

# Heterodyne reception of light by an injection laser

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The injection laser is treated as a heterodyne light receiver. It is shown that extraneous radiation entering into the active region of the laser diode produces a response in its electric circuit at a frequency equal to the difference between the frequencies of the received and generated radiation. The gain, the resolving power, and the sensitivity of such a receiver are obtained.

## 1. INTRODUCTION

In spite of the fact that the first laser was produced 13 years ago, its use for practical communication in the optical band is only now becoming possible. The conditions for this possibility have been created by a number of advances both in generation and reception of coherent optical radiation. On the one hand, many lasers operating with stable operation have by now been developed; on the other hand, much progress was made in the receiver development. There are two types of coherent optical radiation receivers. The first is based on the photodetection principle, i.e., the direct conversion of an optical signal into an electrical signal. These are ordinary photodiodes and photoresistors. The receivers of the second type are based on the principle of optical heterodyning<sup>[1]</sup>.

In an optical heterodyne receiver, the registered radiation is superimposed with the aid of a semitransparent mirror on the radiation from a heterodyne laser. The resultant radiation is directed to an ordinary square-law mixer-photodetector (for example, a photoresistor or a photodiode). If the frequencies of the radiation signal and of the heterodyne laser differ, the resultant flux experiences beats at the difference frequency. These beats cause oscillations of the photoreponse of the detector at the same difference frequency.

The main advantages of heterodyne reception are, first, the exceedingly high sensitivity, which reaches the quantum limit, and second, the exceedingly low ratio of the width of the input band to the carrier frequencies, i.e., the high interference immunity<sup>[2]</sup>. (We recall that the carrier frequency is the frequency of the light).

In this paper we determine the impedance of a semiconductor injection laser in the generation regime and the change is produced in the current flowing through the laser diode under the influence of extraneous radiation incident on its active region. We shall verify that the extraneous radiation produces in the electric circuit of the laser diode an alternating current component with frequency equal to the difference frequency of this radiation and of the radiation generated by the laser. The reason for the onset of the current oscillations consists in the following: The current flowing through the laser diode  $J$  can be divided into two parts. The first part, equal to the threshold generation current  $J_{th}$ , is due to the spontaneous and nonradiative recombinations of the electrons and holes in the active region. The remaining part  $J - J_{th}$  is due to stimulated recombination of the carriers and is proportional to the square of the electric field  $E(t)$  of the radiation in the active region. If the frequency of the extraneous radiation differs from the laser-generation frequency, then  $E^2(t)$  experiences

beats, and consequently the current in the diode circuit also oscillates.

Thus, the injection laser is a heterodyne receiver in which both the heterodyne and the mixer are combined in a single element. Analogs of such a system are used in radio engineering<sup>[1]</sup>. Unlike the radio band, however, in the optical band such a superposition offers a fundamental advantage over the usual optical-heterodyne system. In the case of optical heterodyne receiving, it is necessary that the directions of the wave vectors of the signal and of the heterodyne coincide accurate to the ratio of the wavelength to the dimensions of the photosensitive area of the detector<sup>[1]</sup>. In other words, the signal field and the heterodyne field should be in either the same mode or in very close modes. In the traditional reception scheme, this requirement imposes stringent limitations on the permissible relative displacement of the elements of the system (for example, changes in the orientation of the semitransparent mirror or of the heterodyne relative to the photoreceiver). It is clear that in the case of an injection laser, whose sensitive layer serves simultaneously also as the photosensitive region of the mixer-detector, this difficulty is automatically eliminated. In the injection laser, the active region, owing to the large dielectric constant in comparison with the neighboring regions, is a thin flat dielectric waveguide. The mode generated by the laser is localized on this waveguide<sup>[3-5]</sup>. We shall show that current beats are produced in the laser-diode circuit only if the signal field is in the same mode in which the laser-generated radiation is concentrated. This raises the problem of introducing the signal radiation into this mode. It appears that this can be done most effectively by using diffraction gratings deposited on the waveguide or on a crystal surface close to the waveguide layer<sup>[6,7]</sup> (Fig. 1). In addition, it can be assumed that with respect to the optical matching, the injection laser is well compatible with the optics fibers. This is of particular interest since fiber-optics systems with exceedingly small damping (4 dB per kilometer) have by now been developed<sup>[8]</sup>.

Another important advantage of a receiver based on an injection laser lies in the miniaturization that is traditional for semiconductor devices. We shall verify below that such a receiver has the maximum quantum sensitivity inherent in heterodyne receivers.

The operating speed of an ordinary optical heterodyne receiver meaning also its frequency bandwidth, is determined by the operating speed of the photodetector. For semiconductor photodetectors, the minimum response time is not less than the lifetime of the non-equilibrium carriers. In the case of a receiver based on a laser diode this time can be much shorter. The

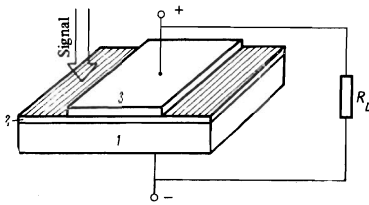


FIG. 1. Schematic diagram of receiver. 1—n-emitter of diode, 2—its active layer with diffraction grating on the outer surface, 3—p-emitter,  $R_L$ —load resistor.

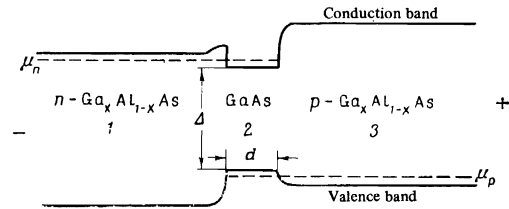


FIG. 2. Band diagram of a forward-biased heterolaser.

reason is that in the lasing regime the optical field in the active regions stimulates electron transitions between bands and by the same token shortens the response time.

The possibility of using an injection laser as a heterodyne receiver becomes particularly attractive in connection with one more advantage. It is known<sup>[9]</sup> that current modulation in the supply circuit of an injection laser leads to a modulation of its radiation, and the modulation band coincides with the reception band. Thus, the same device can be used both as a transmitter and as a receiver.

## 2. CONVERSION COEFFICIENT AND IMPEDANCE OF LASER DIODE

As the concrete model we consider an injection laser with two heterojunctions (heterolaser), the idea of which was first proposed by one of us (Kazarinov) with Alferov in 1963. This laser is a three-layer semiconductor structure in which the central, active layer is a semiconductor with a smaller forbidden-bandwidth than the neighboring electron and hole emitters. The energy scheme of the heterolaser is shown in Fig. 2.

The choice of this model is governed by two circumstances. First, heterolasers, by virtue of a number of physical factors (such as superior waveguide properties of the active layer, limitation of the recombination region, superinjection of the carriers<sup>[5]</sup>) have much lower current-density thresholds than ordinary injection lasers. They can therefore operate in the continuous regime even at room temperature<sup>[10,11]</sup>.

The second important advantage is that the thickness  $d$  of the active layer of the heterolaser can be much smaller than the diffusion length. The carrier-diffusion time across this layer, equal to  $d^2/d\bar{D}$  (where the reduced diffusion coefficient is  $\bar{D} = 2D_nD_p/(D_n + D_p)$ , and  $D_n$  and  $D_p$  are the electron and hole diffusion coefficients), can be made much shorter than the lifetime of the nonequilibrium carriers. If the characteristic frequencies of the variation of the carrier densities satisfy the inequality

$$\omega \ll 2\bar{D}/d^2, \quad (1)$$

then the carrier density in the active region can be regarded as independent of the coordinate. In this case the delay due to the diffusion processes turns out to be negligible.

Our problem is to find the impedance of a laser diode as it generates and when beats are produced in its electric circuit by introduction of an external radiation into the laser resonator. To this end it is necessary first to write down an equation for the electromagnetic field in the resonator. Since we are interested in the single-mode heterodyne-generation regime, and assume that the difference between the frequency of the extraneous radiation and the heterodyne frequency is low, in comparison with the difference between the frequencies of

the neighboring resonator modes, we can confine ourselves to the equation for the field in only this mode in which the laser generation takes place. It takes the form

$$\frac{d^2}{dt^2} E + \Omega^2 E + \frac{d}{dt} \left( \frac{4\pi\sigma}{\kappa} + \frac{\Omega}{2\pi Q} \right) E = \frac{4\pi}{\kappa} \frac{d}{dt} j. \quad (2)$$

Here  $E$  is the coefficient in the expansion of the electric field in the eigenfunctions of the resonator, and corresponds to the generated mode;  $\Omega$  is the natural frequency of this mode;  $\sigma$  is the diagonal matrix element of the real part of the high-frequency conductivity in terms of the eigenfunctions of this mode;  $\kappa$  is the similar matrix element of the dielectric constant. The parameter  $Q$  is the figure of merit of the mode. The quantity  $j$  is the diagonal matrix element in terms of the same eigenfunctions of the high-frequency current that induces the extraneous field.

The real part of the high-frequency conductivity  $\sigma$  depends on two components. The first describes the current connected with the interband transitions of the electrons, while the second describes the absorption of the light by the free carriers. Under inverted-population conditions, when the difference  $\mu_n - \mu_p$  between the Fermi quasilevels of the electrons and holes exceeds the width  $\Delta$  of the forbidden band of the semiconductor of the active region, the first component of  $\sigma$  is negative in the frequency range from  $\Delta/\hbar$  to  $(\mu_n - \mu_p)/\hbar$ <sup>[13]</sup>. The second component of  $\sigma$  is always positive. The necessary lasing condition is that the gain of the light due to the interband transitions exceed the absorption by the free carriers. In other words,  $\sigma$  must be negative. Under these conditions  $|\sigma|$  increases with increasing density of the nonequilibrium carriers in the active region.

Since the negative high-frequency conductivity in injection lasers is smaller by several orders of magnitude than the frequency  $\Omega$ , i.e., the gain per period is very small, we can use the well known Van der Pol method to solve Eq. (2)<sup>[14]</sup>. To this end, we represent  $E(t)$  in the form

$$E(t) = \mathcal{E}(t) \cos[\Omega t + \varphi(t)]. \quad (3)$$

Then the equation for the amplitude  $\mathcal{E}(t)$  and the phase  $\varphi(t)$ , which change little over the period  $2\pi/\Omega = \nu^{-1}$  of the optical oscillations, take the form

$$\frac{d}{dt} \mathcal{E}(t) + \frac{1}{2} \left( \frac{4\pi\sigma}{\kappa} + \frac{\nu}{Q} \right) \mathcal{E}(t) = -F_s(t), \quad (4)$$

$$\mathcal{E}(t) \frac{d}{dt} \varphi(t) - \frac{1}{2\Omega} \left( \frac{4\pi\sigma}{\kappa} + \frac{\nu}{Q} \right) \frac{d}{dt} \mathcal{E}(t) = F_\varphi(t). \quad (5)$$

Here

$$F_s = \frac{4\pi}{\kappa\Omega} \left\langle \frac{dj(t)}{dt} \sin[\Omega t + \varphi(t)] \right\rangle, \quad (6)$$

$$F_\varphi = \frac{4\pi}{\kappa\Omega} \left\langle \frac{dj(t)}{dt} \cos[\Omega t + \varphi(t)] \right\rangle,$$

where the angle brackets denote averaging over the period  $1/\nu$ .

In the absence of extraneous currents ( $F_S = F_C = 0$ ), Eq. (4) has the nontrivial stationary solution ( $\mathcal{E} \neq 0$ ,  $d\mathcal{E}/dt = 0$ ) if

$$4\pi\sigma/\kappa + \Omega/2\pi Q = 0. \quad (7)$$

This equation is the sufficient lasing condition. Since  $\sigma$  depends on the densities of the nonequilibrium carriers, Eq. (7) is in essence the equation for the carrier density in the active region under the lasing conditions.

If the generated power is not too high and the changes of the electron and hole distribution functions due to stimulated interruptions can be neglected<sup>2)</sup>, then  $\sigma$  does not depend on the field amplitude  $\mathcal{E}$ . Therefore, Eq. (7) means that at all currents through the laser diode in excess of the threshold value  $J_{th}$  the carrier density remains fixed and equal to its threshold value. Thus, the excess of the current above threshold leads not to an increase of the carrier density in the active region, but to an increase of the radiation power as a result of the increase in the probability of the stimulated transitions.

Generally speaking,  $\sigma$  is a function of two densities, the electron density and the hole density. In the situation considered here, however, the quasineutrality condition is satisfied with a high degree of accuracy, and the densities of the nonequilibrium electrons and holes are equal. Satisfaction of the quasineutrality condition is ensured by two factors. First, the characteristic times of variation of the current and carrier density of interest to us, as will be shown below, exceed the Maxwellian relaxation time in the active layer by several orders of magnitude. Second, the thickness of the active layer is assumed to be much larger than the Debye screening length in the active layer, which at the actual carrier densities is of the order of several dozen Angstrom units.

Moreover, we assume that the characteristic length over which the width of the forbidden band changes also exceeds the Debye length. In other words, we assume the heterojunction to be smooth, and it is precisely under these conditions that the band diagram shown in Fig. 2 is valid. The height of the potential barrier on the boundary between the n emitter and the active region, which prevents electron injection, is in this case much lower than for an abrupt transition<sup>[5]</sup>. Therefore the resistance of this barrier can be regarded as negligible.

Taking the foregoing into account, we can write down an equation relating the current  $J$  flowing through the laser diode with the total number  $n$  of nonequilibrium electrons and with the amplitude  $\mathcal{E}$  of the electromagnetic field:

$$\partial n/\partial t + R(n) - \sigma \mathcal{E}^2 V/2\Delta = J/e. \quad (8)$$

Here  $R(n)$  is the rate of recombination of the electrons in the entire volume of the active layer, due to the non-radiative spontaneous radiative transitions<sup>3)</sup> ( $e$  is the electron charge). Equation (8) was obtained by integrating the continuity equation for the electrons over the entire volume  $V$  of the active layer. We took into account here the fact that in lasers with a double heterostructure practically the entire recombination of the carriers occurs in the active region<sup>[5]</sup>. The first term in the left-side of (8) describes the current of carrier accumulation in the active region, while the last term describes the change of the recombination current as a result of stimulated transitions.<sup>4)</sup>

The electric field amplitude  $\mathcal{E}$  in the stationary regime and in the absence of extraneous currents ( $F_S = F_C = 0$ ) can be easily obtained from (8). It must be recognized here that starting with the generation threshold, as already mentioned above, the total number  $n_0$  of nonequilibrium electrons (holes) does not depend on the current and is determined as the solution of Eq. (7). Thus,

$$\mathcal{E}_0^2 = 16\pi^2 \hbar (J - J_{th}) Q/\kappa V e \quad (9)$$

where the threshold current is

$$J_{th} = eR(n_0). \quad (10)$$

Our problem consists of finding the change of the current  $\delta J$  under the influence of the extraneous radiation and the linear approximation in the extraneous currents. To this end it is necessary to vary Eqs. (4), (6), and (8) with respect to the deviations  $\delta \mathcal{E}$  and  $\delta n$  of the field amplitude and the concentration from the stationary values  $\mathcal{E}_0$  and  $n_0$ . In the course of variation it must be recognized that the relations (7) and (9) are satisfied under stationary conditions:

$$\frac{d}{dt} \delta \mathcal{E} - \frac{\mathcal{E}_0 \Omega \kappa}{4\pi Q n_0} \delta n = -F_s, \quad (11)$$

$$\mathcal{E}_0 d\varphi/dt = -F_c, \quad (12)$$

$$\left( \frac{d}{dt} + \frac{1}{\tau} \right) \delta n - \frac{\sigma \mathcal{E}_0 V}{\Delta} \delta \mathcal{E} = \frac{\delta J}{e}. \quad (13)$$

We have introduced here the effective carrier lifetime  $\tau$ :

$$\frac{1}{\tau} = \frac{J_{th}}{en_0} \eta + \frac{J - J_{th}}{en_0} \chi, \quad (14)$$

where

$$\eta = \left. \frac{\partial \ln R}{\partial \ln n} \right|_{n=n_0}, \quad \chi = \left. \frac{\partial \ln |\sigma|}{\partial \ln n} \right|_{n=n_0}.$$

Equations (11)–(13) must be supplemented by a relation between the variations of the number of carriers in the active region and the diode voltage  $\delta \Phi$ . The voltage  $\Phi$  across the p-n junction, as seen from Fig. 2, is equal to the difference between the electron and hole Fermi quasilevels, divided by  $e$ :

$$e\Phi = \mu_n - \mu_p. \quad (15)$$

Thus,

$$\delta \Phi = \Phi_0 \frac{\delta n}{n_0}, \quad \Phi_0 = \frac{\partial \mu_n}{\partial \ln n} - \frac{\partial \mu_p}{\partial \ln p}, \quad (16)$$

where  $\Phi_0$  is a quantity on the order of the Fermi energy of the electrons and holes, reckoned from the edges of the conduction and valence band, respectively.

Since we are interested in the linear response of the system, we confine ourselves to a monochromatic signal and choose the extraneous current in the form

$$j(t) = \text{Re } j_0 e^{-i(\omega + \omega)t}. \quad (17)$$

Then averaging in formula (6) yields

$$F_s = \text{Im } j_0 e^{-i\omega t + i\varphi}, \quad F_c = \text{Re } j_0 e^{-i\omega t + i\varphi}, \quad j_0 = 2\pi j_\omega / \kappa \Omega. \quad (18)$$

We shall henceforth assume the power of the extraneous radiation to be so small in comparison with the heterodyne power, that the heterodyne frequency locking does not occur in the range of frequencies  $\omega$  of interest to us. Then the signal does not influence the phase of the heterodyne, which we assume for the time being to be constant in time.

From (11), (13), and (16), taking (18) into account, we

can easily express the complex amplitude of the alternating current in the laser-diode circuit, in terms of the complex amplitudes of the variations of the voltage and of the extraneous current, describing the received radiation:

$$\delta J = e \left[ \left( -i\omega - \frac{\omega_0^2}{i\omega} + \frac{1}{\tau} \right) n_0 \frac{\delta \Phi_0}{\Phi_0} + i\sigma \frac{\mathcal{E}_0 V e^{i\omega t}}{\omega \Delta} f_0 \right]. \quad (19)$$

We have introduced here the symbol

$$\omega_0^2 = \frac{\Omega}{2\pi Q} \chi \frac{J - J_{th}}{en_0}. \quad (20)$$

In the absence of a signal ( $f_0 = 0$ ), formula (19) describes the admittance of the laser diode in the lasing regime. This admittance consists of three terms, equivalent to the capacitance, inductance, and the main conductivity. The first term is the susceptance and is connected with the accumulation of the carriers in the active layer. The second is the inductance due to the delay of the amplitude of the generated electromagnetic field relative to the change of the density (see Eq. (11)). The third, corresponds to the active conductivity and is due to carrier recombination.

Thus, from the electrical point of view the laser diode is equivalent to a high-pass filter made of a capacitor, inductance, and resistor connected in parallel. The resonant frequency of this circuit is  $\omega_0/2\pi$ . At  $(J - J_{th}) \sim J_{th}$  and at parameter values typical of the heterolasers based on solid solutions of gallium arsenide and aluminum arsenide ( $1/\tau \sim 10^{-8}$  sec,  $Q \sim 10^3$ ), this value turns out to be of the order of  $10^9$  Hz.

Formula (19) enables us to find the current at the frequency  $\omega$  produced by extraneous radiation in an electric circuit consisting of a laser diode connected in series with a load resistor  $R_L$ . The expression for this current follows from Kirchhoff's equation

$$\delta \Phi_0 + R_L \delta J_0 = 0 \quad (21)$$

and takes the form

$$\delta J_0 = \frac{-\Phi_0 (\sigma V \mathcal{E} / \Delta n_0 R_L) f_0 e^{i\omega t}}{\omega^2 - \omega_0^2 + i\omega (\tau^{-1} + \Phi_0 / en_0 R_L)}. \quad (22)$$

The power  $W$  delivered to the load resistor, equal to  $(1/2) R_L |J_0|^2$ , is, in accord with formula (22),

$$W = \frac{\Phi_0^2 \omega_0^2 \chi V |f_0|^2}{R_L n_0 4\pi \Delta [(\omega^2 - \omega_0^2)^2 + \omega^2 (\tau^{-1} + \Phi_0 / en_0 R_L)^2]}. \quad (23)$$

The quantity  $f_0$  contained in this formula is proportional to the amplitude  $\mathcal{E}_0$  of the received radiation. It is convenient to choose the proportionality coefficient such that

$$f_0 = \frac{\Theta}{2} \sqrt{\frac{Scv}{\chi V Q}} \mathcal{E}_0. \quad (24)$$

Here  $c$  is the velocity of light in vacuum and  $S$  is the area of the input aperture. The quantity  $\Theta$  characterizes the effectiveness with which the received radiation is introduced into the laser resonator. To explain the meaning of this quantity, let us consider a laser resonator in the absence of absorption and amplification. Under these conditions, the electric field  $E(t)$  takes again the form (3) and its amplitude  $\mathcal{E}(t)$  satisfies Eq. (4), in which  $\sigma = 0$ . Its solution is

$$\mathcal{E}_0 = f_0 / (-i\omega + v/2Q) \quad (25)$$

In the absence of absorption or amplification, the entire power entering the resonator is exactly equal to the power radiated by it. The latter is given by

$$P = \frac{\chi V}{8\pi} |\mathcal{E}_0|^2 \frac{v}{Q} = \frac{|\Theta|^2 P_s}{1 + 4\omega^2 Q^2 / v^2}, \quad P_s = \frac{c}{8\pi} |\mathcal{E}_0|^2 S. \quad (26)$$

where  $P_s$  is the energy flux of the received radiation entering the aperture.

Thus,  $|\Theta|^2$  is the ratio of the energy flux entering the resonator to the energy flux of the radiation incident from the outside in the case of exact resonance ( $\omega = 0$ ).

Using (23), (24), and (26) we can find the power conversion coefficient, which is equal to the ratio of the alternating-current power  $W$  delivered to the load resistor (or, as is customarily said, to the intermediate-frequency channel), to the power of the received signal ( $\lambda = \Phi_0 / en_0 \omega_0 R_L$ )

$$K(\omega) = \frac{W}{P_s} = \frac{|\Theta|^2 e \Phi_0}{4\pi \hbar \omega_0 Q \chi} \frac{\lambda (1 + 4\omega^2 Q^2 / v^2)}{[(\omega/\omega_0)^2 - 1]^2 + (\omega^2/\omega_0^2) (1/\omega_0 \tau + \lambda)^2}, \quad (27)$$

Figure 3 shows plots of  $K(\omega)$  for different values of the load resistor, i.e., the parameter  $\lambda$ . We see that in large values of  $R_L$  ( $\lambda \ll 1$ ) the dependence of the conversion coefficient on the frequency has a clearly pronounced resonant character, and it can be seen from (20) and (14) that the resonant frequency  $\omega_0$  and the resonance half-width  $1/\tau$  increase with increasing current. With decreasing  $R_L$ , i.e., with increasing parameter  $\lambda$ , the resonance spreads out, and when the value  $\lambda = \sqrt{2}$  is reached the frequency characteristic of the receiver becomes practically constant all the way to the frequency  $\omega = \omega_0$ . The conversion coefficient  $K(\omega)$  at typical values of  $\text{Ga}_x\text{Al}_{1-x}\text{As}$  heterolaser parameters, lies in the range from 1 to 10. Under resonance conditions, on the other hand, the maximum conversion coefficient can range from 10 to 100.

The fact that  $K(\omega)$  exceeds unity means that the system in question produces amplification. The reason is that the signal radiation, entering the resonator, finds itself in a medium with negative absorption. Therefore, the field induced by the signal in the resonator is much larger than that in the absence of amplification ( $\sigma = 0$ ).

From (11), (13), (24), and (26) we can easily obtain the corresponding power gain

$$K_1(\omega) = |\Theta|^2 \frac{(\omega^2 + v^2/4Q^2) [\omega^2 + (\tau^{-1} + \omega_0 \lambda)^2]}{(\omega^2 - \omega_0^2)^2 + \omega^2 (\tau^{-1} + \lambda \omega_0)^2}.$$

If  $\lambda \lesssim (\omega_0 \tau)^{-1} \ll 1$ , then  $K_1$  as a function of the frequency has a resonance at  $\omega = \omega_0$ , where it reaches a value  $|\Theta|^2 v^2 \tau^2 / 4Q^2$ . In a  $\text{Ga}_x\text{Al}_{1-x}\text{As}$  injection laser this value can amount to  $10^4 - 10^5$ .

The overall gain, however, is smaller than  $K_1(\omega)$ , since the next conversion stage, from light to current, entails a loss of power. The corresponding power conversion coefficient can be obtained from (13) and (16), and takes the form

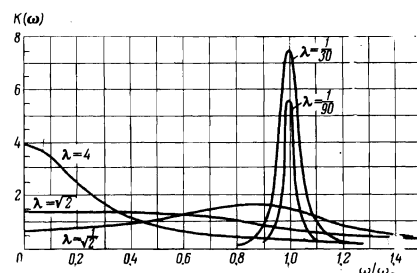


FIG. 3. Power conversion coefficient, in relative units, as a function of the frequency at various values of the load resistor.

$$K_2(\omega) = 2 \frac{e\Phi_0}{\Delta} \frac{(J - J_n)\Phi_0/R_L}{(en_0/\tau + \Phi_0/R_L)^2} \frac{1}{1 + \omega^2(\tau^{-1} + \Phi_0/en_0R_L)^{-2}}$$

This coefficient is much smaller than unity, first because of the small ratio of the energy of the electron and hole produced by the external photon and released in the external circuit (which is the order of  $e\Phi_0$ ) to the photon energy  $\Delta$ . Second, at high frequencies, and particularly at  $\omega = \omega_0$ , when  $K_1$  is maximal, the last factor in  $K_2$  is also small. The reason for the latter is that the carriers do not have time to recombine during the period of the oscillations, and the current cannot "follow" the variation of the number of quanta in the resonator.

Naturally, the total conversion coefficient (27) equals  $K_1K_2$ . In spite of the smallness of  $K_2$ , it can be, as already mentioned above, appreciably larger than unity.

### 3. RESOLVING POWER AND SENSITIVITY OF RECEIVER

We have assumed above that the phase  $\varphi$  of the heterodyne and the amplitude  $\mathcal{E}_0$  are independent of the time. It is well known, however, that both quantities fluctuate in random fashion. The phase fluctuations lead to a nonzero width of the spectral line of the heterodyne emission<sup>[16]</sup>, and consequently limit the frequency resolution of the receiver. The amplitude fluctuations of the heterodyne lead to beats of the quantity  $E^2(t)$ , meaning to the appearance of an alternating current component in the electric circuit of the laser diode. The resultant noise limits the sensitivity of the receiver. In addition, generation-recombination and Johnson noises are present in the same circuit.

Let us consider first the question of the influence of the phase fluctuation on the resolution of the receiver.

If the phase  $\varphi$  depends on the time then, as seen from (4) and (5), with allowance for (18), the response to a monochromatic signal is no longer monochromatic. It is easy to visualize that the spectral amplitude of the current  $\delta J_\omega$ , due to the extraneous radiation with frequency  $\Omega + \bar{\omega}$ , is determined in this case by formula (22) with the substitution

$$e^{i\varphi} \rightarrow (e^{i\varphi(t)})_{\omega-\bar{\omega}} \equiv \int dt e^{i\varphi(t) + i(\omega-\bar{\omega})t}. \quad (28)$$

Naturally, the power delivered to the load resistor is likewise no longer concentrated in one frequency, and is distributed over frequencies having a spectral density

$$W_{\omega,\bar{\omega}} = K(\omega) \overline{|(e^{i\varphi(t)})_{\omega-\bar{\omega}}|^2} P_s(\bar{\omega}), \quad (29)$$

where  $K(\omega)$  is given by (26) and the superior bar denotes averaging over the random forces causing the heterodyne-phase fluctuations.

As shown in<sup>[12]</sup>, if  $|\omega - \bar{\omega}|$  is much smaller than the characteristic reciprocal correlation times of the random forces, then

$$\overline{|(e^{i\varphi(t)})_{\omega-\bar{\omega}}|^2} = \frac{1}{\pi} \frac{2D}{(\omega - \bar{\omega})^2 + D^2}, \quad (30)$$

where  $D$  is the so-called phase diffusion coefficient, to which the half-width of the spectral line of the generator is equal. If it is assumed that the entire noise is due only to thermal and zero-point fluctuations of the field in the passive resonator, then the coefficient  $D$  is given by<sup>[12]</sup>

$$D = \frac{\Omega}{8\pi QN} \left( \frac{1}{e^{h\nu/kT} + 1} + \frac{1}{2} \right), \quad (31)$$

where  $kT$  is the temperature in energy units and  $N$  is the total number of quanta in the mode. For injection lasers with an active region of gallium arsenide at a power  $10^{-1}$  W, this value is of the order of  $10^4$  Hz. It is clear that it is precisely the quantity  $D$  which determines the frequency resolution.

We turn now to the question of the sensitivity of the system. We consider three causes of noise. The first is the thermal and zero-point fluctuations of the field in the passive resonator, the second is the generation-recombination noise, and the third is the Johnson noise in the electric circuit of the laser diode.

To describe the first and second causes, it is convenient to introduce into the right-hand sides of (11) and (13) the "random forces"  $\delta f$  and  $\delta G$ , respectively.

Then Eqs. (11) and (13) lead to equations for the Fourier amplitudes  $\delta \mathcal{E}_\omega$  and  $\delta n_\omega$ :

$$-i\omega \delta \mathcal{E}_\omega - \frac{\mathcal{E}_0 \delta \chi}{4\pi Q n_0} \delta n_\omega = -\delta f_\omega, \quad (32)$$

$$\left( -i\omega + \frac{1}{\tau} + \frac{\Phi_0}{en_0 R_L} \right) \delta n_\omega - \frac{\sigma \mathcal{E}_0 V}{\Delta} \delta \mathcal{E}_\omega = \delta G_\omega. \quad (33)$$

From this we easily obtain the spectral density of the power delivered to the load resistor as a result of the action of the random forces, i.e., the noise power

$$W_n \equiv \frac{1}{2} R_L |\delta J_n|^2 = \frac{K(\omega)}{|\Theta|^2} \frac{\kappa V Q}{2\Omega} \left( \overline{|\delta f_\omega|^2} + \frac{\Delta^2 \omega^2}{\mathcal{E}_0^2 V^2 \sigma^2} \overline{|\delta G_\omega|^2} \right). \quad (34)$$

Our problem now consists of finding the spectral density of the random forces  $|\delta f_\omega|^2$  and  $|\delta G_\omega|^2$ . We start from the assumption that the indicated spectral densities do not differ from those characterizing the noise in the system in the absence of generation ( $\mathcal{E}_0 = 0$ ).

This customarily employed assumption is based on the fact that the generation affects only one degree of freedom of the system, whereas the noise is connected with a macroscopically large number of degrees of freedom.

It is natural to assume that the spectral density of the random currents producing the fluctuation field in the resonator has no singularities at the frequency  $\Omega$ . Therefore the quantity  $|\delta f_\omega|^2$  can be regarded as slowly varying with frequency. To determine it we can use an expression for the mean-squared fluctuation of the field in the resonator in the absence of generation and amplification ( $\sigma = 0$ )

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \overline{|\delta \mathcal{E}_\omega|^2} = \overline{(\delta \mathcal{E}(t))^2} = \frac{8\pi\Delta}{\kappa V} \left( \bar{N} + \frac{1}{2} \right),$$

where  $\bar{N}$  is the average number of photons in the mode in the absence of generation and amplification.

Substituting here  $\delta \mathcal{E}_\omega$  expressed in terms of  $\delta f_\omega$  from Eq. (25), and taking  $|\delta f|^2$  outside the integral sign, we obtain

$$\overline{|\delta f_\omega|^2} = \Delta \left( \bar{N} + \frac{1}{2} \right) \frac{2\Omega}{Q\kappa V}. \quad (35)$$

We can obtain analogously also the quantity  $|\delta G_\omega|^2$ . To this end it is necessary to use the expression for the mean-squared fluctuation of the number of particles in the active region

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \overline{|\delta n_\omega|^2} = \overline{(\delta n(t))^2} = n_0$$

and Eq. (33) at  $\mathcal{E}_0 = 0$ , taking (16) and (21) into account.

As a result we obtain

$$|\delta G_{\omega}|^2 = 2n_0(1/\tau + \Phi_0/en_0R_L). \quad (36)$$

It is convenient to compare the noise power in the electric circuit of the laser diode with the equivalent power at the receiver input

$$NEP = W_n(\omega) \Delta\omega / K(\omega) 2\pi;$$

here  $\Delta\omega/2\pi$  is the reception bandwidth. Taking (24)–(36) into account, as well as the fact that  $W_n$  receives a contribution  $2kT(\Delta\omega/2\pi)$  from the Johnson noise in the electric circuit, we obtain

$$NEP = \frac{\Delta}{|\Theta|^2} \left[ 1 + 2\bar{N} + 2 \left( \frac{Q\omega}{v} \right)^2 \frac{J_{th}\eta + \Phi_0/R_L}{J - J_{th}} + \frac{2kT}{\Delta K(\omega)} \right] \frac{\Delta\omega}{2\pi}.$$

The first term in the square brackets describes the quantum fluctuations of the field and these, naturally, do not make it possible to register fluxes less than one quantum during the measurement time. The second term is due to the spontaneous and thermal radiation in the active medium. The calculation of  $\bar{N}$  is the subject of a separate investigation. Estimates show, however, that it is proportional to the coefficient of interband absorption of the semiconductor at the lasing frequency in the absence of injection. Depending on the concrete conditions, it ranges from  $10^{-1}$  to  $10^2$ .

The third and fourth terms describe respectively the recombination-generation and the Johnson noises. At  $K(\omega) \gtrsim 1$ , the last term is smaller than unity by at least a factor  $\Delta/kT$ . For gallium arsenide at room temperature we have  $\Delta/kT = 55$ .

We see that the generation-recombination term is inessential at

$$\frac{J - J_{th}}{J_{th}} \gg 2 \left( \frac{\omega Q}{v} \right)^2 \left( \eta + \frac{\Phi_0}{J_{th}R_L} \right).$$

Even at small values of  $R_L$ , corresponding to the curve  $\lambda = \sqrt{2}$  in Fig. 3, the right-hand side of this inequality at  $\omega = \omega_0$  is of the order of  $\sim Q/\Omega\tau \ll 1$ .

Thus, an injection laser can be used as a heterodyne receiver having a considerable preamplification of the optical signal and having, if the parameters are suitably chosen, a sensitivity close to the quantum limit.

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<sup>1</sup>In vacuum-tube superheterodyne receivers, for example, the 6A7 tube served as such a heterodyne mixer.

<sup>2</sup>In fact, owing to the small ratio of the interband-transition frequency to the reciprocal carrier-energy relaxation time, this is valid at any reasonable lasing power.

<sup>3</sup>The rate of hole recombination we assume to be equal to the rate of electron combination, inasmuch as at the injection levels needed for lasing the electron and hole densities are so large that the charge of the traps can be neglected.

<sup>4</sup>We have assumed the energy  $\hbar\Omega$  of the emitted quanta to be equal to the width  $\Delta$ , of the forbidden band, an assumption that is valid accurate to the ratio of the amplification bandwidth (which is equal to the sum of the excesses of the quasilevels of the electrons and holes over the edges of the corresponding bands) to the quantum energy. For GaAs lasers this ratio is of the order of  $10^{-2}$ .

<sup>5</sup>The first to point out the possibility of such amplification in solid-state lasers was Oraevskii [13]. A distinguishing feature of the injection laser consists in the dependence of  $K_1$  on the resistance of the external circuit, which enters in  $\lambda$ .

<sup>1</sup>M. Ross, *Laser Receivers*, Wiley, 1966.

<sup>2</sup>M. S. Teich, *Semicond. and Semimet.*, ed. by R. K. Willarson and A. C. Beer, Vol. 5, New York (1970), p. 361.

<sup>3</sup>A. L. McWorter, *Sol. State Electr.* 6, 417 (1963).

<sup>4</sup>R. F. Kazarinov, O. V. Konstantinov, V. I. Perel', and A. L. Effros, *Fiz. Tverd. Tela* 7, 1506 (1965) [*Sov. Phys.-Solid State* 7, 1210 (1965)].

<sup>5</sup>R. F. Kazarinov, *Fiz. Tekh. Poluprovodn.* 7, 735 (1973) [*Sov. Phys.-Semicond.* 7, 508 (1973)].

<sup>6</sup>M. L. Dakss, L. Kunn, R. F. Heidrich, and R. A. Scott, *Appl. Phys. Lett.* 16, 12 (1970).

<sup>7</sup>R. F. Kazarinov and R. A. Suris, *Fiz. Tekh. Poluprovodn.* 6, 1359 (1972) [*Sov. Phys.-Semicond.* 6, 1184 (1973)].

<sup>8</sup>*Electron. Design*, No. 25 (1972), pp. 30–32.

<sup>9</sup>T. L. Paoli and J. E. Ripper, *Proc. IEEE* 58, No. 10 (1970).

<sup>10</sup>Zh. I. Alferov, in: *Poluprovodnikovye pribory i ikh primeneniya*, (Semiconductor Devices and their Applications), Ya. A. Fedotov, ed., No. 25 (1970), p. 204.

<sup>11</sup>I. Hayashi, M. V. Panish, and F. K. Reinhart, *J. Appl. Phys.* 42, 1929 (1971).

<sup>12</sup>W. L. Lamb, *Theory of Optical Masers*, transl. in: *Kvantovaya Optika i Radiofizika* (Quantum Optics and Radiophysics), Mir, 1966.

<sup>13</sup>N. G. Basov, O. N. Krokhin, and Yu. M. Popov, *Zh. Eksp. Teor. Fiz.* 40, 1879 (1961) [*Sov. Phys.-JETP* 13, 1320 (1962)].

<sup>14</sup>A. A. Andronov, A. A. Vitt, and S. Kh. Khaikin, *Teoriya kolebaniy* (Theory of Oscillations), Fizmatgiz (1959).

<sup>15</sup>A. N. Oraevskii, *Tr. FIAN, Kvantovaya elektronika* (FIAN Proc., Quantum Electronics), 1965.

<sup>16</sup>S. M. Rytov, *Vvedenie v statisticheskuyu radiofiziku* (Introduction to Statistical Radiophysics), Nauka (1966).

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110