Instability of spin waves in a sample with domain structure

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(Submitted July 8, 1973)
Zh. Eksp. Teor. Fiz. 66, 679–685 (February 1974)

The instability conditions for first-order spin waves in a sample with laminar domain structure are calculated. Equations for the spin wave amplitudes, the dispersion relation, and an expression for the threshold field are obtained for an arbitrary orientation of the microwave field. The threshold conditions for a perpendicular domain structure with various forms of pumping are analyzed in detail. The dependence of the threshold field on the stationary field is discussed for various pumping conditions and sample properties.

Theoretical and experimental studies of magnetic resonances in ferrite samples with domain structure[1-5] have shown that the resonance conditions largely depend on the demagnetizing fields of the sample and the domains, and therefore on the form of the domain structure.

The presence of domain structure should also alter significantly the conditions of parametric spin-wave excitation. In fact, the experimental results [6,7] obtained with single-crystal and polycrystalline ferrite samples with domain structure have exhibited a number of interesting features in the dependence of the threshold fields on the stationary external magnetic field (frequency dependence, dependence on the saturation magnetization, and the anisotropy field). From these results we can infer the significant influence, on the conditions of parametric spin-wave excitation, of changes in the orientations of the domain magnetic moments and the proximity of the ferromagnetic resonance frequency to the pumping frequency (an analog to the proximity effect of the secondary to the primary frequency in samples magnetized to saturation [8]).

Courtney [9] calculated the threshold fields with parallel pumping for the perpendicular domain structure discussed in [2]. However, in [9] he considered only the parametric excitation of spin waves due to the microwave field components parallel to the equilibrium orientation of the domain magnetic moments, and did not take into account the perpendicular components of the field, thus obtaining results that contradicted the experimental data [6,7].

It is the aim of the present work to calculate the threshold fields for spin-wave instability in the domains of a single-crystal sample, taking into account the interaction of the magnetic moments with all the components of the external microwave field. We consider a spherical single crystal with cubic symmetry and a negative first anisotropy constant ($K_1 < 0$). The stationary external field $\mathbf{H}_0$ is directed along the [110] axis. In this case there can exist two types of domain (the two nearest directions of easy magnetization) with magnetic moments at the same angle to the external field direction (M_1 and M_2 in Fig. 1). We assume that the domains are in the form of laminas perpendicular to the (001) plane, and the domain walls make an angle α with the [110] axis [4,5].

The calculation is performed with the following simplifying assumptions: 1) the domain walls are immobile; 2) the interactions of spin waves in neighboring domains are neglected; 3) in each domain the spin waves are regarded as plane waves, and the boundary conditions at the sample and domain boundaries are not taken into account.

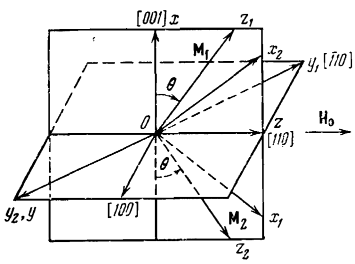

FIG. 1. General and local coordinate systems.

The solution of the static problem can be written in the form [2,4]

$$\theta_1=\theta,\ \theta_2=\pi-\theta,\ \varphi_1=\varphi_2=\pi/4,\quad \nu_1=\nu_2=1/2,$$
$$H_0=\sin\theta\left[\frac{4\pi M}{3}+\frac{|K_1|}{M}(3\sin^2\theta-2)\right], \tag{1}$$

where ν_i is the relative volume of the i-th domain.

The following coordinate systems are used in the calculation (Fig. 1):

1) the general coordinate system x, y, z;

2) the local coordinate system x_i, y_i, z_i for each domain, where z_i is parallel to the equilibrium direction of the magnetic moment of the given domain, x_i lies in the (1$\bar{1}$0) plane, y_1 is along the [$\bar{1}$10] axis, and y_2 is along the [1$\bar{1}$0] axis.

We denote the relative magnetic moment of the first domain by $\boldsymbol{\alpha}=\mathbf{M}_1/M$, that of the second domain by $\boldsymbol{\beta}=\mathbf{M}_2/M$; the cyclic variables are

$$\alpha^+=\alpha_{x_1}+i\alpha_{y_1}=\sum_k \alpha_k e^{i\mathbf{kr}},\qquad \beta^+=\beta_{x_2}+i\beta_{y_2}=\sum_k \beta_k e^{i\mathbf{kr}}; \tag{2}$$

and the components of the external microwave field

$$h_x=h_0a_x\cos\omega t,\ h_y=h_0a_y\cos\omega t,\ h_z=h_0a_z\cos\omega t. \tag{3}$$

The components of the effective fields acting on the magnetic moment of the first domain are given in the local coordinate system below.

1. External field:

$$H_{\text{ex}\,x_1}=-h_x\sin\theta+(H_0+h_z)\cos\theta,$$
$$H_{\text{ex}\,y_1}=-h_y, \tag{4}$$
$$H_{\text{ex}\,z_1}=h_x\cos\theta+(H_0+h_z)\sin\theta,$$

2. Anisotropy field, including the nonlinear terms:

$$H_{ax_1}=-(N_{xxz_2}\alpha_x\alpha_z^2+N_{xz_3}\alpha_z^3+N_{xxz_2}\alpha_x^2\alpha_z+N_{xyz}\alpha_y^2\alpha_z),$$
$$H_{ay_1}=-(N_{yyz_2}\alpha_y\alpha_z^2+N_{yxyz}\alpha_x\alpha_y\alpha_z), \tag{5}$$
$$H_{az_1}=-(N_{zxz_2}\alpha_x\alpha_z^2+N_{zz_3}\alpha_z^3),$$

where N_{xxz_2}, N_{xz_3}, ..., are functions of θ and $|K_1|/M$

and are determined from the expression for the anisotropy energy.

We include here only the anisotropy-field nonlinear terms that affect the condition of parametric excitation of first-order spin waves ($\omega_k = \omega/2$).

3. Demagnetization field of the sample:

$$\begin{gathered} H_{s x_1} = -{}^1/_3 \pi M[\alpha_0 + \alpha_0^* + (\beta_0 + \beta_0^*)\cos 2\theta + 2\sin 2\theta], \\ H_{s y_1} = -{}^1/_3 \pi M[\alpha_0 - \alpha_0^* - \beta_0 + \beta_0^*), \\ H_{s z_1} = -{}^1/_3 \pi M[(\beta_0 + \beta_0^*)\sin 2\theta + 4\sin^2\theta]. \end{gathered} \tag{6}$$

4. Demagnetization field of the domains:

$$\begin{gathered} H_{d x_1} = -{}^1/_4 N_\gamma M \mathscr{L} \cos\theta \sin\alpha, \\ H_{d y_1} = {}^1/_4 N_\gamma M \mathscr{L} \cos\alpha, \\ H_{d z_1} = -{}^1/_4 N_\gamma M \mathscr{L} \sin\theta \sin\alpha, \end{gathered} \tag{7}$$

where

$$\begin{gathered} \mathscr{L} = p_1 \cos\theta \sin\alpha + i p_2 \cos\alpha, \\ p_1 = \alpha_0 + \alpha_0^* - \beta_0 - \beta_0^*, \quad p_2 = \alpha_0 - \alpha_0^* + \beta_0 - \beta_0^*, \end{gathered} \tag{8}$$

and N_γ is the demagnetization factor of the domain.

The exchange field and the field of the dipole-dipole interaction in the spin wave for the first domain can be written in the same form as for a sample magnetized to saturation [8].

The equation of motion for the first domain is

$$-\frac{i}{\gamma}\dot{\alpha}^+ = \alpha^+ H_{z_1} - \alpha_z (H_{x_1} + i H_{y_1}). \tag{9}$$

After the expressions (4)–(8) for the effective fields are substituted in Eq. (9) we obtain a general equation of motion for the magnetic moment in the first domain. A similar equation is obtained for the second domain. From these equations and their complex conjugates we obtain a system of equations for the uniform precession and for the spin wave amplitudes in each domain.

Since we shall be interested below in calculating the instability threshold of first-order spin waves, we must separate from the equation of motion of the spin-wave amplitudes the part linear in α_k, α_0, β_0, and h, which can be written in the form [10]

$$\dot{a}_k = i[(A_k + C_k)\alpha_k + (B_k + D_k)\alpha_{-k}^*] - i\alpha_k(h_x \cos\theta + h_z \sin\theta), \tag{10}$$

where

$$\begin{gathered} A_k = N_a + \omega_{06} k^2 + {}^1/_2 \omega_M \sin^2\theta_k, \quad B_k = {}^1/_2 \omega_M \sin^2\theta_k e^{2i\varphi_k} + N_{1^\bullet}, \\ C_k = -{}^1/_2 \omega_M \sin\theta_k \cos\theta_k\, (\alpha_0 e^{-i\varphi_k} + \alpha_0^* e^{i\varphi_k}) \\ + 2N_{11}(\alpha_0 + \alpha_0^*) - {}^1/_{12}\omega_M(\beta_0 + \beta_0^*)\sin 2\theta \\ - {}^1/_4 N_\gamma \hat{\omega}_M \mathscr{L} \sin\theta \sin\alpha, \\ D_k = -\omega_M \sin\theta_k \cos\theta_k e^{i\varphi_k}\alpha_0 + 2N_{11}\alpha_0 + N_{1^\bullet 1^\bullet}\alpha_0^*, \\ N_a = \omega_a(-2 + 8\sin^2\theta - {}^9/_2 \sin^4\theta), \quad N_{1^\bullet} = {}^3/_2 \omega_a \sin^2\theta(2 - 3\sin^2\theta), \\ N_{11} = {}^9/_8 \omega_a \sin 2\theta(2 - 3\sin^2\theta), \quad N_{1^\bullet 1^\bullet} = -{}^3/_8 \omega_a \sin 2\theta(2 + 3\sin^2\theta), \\ \omega_M = 4\pi\gamma M, \quad \hat{\omega}_M = \gamma M, \quad \omega_{06} = \gamma D, \quad \omega_a = \gamma|K_1|/M, \end{gathered} \tag{11}$$

$\mathbf{k}(k, \theta_k, \varphi_k)$ is the wave vector of the spin wave in the local coordinate system of the first domain; and θ_k and φ_k are the polar and azimuthal angles of $\mathbf{k}$ (φ_k is measured from the x_1 axis in the $x_1 y_1$ plane).

The presence of domain structure significantly alters the equation of motion: additional terms appear due to the demagnetization fields of the domains (the term with N_γ in C_k) and the demagnetization fields of the sample, related to uniform precession in the second domain (the term with $\beta_0 + \beta_0^*$ in C_k); instead of the external field, the demagnetization field of the sample at constant magnetization, and the anisotropy field, there appears in A_k the term N_a specified by the conditions of static equilibrium (1).

The inclusion of the nonlinear terms in the anisotropy field (5), as in the case of a sample magnetized to saturation, leads to the appearance of an additional anisotropic coupling between the spin wave and the uniform precession.

Using the Holstein-Primakoff transformation for anisotropic media [11], we obtain from Eq. (10) an equation involving B_k and the dispersion relation for spin waves in the domain:

$$\begin{gathered} \omega_k^2 = A_k^2 - |B_k|^2 = (N_a + \omega_{06}k^2 + \omega_M \sin^2\theta_k)(N_a + \omega_{06}k^2) \\ - N_{1^\bullet}(\omega_M \sin^2\theta_k \cos 2\varphi_k + N_{1^\bullet}), \end{gathered} \tag{12}$$

which agrees with the dispersion relation obtained in [9].

From the equation for B_k, setting $\omega_k = \omega/2$, we obtain in the usual manner [8,10] an expression for the threshold field:

$$h_{\text{thr}} = \frac{\Delta H_k}{2}\frac{\omega}{|W|}, \tag{13}$$

where ΔH_k is related to the spin-wave attenuation parameter η_k (which is introduced by replacing ω_k with $\omega_k + i\eta_k$) by $\Delta H_k = 2\eta_k/\gamma$.

Further calculation of the threshold is a matter of solving the system of equations for uniform precession for specific models of the domain structure (angle α) and minimizing the threshold field (13) with respect to θ_k and φ_k (using $\omega_k = \omega/2$) for each value of the field (the corresponding value of θ is then given by Eq. (1)).

Using a solution of the system of equations for uniform precession in the form

$$\alpha_0 = (q_L e^{i\omega t} + q_A e^{-i\omega t})\frac{\gamma h_0}{2}, \qquad \beta_0 = (p_L e^{i\omega t} + p_A e^{-i\omega t})\frac{\gamma h_0}{2},$$

we obtain

$$\begin{gathered} W = \frac{\omega_M}{2}\sin\theta_k \cos\theta_k \Big[\Big(A_k + \frac{\omega}{2}\Big)e^{i\varphi_k}q_L + \frac{B_k^2}{|B_k|^2}\Big(A_k - \frac{\omega}{2}\Big)e^{-i\varphi_k}q_A^* \\ - B_k(q_L e^{-i\varphi_k} + q_A^* e^{i\varphi_k})\Big] - \Big(A_k + \frac{\omega}{2}\Big)(N_{11}q_L + N_{1^\bullet 1^\bullet}q_A^*) - \frac{B_k^2}{|B_k|^2}\Big(A_k - \frac{\omega}{2}\Big) \\ \times (N_{11}q_A^* + N_{1^\bullet 1^\bullet}q_L) + B_k\Big[2N_{11}(q_L + q_A^*) - \frac{\omega_M}{12}(p_L + p_A^*)\sin 2\theta \\ - \frac{N_\gamma \hat{\omega}_M}{4}\mathscr{L}_{pq}\sin\theta\sin\alpha\Big] + B_k(a_x \cos\theta + a_z \sin\theta), \end{gathered} \tag{14}$$

$$\mathscr{L}_{pq}(q_L + q_A^* - p_L - p_A^*)\cos\theta\sin\alpha + i(q_L - q_A^* + p_L - p_A^*)\cos\alpha. \tag{15}$$

Let us consider the simplest type of perpendicular domain structure [2,4,5]. The solution of the equations for uniform precession under various excitation conditions [4] is given in the table.

	Longitudinal excitation $h_z \neq 0,\ h_x = h_y = 0$	Antisymmetric transverse excitation $h_x \neq 0,\ h_y = h_z = 0$	Symmetric transverse excitation $h_y \neq 0,\ h_x = h_z = 0$
q_L	$\frac{\omega + d_L}{\omega_2^2 - \omega^2}\cos\theta$	$-\frac{\omega + d_A}{\omega_1^2 - \omega^2}\sin\theta$	$-i\frac{\omega + d_S}{\omega_1^2 - \omega^2}$
q_A^*	$-\frac{\omega - d_L}{\omega_2^2 - \omega^2}\cos\theta$	$\frac{\omega - d_A}{\omega_1^2 - \omega^2}\sin\theta$	$-i\frac{\omega - d_S}{\omega_1^2 - \omega^2}$
p_L	q_L	$-q_L$	$-q_L$
p_A^*	q_A^*	$-q_A^*$	$-q_A^*$

Note: the following notation is used in the table: $d_P = \omega_a(5\sin^2\theta - 2)$, $d_A = {}^1/_3\omega_M + \omega_a(5\sin^2\theta - 2)$, $d_S = {}^1/_3\omega_M \sin^2\theta + (1 - \sin^2\theta)[\omega_a(9\sin^2\theta - 2) + N_\gamma\omega_M]$, $\omega_1^2 = \{(1 - \sin^2\theta)[\omega_a(9\sin^2\theta - 2) + N_\gamma\omega_M] + {}^1/_3\omega_M\sin^2\theta\}[{}^1/_3\omega_M + \omega_a(5\sin^2\theta - 2)]$, $\omega_2^2 = \omega_a(1 - \sin^2\theta) \times (5\sin^2\theta - 2)[{}^1/_3\omega_M + \omega_a(9\sin^2\theta - 2)]$, and ω_1 and ω_2 are the ferromagnetic resonance frequencies under transverse and longitudinal excitation [2,4,5].

By making use of the expressions given in the table for the amplitude of the uniform precession, we can write the corresponding expressions for the W function under various pumping conditions.

For parallel pumping: $h_z \neq 0$, $h_x = h_y = 0$,

$$W = \frac{T_1\omega + T_2 d_P}{\omega_2^2 - \omega^2}\cos\theta + B_k \sin\theta. \tag{16}$$

For antisymmetric perpendicular pumping: $h_x \neq 0$, $h_y = h_z = 0$,

$$W = -\frac{T_1\omega - T_2^A d_A}{\omega_1^2 - \omega^2}\sin\theta + B_k\cos\theta. \tag{17}$$

For symmetric perpendicular pumping: $h_y \neq 0$, $h_x = h_z = 0$,

$$W = i\frac{T_2^A\omega - T_1 d_C}{\omega_1^2 - \omega^2}. \tag{18}$$

Here

$$T_1 = \frac{\omega_M}{2}\sin\theta_k\cos\theta_k\left[\left(A_k + \frac{\omega}{2}\right)e^{i\varphi_k} - \frac{B_k^2}{|B_k|^2}\left(A_k - \frac{\omega}{2}\right)e^{-i\varphi_k} + 2iB_k\sin\varphi_k\right] - N_1\left[A_k + \frac{\omega}{2} - \frac{B_k^2}{|B_k|^2}\left(A_k - \frac{\omega}{2}\right)\right],$$

$$T_2 = T' + B_k\left(N_3 - \frac{\omega_M}{6}\sin 2\theta\right),$$

$$T' = \frac{\omega_M}{2}\sin\theta_k\cos\theta_k\left[\left(A_k + \frac{\omega}{2}\right)e^{i\varphi_k} + \frac{B_k^2}{|B_k|^2}\left(A_k - \frac{\omega}{2}\right)e^{-i\varphi_k} + 2B_k\cos\varphi_k\right] - N_2\left[A_k + \frac{\omega}{2} + \frac{B_k^2}{|B_k|^2}\left(A_k - \frac{\omega}{2}\right)\right] \tag{19}$$

$$T_2^A = -T' - B_k[N_3 + \hat{\omega}_M \sin 2\theta\,(^2/_3\pi - {}^1/_2 N_\gamma)],$$

$$N_1 = {}^3/_4\omega_a \sin 2\theta\,(4 - 3\sin^2\theta);\quad N_2 = {}^3/_2\omega_a\sin 2\theta\,(1 - 3\sin^2\theta),$$

$$N_3 = {}^3/_2\omega_a\sin 2\theta\,(2 - 3\sin^2\theta).$$

From Eqs. (16)–(18) for W it is clear that in the cases of parallel and antisymmetric perpendicular pumping we are dealing essentially with oblique pumping, when the instability threshold depends not only on the interaction of the microwave field with the longitudinal component of the spin-wave magnetic moment (terms $B_k \sin\theta$ in Eq. (16) and $B_k\cos\theta$ in Eq. (17)), but also in the interaction of the spin wave with the uniform precession in the domain (first terms in Eqs. (16) and (17)).

For parallel pumping in sufficiently strong fields (near the saturation value) the threshold is essentially specified by parallel pumping ($\sin\theta \sim 1$, $\cos\theta \sim 0$); as the field strength is reduced, the contribution from perpendicular pumping increases and that from parallel pumping decreases. In this effect the proximity of the resonance frequency ω_2 to the pumping frequency is significant (coincidence effect of the secondary and primary resonances).

When the values of ω_{pum} and $\omega_{2\,max}$ are sufficiently close, a decrease in the threshold field can be expected; then in the fields corresponding to $\omega_{2\,max}$ there will be a threshold-field minimum (Fig. 2, curve 1). As the pumping frequency rises the depth of this minimum decreases, and at comparatively high frequencies the threshold field rises monotonically as H_0 decreases.

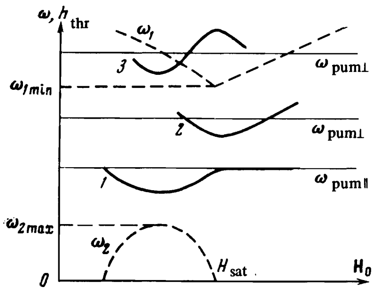

FIG. 2. The resonance frequencies (dashes) and threshold fields (solid lines) as functions of the stationary field H_0 under various excitation conditions.

A similar effect can be observed in samples with different $\omega_{2\,max}$. This conclusion is in qualitative agreement with the experimental results [6,7]. In contrast to the parallel case, for antisymmetric perpendicular pumping the threshold in strong fields is specified essentially by the perpendicular pumping; as the stationary field is reduced, the relative weight of the latter compared to the parallel pumping is decreased. However, we must remember that the relative weights of perpendicular and parallel pumping depend on the relation between the pumping frequency and $\omega_{1\,min}$. If $\omega_{pum} < \omega_{1\,min}$, then in the vicinity of the saturation field the threshold field should exhibit a minimum followed by a monotonic rise (Fig. 2, curve 2). If $\omega_{pum} > \omega_{1\,min}$, then in this same vicinity a threshold-field maximum should occur; as H_0 decreases and ω_1 approaches the pumping frequency, the threshold field should decrease and reach a minimum at $\omega_1 = \omega_{pum}$ (Fig. 2, curve 3).

In the case of symmetric perpendicular pumping, purely perpendicular pumping occurs in the domains. Qualitatively, we should expect the threshold fields to depend on the stationary field in a manner similar to the case of antisymmetric pumping. However, the difference in the conditions of spin-wave excitation (perpendicular and oblique pumping) can introduce significant differences in the values of the threshold fields and their dependence on the stationary field. The dependence of the threshold field on H_0 can be determined more accurately by minimizing Eq. (13) for the various pumping conditions (16)–(18).

From the preceding analysis of the spin-wave excitation conditions for a perpendicular domain structure it is clear that oblique or perpendicular pumping always occurs in the domains for every orientation of the pumping field; accordingly, the proximity effect of the secondary to the primary resonance, together with a sharp drop in the threshold field, is always possible. Evidently, this effect will occur for any domain structure in which ferromagnetic resonance of the uniform precession of the domain magnetic moment is possible. As a confirmation we may cite the results obtained in polycrystalline ferrite samples [6,7].

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Translated by R. Rutherford III
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