An upper limit on the density of gravitational radiation of extraterrestrial origin

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Experiments are described which have the purpose to verify the results obtained by Weber [1] in search for bursts of gravitational radiation from cosmic space. In distinction from Weber we have not observed coincidences in two independent gravitational antennas approximately 20 km apart. From the results of our measurements of gravitational radiation pulses with a frequency of at least one pulse per day it follows that the admissible spectral energy density does not exceed $H(\omega)\simeq 10^6$ erg/cm^2 Hz near a frequency of 1640 Hz. Reduced to the center of our galaxy, the energy of one such pulse of duration τ cannot exceed the value $(5\times 10^{-3}/\tau)M_{\odot}c^2$. In addition to a straight repetition of the Weber experiment proper, we have developed a method of statistical treatment of the data which is necessary for the detection of random bursts. This procedure will be employed in future experiments of this type with a higher sensitivity. The conclusion contains some remarks on the outlook for increasing the sensitivity of gravitational antennas.

1. THE MEASUREMENT METHOD

a) The Potential Sensitivity of the Antennas

A gravitational wave, representing a field of variable accelerations[1,2], will produce mechanical vibrations in extended solids. In principle these vibrations can be detected if they exceed the Brownian motion. The simplest detector for gravitational radiation consists of a dumbbell of length l, consisting of two masses m connected by a rod of rigidity $K = m\omega^2$. If the only source of noise are the thermal vibrations of this quadrupole oscillator, then in the classical approximation the smallest flux density I which can be detected is

$$I_{min}\approx c^3\varkappa T/2\pi G m\omega_0 Q l^2\tau, \qquad (1)$$

where κ is the Boltzmann constant, T is the temperature, c is the speed of light, G is the gravitational constant, τ is the duration of the action of gravitational radiation of frequency $\omega_{gr}\simeq\omega_0$, $Q=\omega_0\tau^*/2$ is the quality-factor of the oscillator and τ^* is the relaxation time. The relation (1) is valid, and if $\tau\ll\tau^*$, it is necessary for the detection of gravitational radiation that, e.g., the variation of the amplitude of vibrations of the antenna produced by the wave

$$\Delta A = l\tau[2\pi GI/c^3]^{1/2}$$

should be larger than the variation of the amplitude produced by the Brownian motion

$$\Delta A_{Br}=[2\varkappa T\tau/m\omega_0^2\tau^*]^{1/2}$$

(cr. [2]).

If in place of the dumbbell one uses a segment of a cylinder of length L and mass M, then I_{min} will differ insignificantly from (1): it is necessary to replace m by M/4 and l by $8L/\pi^2$. In the cylinders we have used $M\simeq 1.2\times 10^6$ g, L = 150 cm, $f_0=\omega_0/2\pi = 1640$ Hz, $Q = 1.8\times 10^5$ and $\tau = 1$ sec. This yields $I_{min}\simeq 4\times 10^5$ erg/sec-cm^2 at T = 300°K. These parameters are close to the parameters of the antennas used by Weber. The potential sensitivity of the antenna as determined by the relation (1), will be called "sensitivity (A)". This sensitivity can be attained only for a determined signal under the condition that the measurement can be repeated.

Earlier [3] the quantity $H(\omega)$—the spectral density of the gravitational energy-momentum of a pulse—was introduced for estimating the action on the detector. This quantity is convenient for characterizing astrophysical sources of radiation. For a short train of gravitational waves of duration $\tau\ll\tau^*$ of carrier frequency $\omega\lesssim\omega_0\pm\tau^{-1}$, the variation of the amplitude of the detector determines $H(\omega)$ uniquely for the train:

$$H(\omega)=(c^3/8\pi^2 Gl)(\Delta A)^2.$$

Measuring ΔA in units of the Brownian standard deviation $\Delta A = k\sigma_{Br} = l(\kappa T/m\omega_0^2)$, we obtain for our detector the estimate

$$H(\omega)=\frac{c^3}{8\pi^2 Gl}\frac{\varkappa T}{m\omega_0^2}k^2\approx 4.5\cdot 10^5\, k^2\ \text{erg/cm}^2\cdot\text{Hz}.$$

The minimal value of $H(\omega)$ which can be recorded by our detector corresponds to $k=(2\tau/\tau^*)^{1/2}$. Thus, $H_{min}(\omega)$ depends on the duration of the pulse. Making use of this value of k, as well as of the relation between $H(\omega)$ and the density of gravitational radiation $I=(4\pi/\tau^2)H(\omega)$, we again obtain the relation (1) for I_{min}.

b) Peculiarities of the Geometry and the Interference Immunity of the Antennas

In order to record small mechanical vibrations of the cylinder, corresponding to the first quadrupole mode ($f_0 = 1640$ Hz), we have used a capacitive transducer, the advantages of which over a piezoelectric transducer have been discussed already [4]. The use of a capacitive transducer have led to a complication of the antennas (Fig. 1). In order to fix the plate of the working capacitance of the transducer to the body of the cylinder, cantilever bars ("horns") have been machined into the body. The oscillation amplitude of the first mode can be determined from the change of the distance between the ends of the horns. In our case, with the length of each horn 33 cm, and the total antenna length 150 cm, the amplitude of oscillation between the ends of the horns was approximately 0.8 of the amplitude of oscillation between the ends of the cylinder, near the frequency 1640 Hz.

For protection against acoustic interference the cylinder was placed in a vacuum chamber. A simple calculation shows that in a vacuum with $p\lesssim 10^{-3}$ Torr the acoustic interference would not exceed the Brownian-motion fluctuations. During the measurements the pressure did not exceed 10^{-3} Torr. The steel vacuum chamber with

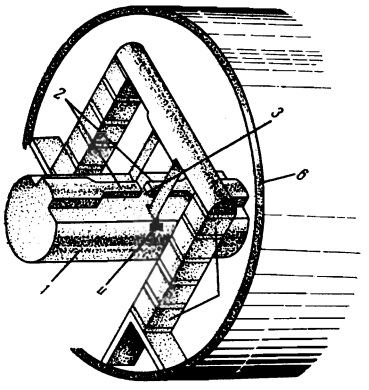

FIG. 1. A general view of the gravitational detector: 1—the aluminum cylinder-detector, 2—the cantilever bar "horns", 3—the place where the measuring capacitor of the displacement transducer is attached, 4—"dovetail" attachment of the ribbons, 5—the antiseismic filter, 6—vacuum chamber.

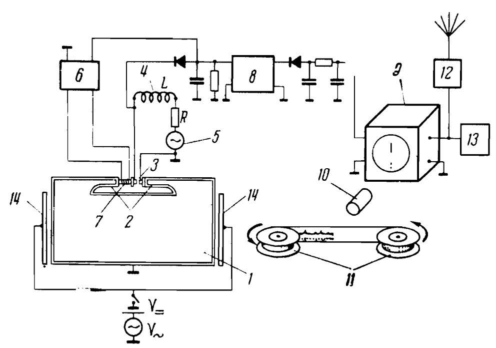

FIG. 2. Block-diagram of the measuring system: 1—the aluminum cylinder, 2—the "horns", 3—the measuring capacitor, 4—the inductance of the transducer LC circuit, 5—pumping quartz generator, 6—servo system, 7—correcting element of the servo system (electric oven), 8—narrow-band amplifier, 9—dual-trace oscilloscope, 10—objective lens, 11—film transport mechanism, 12—receiver for standard time radio signals, 13—time marker quartz generator (quartz clock), 14—calibration system.

walls 2 cm thick also served as a screen from possible ponderomotoric interference (cf., e.g., [5]), since the thickness of the skin-layer for steel is about 10^{-2} cm at $f_0 \simeq 1640$ Hz. Figure 1 also shows an antiseismic filter, similar to the one used by Weber [1]. The filter guaranteed a weakening of seismic interference by at least a factor of 10^{-13} at the eigenfrequency of the cylinder. The cylinder was suspended on thin aluminum plates (0.3 cm thick, 10 cm wide); the plates were attached to the cylinder and to the beam which rests on the filter by means of "dovetail" junctions.

The possible influences of dynamic gravitational fields in the induction zone, as well as the influence of cosmic rays on the antenna have already been discussed [3]; the simulation of the action by electric fluctuations in the transducer on the antenna will be discussed below. As was already noted in the brief description of the first series of observations [6], two such antennas, separated by 20 km, were used. In the same manner as in Weber's experiments, the axes of the antennas were oriented in the East-West direction.

c) The Capacitive Transducer and the Recording System

The plates of the parallel-plate capacitor making up the capacitive transducer were fixed between the ends of the horns (cf. Figs. 1 and 2). Together with the inductance L this capacitance formed an LC circuit with $Q_e \simeq 500$ and resonant frequency $f_e \simeq 5$ MHz. The frequency f of the driving generator (5 in Fig. 2) was tuned to coincide with the lateral slope of the resonance curve of the LC circuit. In this case a small change ΔA of the distance d between the plates leads to a change $\Delta U_\sim$ in the oscillation amplitude on the capacitor, equal to

$$\Delta U_\sim \approx {}^1/_2 U_\sim Q_e \Delta A/d, \qquad (2)$$

where $U_\sim$ is the capacitor voltage amplitude. In the normal regime of the transducer $U_\sim = 10-20$ V and $d \simeq 2\times 10^{-4}$ cm. For these values of the parameters the root mean square amplitude of Brownian oscillations (taking into account the transfer coefficient of the horn) is $(\overline{A^2_{Br}})^{1/2} = \sigma_{Br} = 6\times 10^{-14}$ cm and leads to $\Delta U \simeq 0.7-1.4$ μV. The driving generator noise allowed us to record in a band of 0.5 Hz width variations of the Brownian oscillations with an amplitude of $\Delta A \simeq \sigma_{Br}/4 = 1.5\times 10^{-14}$ cm, corresponding to a probable amplitude variation $\Delta A_{Br} \simeq \sigma_{Br}(2\tau/\tau^*)^{1/2}$ with $\tau^* \simeq 33$ sec and $\tau = 2$ sec. The distance between the plates of the transducer capacitor was maintained at a level of $d \simeq 2\times 10^{-4}$ cm by means of a servo system. The rectified voltage $U_\sim$ was fed to an amplifier (Fig. 2), to the output terminals of which was connected a small electric oven which heated by several degrees an aluminum bar glued between the end of one horn and one of the capacitor plates. The highest frequency of the servo system was of several tenths of a Hertz, and thus the servo system did not influence on the recording of the Brownian vibrations in the main part of the spectrum.

After detection of the r.f. signal its ac component with a bandwidth of $\Delta f = 1$ Hz near $f_0 = 1640$ Hz was amplified. The bandwidth of 1 Hz was produced by an electro-mechanical tuning-fork filter. This was followed by an amplitude detector and an active RC filter with a bandwidth $\Delta f = 0.5$ Hz. A more detailed description of the capacitive transducer can be found elsewhere [7]. The signal at the output of the filter, corresponding to the amplitude of Brownian oscillations, had the form of a slowly varying voltage with an average variation amplitude of approximately 1 V and a characteristic time of approximately 30 seconds. This voltage was fed to the input of the oscilloscope and the trace of the oscilloscope was photographed on a film traveling in front of the tube with a speed of 0.6 mm/sec (approximately 50 meters of film per 24 hours). The thickness of the trace on the film was 0.1 mm, allowing us to resolve time intervals $\Delta\tau \simeq 0.3$ sec. At the same time markers at intervals of one and 10 seconds were recorded on the film from a quartz clock and (for an exact synchronization of the two antennas) hourly signals of standard time. The whole recording equipment (in distinction from Weber's experiment) was situated next to the antenna. The radio frequency generator, the servo system and the amplifier were powered from storage batteries.

In distinction from the first series of measurements (briefly described in [6]) here all the elements of the capacitive transducer including the r.f. generator and the preamplifier, were placed inside the vacuum chamber and fixed on the antiseismic filter. This allowed us to get rid completely of microphonic noises and also to lower considerably the frequency and amplitudes of rare outbursts of non-Brownian origin, the sources of which seemed to be flicker-noises of the r.f. generator. For a force-calibration of the antennas use was made of the Coulomb force produced by a voltage applied between the ends of the aluminum cylinder and two discs parallel to these ends (cf. [7] for more details).

d) The Technique of Extracting the Information

The recordings of signals from both antennas were processed independently. First the standard deviation σ_{exp} was compared with the expected one, derived by means of the force calibration, using a χ^2 test on 50 points. The differences between σ_{exp} and σ_{cal} reached a relative value of 40%. This is caused mainly by the fact that the calibration was carried out at levels close to the noise level. The quantity ΔA_{cal} was usually selected of the order $(2-3)\sigma_{Br}$; in addition the χ^2 test yields relatively wide fiducial limits for the estimate of the variance. An operator would then scan the film, determining the exact time where over $\tau = 2$ sec the amplitude A had a variation $\Delta A \gtrsim k\sigma_{Br}$.

The quantity k was selected in such a manner that one could obtain several accidental coincidences per day. The films were scanned by different operators several times. The instants of time (corresponding to the middle of the $\tau = 2$ sec intervals where $A \gtrsim k\sigma_{Br}$) obtained independently by several operators on the two recordings from the two antennas were compared, and the number of coincidences and their absolute times were determined (the absolute time was fixed at the middle of the interval τ).

In order to bring the analysis scheme to a form as close as possible to the one used by Weber, in the second series of measurements described below the operators have not taken into account the shape (the quantity $\Delta A/\tau$) of rare bursts.

2. THE METHOD OF ANALYSIS OF COINCIDENCE

In this section we discuss the statistical analysis of the recorded signals from the gravitational antennas. In spite of the considerable time elapsed since the first experiments of Weber, the methods of treating the information extracted from such experiments have not been very systematic, which led to insufficiently founded conclusions in a number of papers [8,9]

As was noted in Sec. 1, the relation (1) represents the optimal sensitivity for a determined signal (with exactly known time of action or the possibility of repetition), if the only interference are Brownian fluctuations (the sensitivity (A)). Another definition has to be introduced for the detection of a random pulsed signal, about which no a priori information is available, and if in addition there is a possibility of interference of a random non-Brownian origin (which the experimenter cannot in principle ever exclude). Such a sensitivity (B) can be defined on the basis of the theory of optimal (radio) reception. In fixing the coincident variations of the signal in the two antennas, the operator must conclude on the presence (or absence) of a simultaneous action on the two antennas. In the absence of a priori information on the signal the optimal method of solving this problem is related to the Neyman-Pearson criterion. Assume that the operator has established that over the same time interval τ^{**} the average number of variations of the signal at a certain level is respectively $\bar{n}_1$ and $\bar{n}_2$. In our case we consider as variation of the signal a change in amplitude $\Delta A \gtrsim k\sigma_{Br}$ for $\tau = 2$ s. By changing the threshold k one could change the numbers $\bar{n}_1$ and $\bar{n}_2$ per 24 hours within a range from zero to several hundred. If such changes ΔA are independent, the expected value for purely accidental coincidences $\bar{N}$ with a resolution time τ_r is obviously equal to

$$\bar{N} = 2\tau_r \bar{n}_1 \bar{n}_2 / \tau^{**}. \quad (3)$$

If the operator (scanner) notices for some realization that N exceeds considerably some threshold value $N_{1-\alpha}$ then with a probability of error α (probability of false alarm or statistical error of the first kind) he can assert that a coincident action has been observed. For rare events the quantity $N_{1-\alpha}$ is simply related to $\bar{N}$:

$$\sum_{m=0}^{N_{1-\alpha}} \frac{(\bar{N})^m}{m!} \exp(-\bar{N}) = 1-\alpha. \quad (4)$$

To estimate the magnitude of the action, one introduces in the Neyman-Pearson test the omission probability β (the statistical error of the second kind).

In our case it is necessary to introduce the hypothesis that the antennas are subject to a sequence of signals of frequency N_0. Then

$$\beta(N_0) = \sum_{m=0}^{N_{1-\alpha}} \frac{(\bar{N}+N_0)^m}{m!} \exp[-(\bar{N}+N_0)]. \quad (5)$$

The values usually selected are $\alpha = 0.05$ and $\beta \lesssim 0.3$. Thus, the "gravitational signal" is characterized by two numbers N_0 and k; for a known σ_{Br} and other parameters of the antenna the latter yields a regular quantity, viz., the average value of the flux density I over the time interval τ.

However, the Neyman-Pearson scheme does not give any indications as to the selection of the quantity N_0. This quantity has to be sought from the outside. We have used the result of Weber as an indication: in his experiments coincident bursts were observed at least with a frequency of $N_0 = 1$ per 24 hours. This quantity has also determined the duration of our second series of measurements, 10 days. Setting $N_0 = 1$ day^{-1}, $\alpha = 0.05$ and $\beta \leq 0.3$ we could indicate the level k which determines the sensitivity of our installation. It is easy to show that the method used by us (i.e., to accept pulses if the change in amplitude of $\Delta A \gtrsim k\sigma_{Br}$ over the time interval τ) is considerably more sensitive than that used by Weber in his first series of measurements (where the intersection from below of the level $k\sigma_{Br}$ by the amplitude was fixed, counting from zero level).

If only Brownian oscillations are recorded in the antenna, the average $\bar{n}^+$ of intersections of the level $k\sigma_{Br}$ during the interval τ^{**} equals

$$\bar{n}^+ \approx \frac{\tau^{**}}{2\tau^*} \exp\left(-\frac{k^2}{2}\right), \quad (6)$$

and the average number $\bar{n}$ of changes of the amplitude by $k\sigma_{Br}$ during τ is:

$$\bar{n} \approx \frac{\tau^{**}}{2\tau}\left\{1-\Phi\left[k\left(\frac{\tau^*}{2\tau}\right)^{1/2}\right]\right\}, \quad (7)$$

where Φ is the probability integral. If $\tau^{**} = 1$ day, $\tau^* = 20$ sec, and $k = 2$, then $\bar{n}^+ = 10^4$, $\bar{n} = 10$ and consequently, short bursts are more easily detected on the background of rare accidental coincidences, corresponding to a change ΔA in the amplitude. The selection of this scheme is also justified by the circumstance that astrophysical estimates do not predict durations of the radiation of $\tau > 1-2$ sec (cf, e.g., [3]).

There is, in principle, another possibility of sorting out the bursts: large bursts of oscillations of the cylinder must have a relaxation tail, whereas electrically induced ones do not have it. However, such a selection is admissible only at relatively high levels $k>2$. For small pulses this rule is not valid: the duration of the tail $\tilde{\tau}$ for a pulse with a relative amplitude k equals $\tilde{\tau}\lesssim k^2\tau^*/2$. In the experiments of Tyson [8] $\tau^*\simeq 100$ sec; for $k=1/2$ he should have observed around 400 bursts with an average length of the tail of approximately 12 seconds (for the case of purely Brownian fluctuations).

One can show that the introduction of a delay τ_d in one of the channels is equivalent in effectiveness to the estimation of N_0 by the excess of N over the threshold $N_{1-\alpha}$.

Instead of simply following the number of coincident bursts the operator can go over to a more complicated experimental strategy, delaying the signal from one of the antennas by the time τ_d. In this case the parameter containing the information becomes the difference between the number of coincidences without delay and those with delay: $\Delta=N(0)-N(\tau_d)$. The average of Δ equals zero in the absence of a delay and tends to the average frequency N_0 of signal pulses for $\tau_d\gg\tau_r$. The random variable Δ is the difference between two Poisson random variables, which with good accuracy may be considered as independent for $\tau_d\gg\tau_r$. The characteristics of the detection which the operator has to use while following the values of Δ are defined by the equations

$$1-\alpha=\sum_{m=0}^{\infty}\frac{\bar{N}^m}{m!}\exp(-\bar{N})\frac{\Gamma(\Delta_{1-\alpha}+m+1,\bar{N})}{\Gamma(\Delta_{1-\alpha}+m+1)},$$

$$\beta(N_0)=1-\sum_{m=0}^{\infty}\frac{\bar{N}^m}{m!}\exp(-\bar{N})\frac{\Gamma(\Delta_{1-\alpha}+m+1,\bar{N}+N_0)}{\Gamma(\Delta_{1-\alpha}+m+1)}. \quad (8)$$

Here $\Delta_{1-\alpha}$ is the threshold value of Δ; $\Gamma(m, x)$ and $\Gamma(m)$ are respectively the incomplete and the ordinary gamma-functions. For large averages $\bar{N}$ the quantity Δ tends asymptotically to a normally distributed variable with dispersion $N(0)+N(\tau_d)$ and mean $N(0)-N(\tau_d)$. We note that the method of delayed coincidences does not yield any gain in sensitivity compared to the usual counting of coincidences; one may only consider it as an additional independent test.

In concluding this section we stop to consider the problem of estimating the absolute value of the gravitational action from the response of the antenna. In the majority of papers published on this subject one sees a tendency to estimate the energy of the pulse of gravitational radiation which excites the antenna in terms of the change in energy of the gravitational detector, expressed in terms of κT, i.e., as a fraction of the average energy of Brownian motion of the gravitational detector. In our opinion this method is not wholly correct for the following reasons:

1. For $\tau\ll\tau^*$ the change of the energy of the gravitational detector is

$$\Delta\varepsilon=m\omega_0^2(\Delta A_{gr}^2+2A_{fl}\Delta A_{gr}\cos\varphi),$$

where A_{fl} is the random amplitude of the gravitational detector at the time of arrival of the signal, ΔA_{gr} is the change in amplitude under the action of the signal in the absence of Brownian fluctuations and φ is the phase shift between the Brownian fluctuations and the signal.

Thus, only for intense signals, $\Delta A_{gr}\gg A_{fl}$, is the change in energy uniquely related to the magnitude of the gravitational action. In the opposite case, $\Delta A_{gr}\lesssim A_{fl}$ (which is of particular interest for measurements on antennas with a long relaxation time), $\Delta\epsilon$ will depend strongly on the quantities A_{fl} and φ. Under the most optimal conditions (for $\varphi=0$) the signal can change the energy of the antenna by the amount

$$|\Delta\varepsilon|=2m\omega_0^2A_{fl}\Delta A_{gr},$$

which is undetermined, owing to the random character of A_{fl}.

2. The usual concept of "effective cross section" of an antenna, $S=\Delta\epsilon/W$ (where W is the density of incident energy of the gravitational wave) loses its absolute meaning in the measurement of short actions $\tau\ll\tau^*$ owing to what was already said. It was shown [3] that in place of S one may introduce the quantity $A=\pi Gml^2\omega^2/c^3$ ($cm^2\cdot Hz$), which relates the change in energy of the detector to $H(\omega)$, namely $\Delta\epsilon=AH(\omega)$ for zero initial amplitude of the detector. Thus, the use of the variation of the energy of the antenna, $\Delta\epsilon$, as a measure of weak actions on a detector with large τ^* is incorrect.

An estimate of the minimally observable intensity of the gravitational radiation that excites the antenna can be obtained from the magnitude of the observable variation of the detector amplitude, ΔA over a prescribed time interval τ.

3. RESULTS OF THE OBSERVATIONS

We have carried out two series of measurements which are compatible. The first series (Series A) from January through March 1972 contained 20 days of clean time. The results of this series have been published earlier [6]. The basic conclusion was the assertion that there are no short bursts of gravitational radiation exceeding in energy flux density the value 10^7 erg/cm$^2\cdot$sec.

Below we list the results of a second series of measurements (Series B) with improved sensitivity. This series was carried out in February and March 1973 and contained approximately 10 days of clean measuring time.

The control tests which were carried out before the coincidence measurements were started showed that in the recording of the output signal of the antenna the operator measures the envelope of the Brownian oscillations of the cylinder. The following tests were carried out:

A. A comparison of the calculated dispersion of the output signal with the results of the force calibration (the averages over ten days of measurements did not differ by more than 20%).

B. A test of the hypothesis of Rayleigh distribution of the output signal with respect to a discrete sampling over a prolonged observation interval; on a significance level of 0.5 no deviation was noted within the $K(\lambda)$ test.

C. A comparison of the number of intersections n^+ per day for high levels $k\geqq 3$ with the calculated number, according to Eq. (6); agreement was within one standard deviation $\sim(n^+)^{1/2}$.

D. A comparison of the total variance of the output noise with the variance measured with a constant capacitance standard in place of the working transducer capacitance.

For the series B the noise standard deviation of electric fluctuations was approximately four times smaller than the Brownian standard deviation of thermal

k	Detector*		Detector**	
	IKI	MGU	IKI	MGU
1.5	280	360	200	230
2	280	170	40	50
3	90	90	20	20
4	35	35	15	15

Notes (to table). IKI denotes the detector at the Institute for Cosmic Research (IKI) and MGU denotes the detector at Moscow State University (MGU).

k is the threshold level in units of σ_{Br}.

*The number of bursts obtained by averaging over all recordings making up 10 days of pure measuring time; the possible deviation for an individual recording is ± 30%.

**The frequency of bursts obtained by selecting the quietest portions of the recordings, totaling four days.

mechanical oscillations of the gravitational detector: $\sigma_e \simeq 0.25\ \sigma_{Br}$. This relation was maintained on the average during the ten days of measurement. However, the nonideal nature of the insulation of the antennas led to the appearance of short-term random excesses over σ_{Br}. Bursts with durations of several seconds were observed with a frequency of 5–10 per hour. We note that such rare perturbations do not practically affect the results of the control tests. Under these conditions one can record on a quiet portion the variation of the oscillation amplitude of the detector by a magnitude ~0.25 σ_{Br}. For a fixed receiver bandwidth $\Delta f \simeq 1$ Hz this resolution corresponds to the potential sensitivity of the detector (the sensitivity (A) at T = 300°K and is of the order 4×10^5 erg/cm$^2\cdot$sec in terms of gravitational radiation flux; the change of amplitude is $\Delta A \simeq 1.2\times10^{-14}$ cm; the estimate for $H(\omega)$ is $\simeq 3\times10^4$ erg/cm^2 Hz ($\tau = 1$ s).

The pulse noise in our experiment is characterized by the data of the table. There can be found the average frequency $\bar{n}$ of noise bursts per day. We have selected bursts which satisfy the requirement: over the period $\tau = 2$ sec the decrease of the amplitude must exceed a given quantity $\Delta A/\sigma_{Br}$, equal to 1.5, 2, 3, and 4. Two values for the frequency of burst are listed: the average over the ten days of observation and the average over the quietest periods of the recording, adding up to approximately four days.

Figure 3 shows the dependence of $N_{1-\alpha}$ (the threshold number of coincidences for ten days) as a function of the "noisiness" n/10 at the level, i.e., the number of pulses at that level per day. This graph was computed according to Eq. (4) for $\alpha = 0.05$ and $\tau_r = 1$ sec. From given α and $N_{1-\alpha}$ we determined the average number of coincidences at the level $\bar{N}$. Further we determined $\bar{n}$, the average number of pulses at the level, assuming $\bar{n}_1 = \bar{n}_2 = \bar{n}$. The averages refer to the total observation time during 10 days. From the $\bar{n}$ determined this way we formed, by taking into account the scatter of a Poisson variable, the quantity $n = \bar{n} - 2(\bar{n})^{1/2}$, which determines within 5% the admissible realization for an average $\bar{n}$. This quantity, divided by 10 (i.e., referred to one day) is in the horizontal axis of the graph of Fig. 3. The passage from $\bar{n}$ to n increases the confidence in the following conclusions.

Before discussing the results on coincidences, we

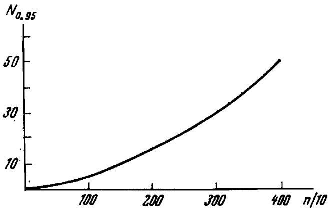

FIG. 3

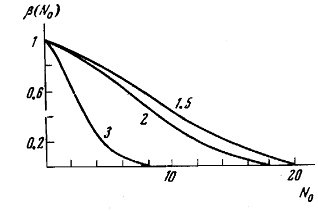

FIG. 4

FIG. 3. The dependence of the threshold number of coincidences over 10 days ($1-\alpha = 0.95$) on the "noisiness" of the level, n/10 (number of pulses at the level per day).

FIG. 4. The error of the second kind, $\beta(N_0)$, (the numbers in parentheses indicate the values of k).

discuss our estimates of the sensitivity with regard to Weber's results. In Weber's experiments the effect over 10 days consists of $\gtrsim 10$ signal pulses. This was observed by an operator operating with a threshold $N_{1-\alpha} \leq 10$. The corresponding value of the "noisiness" of the level n from the graph of Fig. 3 is approximately 150. For an ideally insulated antenna such a noisiness belongs to a level of $\Delta A \simeq 0.9\sigma_{Br}$, if the detection takes place on the background of steep pulses: $\tau = 2$ sec for $\tau^* = 33$ sec (Eq. (7)). If, as in the experiments of Weber, one takes the number of arbitrary intersections of a given level (Eq. (6)), one obtains the value $A \simeq 2.5\sigma_{Br}$ for $\tau^* = 33$ sec (~$2.8\sigma_{Br}$ for $\tau^* = 20$ sec). These figures stress the effectiveness of the transition to steep pulses. In addition, it should be stressed again [10] that even under ideal conditions in the experiments of Weber one can talk about a reliable recording only for high pulses with $A \gtrsim 3\sigma_{Br}$, i.e., $I \gtrsim 10^7$ erg/sec$\cdot$cm^2.

A comparison of the value n = 150 with the table shows that in our experiment the control level for the registration of the Weber effect on the best films is situated between $1.5\sigma_{Br}$ and $2\sigma_{Br}$. For an experimental bandwidth $\Delta f = 0.5$ Hz this corresponds to a flux of gravitational radiation not exceeding $I \simeq 5\times10^6$ erg/sec$\cdot$cm^2. From the graph of Fig. 3 and the table the threshold numbers of coincidences for 10 days at the levels 1.5; 2; $3\sigma_{Br}$ are respectively equal to $N_{0.95} = 34$; 18; 4. Experimentally at these levels we have observed in 10 days 26; 14; 3 coincidences with a resolution $\tau = 1$ sec. Thus, at neither of these levels was the control threshold exceeded. On the contrary, the observed numbers of coincidences agree well with the expected values 25; 11; 2. The errors of second kind $\beta(N_0)$ calculated according to Eq. (5) for the most noisy conditions (we took the maximal value of the average frequency of the background) are represented in the graph of Fig. 4. It can be seen that the second kind error for one pulse per day was smaller than 0.05 for an amplitude $3\sigma_{Br}$ and smaller than 0.3 for an amplitude $2\sigma_{Br}$. Reducing the resolution time to 0.5 s the number of coincidences at the level 1.5 σ_{Br} decreased to 11, in agreement with the calculation.

We have also carried out an experiment with delayed coincidences. The calculation of the threshold for the difference $\Delta = N(0) - N(\tau_d)$ at three preselected levels (according to Eq. (8)) yielded the following values for $\alpha \simeq 0.05$ and $\tau_r = 1$ s: 12; 7; 3. The experimental data for the level 1.5 are the following

Delay time:	2	4	8	10	14	16
Δ_{exper}:	−7	−1	+2	+2	−1	+5

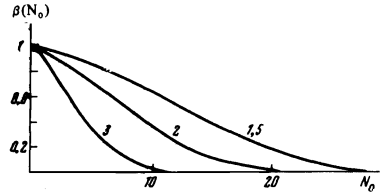

FIG. 5. The error of the second kind $\beta(N_0)$, if one watches the quantity Δ (the numbers in parentheses denote the values of k).

In all cases (different $\tau_d \gg \tau_r$) the threshold 12 was not exceeded. The second kind error β_Δ if one follows the quantity Δ is represented in Fig. 5. As expected, the curves are close to the analogous curves in Fig. 4, although the second kind error has increased somewhat.

Summarizing what was said in this section we can make the following conclusions:

1. In our experiments we have not detected the Weber effect. With a confidence level of 0.95 we have found no difference between the observed number of coincidences and the one expected statistically. The error of the second kind of correlated pulses from the two antennas with a frequency of one per day did not exceed 0.3 for a change in amplitude of $2\sigma_{Br}$ and was smaller than 0.05 for a change in amplitude of $3\sigma_{Br}$.

2. From the results of our measurements one can establish the following upper limit on the pulsed gravitational radiation from the Universe. Short bursts with a frequency of one per day or more have a spectral energy density not exceeding the value $H(\omega) \lesssim 10^6$ erg/cm² Hz near the frequency $f_0 = 1640$ Hz with a bandwidth $\Delta f \simeq 1/\pi\tau$. For a pulse duration $\tau \simeq 2$ sec the energy flux in the pulse does not exceed $I \lesssim 3\times 10^6$ erg/sec · cm². Finally, reduced to the galactic center the energy of such a pulse will yield not more than $E \sim (5\times 10^{-3}/\tau)$ $M_\odot c^2$.

4. PERSPECTIVES FOR INCREASING THE SENSITIVITY OF GRAVITATIONAL ANTENNAS

As can be seen from the relation (1) there are essentially two methods of increasing the sensitivity of solid-body detectors of gravitational waves. The first consists in a considerable lowering of the temperature (Fairbank and Hamilton [11] propose a lowering of T to 3×10^{-3} °K for $m \sim 2.6\times 10^6$ g). The second possibility is to select a material with the highest possible $\omega_0 Q$.

Preliminary experiments (carried out by Bagdasarov, Mitrofanov and one of the present authors, cf. [12]) with a cylinder made out of a sapphire single crystal ($m = 1.1\times 10^3$ g, $l = 15$ cm) yielded $\omega Q = 18\times 10^{12}$ rad/sec for $T = 300$°K and $\omega Q = 2.6\times 10^{13}$ rad/sec at $T = 80$°K (Q equals 4×10^7 at 300°K and 1.3×10^8 at 80°K). In antennas of the Weber type made out of aluminum $\omega Q = 2\times 10^9$. Thus, even for a mass of only $m = 10^3$ g, the quantity $m\omega Q$ is larger for this relatively small cylinder than the $m\omega Q$ for an aluminum antenna with $m \simeq 10^6$ g.

A lowering of the temperature, an improvement of the suspension and a reduction of the surface losses due to a thorough polishing should lead to a considerable improvement of the quantity ωQ. Thus, for $T = 0.4$°K the purely internal losses of the material (sapphire) correspond to a factor of $\omega Q \simeq 9\times 10^{20}$ rad/sec. If one could attain this limit for the ωQ factor, then for $m = 10^4$ g, $l = 20$ cm, $T = 0.4$°K one could record signals of the order of 10^{-3} $M_\odot c^2$ at a distance of 1000 Mpc. For $\omega Q = 10^{15}$ rad/s and the same T, m, l, one could expect a response to bursts of gravitational radiation from the nearest galaxies. This method of raising the sensitivity seems to us to be quite promising [1)].

In conclusion, the authors would like to use this occasion to express their gratitude for valuable discussions and help to Ya. B. Zel'dovich, G. I. Petrov and M. A. Sadovskiĭ, and for help with the measurements to V. P. Rostov and V. K. Martynov.

Translator's note. The Munich-Frascati experiment has recently (March 7–24, 1974) reported coincidences at a level of 3 standard deviations. (H. Billing, P. Kafka, K. Maischberger, F. Meyer, and W. Winkler, Munich Preprint, June 1974.)

[1)] Added in proof (16 January 1974). Recently, on a cylinder made out of a sapphire single crystal $Q = 6.9 \times 10^8$ has been obtained at $T \approx 7$°K.

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