest effect should be expected at temperatures close to T_c, when the depth of penetration δ is relatively large, and for this reason the thickness of the sample can be chosen to be not too small.

Let us explain the qualitative nature of this effect, for example for the case of bulky superconductors. Inasmuch as the thermoelectric current $j^T = -\eta \nabla T$ in the interior of a superconductor with a temperature difference at the its terminals is cancelled out by the superconducting current, an additional phase difference $\theta_{II} - \theta_I$ is produced on going along a closed circuit passing through the interior of the superconductors. On the other hand, however, the total change of phase on going along the circuit should be equal to $2\pi n$, as before. This means that an additional magnetic flux must be produced by the surface currents and lead to cancellation of this thermoelectric phase difference.

We discuss in conclusion the following question. What is the value of the additional magnetic flux produced in the circuit of the Josephson quantum interferometer shown in Fig. 1 when a temperature difference is produced? For simplicity we consider again the case when the critical currents J_c of the two Josephson junctions are equal. The circulating current flowing through the interferometer and producing the additional magnetic flux is

$$-2J_c \sin\left(\pi\frac{\Phi}{\Phi_0} - \frac{\theta_I-\theta_{II}}{2}\right)\cos\Delta\varphi. \qquad (4.8)$$

If we denote by Φ_e the flux produced by the external sources of the magnetic field, then the total flux is

$$\Phi = \Phi_e - \frac{2LJ_c}{c}\sin\left(\pi\frac{\Phi}{\Phi_0} - \frac{\theta_I-\theta_{II}}{2}\right)\cos\Delta\varphi. \qquad (4.9)$$

This equation determines implicitly the function $\Phi(\Phi_e)$, i.e., the dependence of the total magnetic flux on the external flux.

If the critical current is so small that

$$2\pi LJ_c/c\Phi_0 \ll 1, \qquad (4.10)$$

then we have in the lowest order approximation $\Phi = \Phi_e$. In the next higher approximation

$$\Phi = \Phi_e - \frac{2LJ_c}{c}\sin\left(\pi\frac{\Phi_e}{\Phi_0} - \frac{\theta_I-\theta_{II}}{2}\right)\cos\Delta\varphi. \qquad (4.11)$$

The temperature dependence enters, as we see, only in the second term in the right-hand side of (4.11), which is a correction term that amounts, by virtue of (4.10), to a small fraction of the flux quantum Φ_0.

We proceed to the case when the parameter $2\pi LJ_c/c\Phi_0 \gtrsim 1$. If at the same time the measuring current (3.6) is such that even at

$$\frac{2\pi LJ_e}{c\Phi_0}|\cos\Delta\varphi| \gtrsim 1, \qquad (4.12)$$

then the dependence of Φ on Φ_e and $\theta_I - \theta_{II}$ can become multiply valued (see the book of Kulik and Yanson[10], p. 126), so that definite values of Φ_e and $\theta_I - \theta_{II}$ can correspond to several values of the flux Φ. The function Φ can then experience finite discontinuities, when its arguments are varied.

Thus, the maximum current in a quantum interferometer has a simple dependence on the difference of the thermoelectric angles only if the inequality (4.10) is satisfied. Otherwise the connection between these quantities becomes much more complicated.

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