X-ray transition radiation produced in an irregular medium

G. M. Garibyan, L. A. Gevorgyan, and C. Yang

Erevan Physics Institute
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A formula is obtained for the spectral distribution of the number of transition x-ray quanta produced in an irregular medium consisting of randomly disposed parallel plates with arbitrary thicknesses. The formula takes into account the absorption of the radiation by the plates. This formula is averaged over the thicknesses and over the distances between the plates for random distributions of these quantities. In the limiting case when there is no irregularity, the general formula leads to the well-known formula for the transition x rays in a regular stack. In another limiting case, when the medium is extremely irregular (i.e., the scatters of the plate thicknesses and of the distances between them are much larger than the respective formation bands in the matter and in vacuum), the number of emission quanta produced in such a medium is the additive sum of the numbers of the radiation quanta formed on all the boundaries. A numerical calculation is given for a model in which the thicknesses of the plates and the distances between them are described by a gamma distribution.

1. INTRODUCTION

Transition radiation is produced whenever a charged particle passes through an inhomogeneity of a medium. Many papers have been devoted to theoretical and experimental research on transition radiation produced in a medium with regularly (periodically) distributed inhomogeneities (see[1,2]). Under real conditions, however, various deviations from ideal regularity are unavoidable. In addition, it is perfectly natural to raise the question of the appearance of transition radiation in an arbitrary irregular medium. A readily accessible medium of this type is foamed plastic, in which transition radiation was recently observed in experiment[3].

We consider a system of randomly distributed plates of arbitrary thickness, all parallel to one another. Such a system, besides being of independent interest, is also a good model for a theoretical description of the formation of x-ray transition radiation in foamed plastic and in similar media. Indeed, for x-ray frequencies the effective transverse dimensions of the electromagnetic field of an ultrarelativistic charged particle are much smaller than the dimensions of the pores in the foamed plastic. In addition, the intensity of the transition x-rays is practically independent of the entrance angle of the charge into the medium[4].

In this formulation, a preliminary analysis of this problem was presented by us for the case when the medium is actually regarded as weakly irregular[5]. In addition, no account was taken of the absorption of the radiation in the medium. In a somewhat different formulation, but again for a weakly-irregular medium, the problem was considered by Ter-Mikaelyan[6]. In the present paper we consider this problem in the general case without any assumption of weak irregularity, and also take into account the absorption of the radiation in the medium itself.

2. GENERAL THEORY

Assume that a particle with charge e travels with velocity v perpendicular to N plates of identical matter and with thicknesses a_n (n is the number of the plate). The plates are arranged in a vacuum, the distance between the n-th and (n + 1)-st plates being b_n. It is very difficult to find an exact solution of this problem, and we therefore seek immediately an approximate solution for ultrarelativistic particles and transition x-rays. The approximation consists in the fact that we neglect reflections, i.e., we assume the condition

$$|Nr_1|^2 \ll 1 \tag{1}$$

to be satisfied, where r_1 is the coefficient of reflection from the plate and is given by the formula

$$r_1 = (\varepsilon\lambda_0 - \lambda)/(\varepsilon\lambda_0 + \lambda);$$
$$\lambda_0 = (\omega/c)\cos\vartheta, \quad \lambda = (\omega/c)(\varepsilon - \sin^2\vartheta)^{1/2}. \tag{2}$$

Here $\epsilon = \epsilon' + i\epsilon''$ is the dielectric constant of the plate material, and ϑ is the radiation angle.

In the x-ray frequency region we have $|r_1| \approx \omega_0^2/4\omega^2$ (ω_0 is the plasma frequency of the plate material), and consition (1) is well satisfied for a sufficiently large number N. The transverse Fourier component of the radiation field produced when the particle passes through the interface z = 0 between the medium and the vacuum is then given by

$$\mathbf{E}_{1t}(\mathbf{k}, \omega) = \pm \frac{ei\varkappa}{2\pi^2}(\Lambda^{-1} - \Lambda_0^{-1}),$$
$$\Lambda_0 = k^2 - \omega^2/c^2, \quad \Lambda = k^2 - \omega^2\varepsilon/c^2, \tag{3}$$

where κ is the transverse component of the wave vector k, while the plus and minus signs correspond to passage of the particle from the vacuum to the medium and from the medium to the vacuum, respectively.

By a method described by one of us[2], or by a consecutive solution of the equations for matching on the boundary, one can find that the Fourier components of the radiation fields in the plates and in the vacuum segments are superpositions of the quantities (3) with appropriate phase factors. In particular, the transverse Fourier component of the radiation field beyond the last (N-th) plate is given by

$$\mathbf{E}_{Nt}(\mathbf{k}) = \frac{ei\varkappa}{2\pi^2}\exp\left\{i\varphi_0\left[\sum_{k=1}^{N-1}(a_k+b_k)+a_N\right]\right\}\cdot$$
$$\times(\Lambda_0^{-1}-\Lambda^{-1})\sum_{m=0}^{N-1}[1-\exp(i\delta_m)]\exp(iA_m), \tag{4}$$

$$\delta_m = -\varphi a_{m+1}, \quad A_m = -\varphi\sum_{k=m+2}^{N} a_k - \varphi_0\sum_{k=m+2}^{N} b_{k-1},$$

$$\varphi_0 = \frac{\omega}{v} - \left(\frac{\omega^2}{c^2} - \varkappa^2\right)^{1/2}, \quad \varphi = \frac{\omega}{v} - \left(\frac{\omega^2}{c^2}\varepsilon - \varkappa^2\right)^{1/2}. \tag{5}$$

The number of transition-radiation quanta beyond the N-th plate is determined by the formula

$$\frac{dN_{qu}}{d\omega}=\frac{2}{137\pi\omega}\int\left|\frac{1}{1-\beta^2\varepsilon+\vartheta^2}-\frac{1}{1-\beta^2+\vartheta^2}\right|^2\times\left|\sum_{m=0}^{N-1}\exp(iA_m)[1-\exp(i\delta_m)]\right|^2\vartheta^3\,d\vartheta, \tag{6}$$

where $\beta = v/c$; we put $\kappa = (\omega/c)\sin\vartheta$ in the quantities A_m and δ_m.

We note that, by their definition, the quantities a_m and b_m in (4)—(6) are positive. In addition, in the derivation of these formulas (for example, by solving the matching equations), explicit use was made of the positiveness of a_m and b_m. If a_m and b_m are random quantities, then the number of transition-radiation quanta (6) is also a random quantity with a distribution determined by the distributions of a_m and b_m.

3. AVERAGE NUMBER OF RADIATION QUANTA WITHOUT ALLOWANCE FOR ABSORPTION

Let a_m and b_m be independent random quantities. We assume here that a_m and also b_m are independent for different numbers m, and calculate the mean value of (6). To this end it suffices to calculate

$$I\equiv\left\langle\left|\sum_{m=0}^{N-1}\exp(iA_m)[1-\exp(i\delta_m)]\right|^2\right\rangle=\left\langle\sum_{n=0}^{N-1}\sum_{m=0}^{N-1}\exp[i(A_m-A_n^*)][1-\exp(i\delta_m)][1-\exp(-i\delta_n^*)]\right\rangle \tag{7}$$

where the angle brackets denote averaging.

For simplicity, we consider first the case when the absorption can be neglected, i.e., when the quantity ϵ, and consequently also the quantities λ, φ, and A_n, and δ_n can be regarded as real. The allowance for absorption will be considered in the next section.

We break up the sum over m in (7) into three parts: into a sum from zero to $n-1$, a separate term $m = n$, and a sum from $n+1$ to $N-1$. For the first part we have, taking (5) into account

$$\left\langle\sum_{m=0}^{n-1}\exp\left[-i\left(\varphi\sum_{k=m+2}^{n+1}a_k+\varphi_0\sum_{k=m+2}^{n+1}b_{k-1}\right)\right][1-\exp(-i\varphi a_{m+1})]\times[1-\exp(i\varphi a_{n+1})]\right\rangle=\left\langle\sum_{m=0}^{n-1}\exp\left[-i\left(\varphi\sum_{k=m+2}^{n}a_k+\varphi_0\sum_{k=m+2}^{n+1}b_{k-1}\right)\right]\times[1-\exp(-i\varphi a_{m+1})][\exp(-i\varphi a_{n+1})-1]\right\rangle. \tag{8}$$

In the last expression, all the a_i, and also the b_i, have different numbers and therefore are independent. According to a well known theorem, the mean value of the product of independent quantities is equal to the product of the mean values of these quantities. If we make the natural assumption that all the a_i have the same distribution, independent of the number i (and an analogous assumption is made for b_i), then we get in place of (8)

$$\sum_{m=0}^{n-1}h_a^{n-m-1}h_b^{n-m}(1-h_a)(h_a-1)=-(1-h_a)^2h_b\frac{1-(h_ah_b)^n}{1-h_ah_b} \tag{9}$$

where

$$h_a=\langle\exp(-i\varphi a_i)\rangle,\quad h_b=\langle\exp(-i\varphi_0 b_i)\rangle \tag{10}$$

are independent, by assumption, of the number i.

For the second part of the sum over m in (7), i.e., for the term $m = n$, we have

$$2(1-\operatorname{Re}h_a). \tag{11}$$

For the third part, i.e., for the sum over m from $n+1$ to $N-1$, in analogy with the procedure in the first part, we obtain

$$\left\langle\sum_{m=n+1}^{N-1}\exp\left[i\left(\varphi\sum_{k=n+2}^{m}a_k+\varphi_0\sum_{k=n+2}^{m+1}b_{k-1}\right)\right][\exp(i\varphi a_{m+1})-1]\times[1-\exp(i\varphi a_{n+1})]\right\rangle=\sum_{m=n+1}^{N-1}h_a^{*m-n-1}h_b^{*m-n}(h_a^*-1)(1-h_a^*)=-(1-h_a^*)^2h_b^*\frac{1-(h_a^*h_b^*)^{N-n-1}}{1-h_a^*h_b^*}. \tag{12}$$

Gathering all three parts (9), (11), and (12) and summing over n, we get

$$I=2N\operatorname{Re}\frac{(1-h_a)(1-h_b)}{1-h_ah_b}+2\operatorname{Re}\frac{(1-h_a)^2h_b(1-h_a^Nh_b^N)}{(1-h_ah_b)^2}. \tag{13}$$

For the average number of transition x-ray quanta in an irregular medium we have

$$\left\langle\frac{d^2N_{qu}}{d\omega\,d\vartheta}\right\rangle=\frac{2}{137\pi\omega}\vartheta^3\left|\frac{1}{1-\beta^2\varepsilon+\vartheta^2}-\frac{1}{1-\beta^2+\vartheta^2}\right|^2I,\qquad\left\langle\frac{dN_{qu}}{d\omega}\right\rangle=\int\left\langle\frac{d^2N_{qu}}{d\omega\,d\vartheta}\right\rangle d\vartheta. \tag{14}$$

We note that in the case of a weakly irregular medium, when the higher moments can be neglected, we have

$$h_a\approx\exp(-i\varphi\bar a)(1-{}^1\!/_2\varphi^2\langle\Delta a^2\rangle),\qquad h_b\approx\exp(-i\varphi_0\bar b)(1-{}^1\!/_2\varphi_0^2\langle\Delta b^2\rangle), \tag{15}$$

where $\bar a\equiv\langle a\rangle$, $\bar b\equiv\langle b\rangle$, a $\langle\Delta a^2\rangle=\langle(\bar a-a_i)^2\rangle$, $\langle\Delta b^2\rangle=\langle(\bar b-b_i)^2\rangle$ are the corresponding mean-squared deviations. If at the same time

$${}^1\!/_2N(\varphi^2\langle\Delta a^2\rangle+\varphi_0^2\langle\Delta b^2\rangle)\ll1, \tag{16}$$

then we readily obtain from (13)

$$I\approx4\sin^2\frac{\varphi\bar a}{2}\left[\frac{\sin^2NX}{\sin^2X}-\frac{1}{2}(\varphi^2\langle\Delta a^2\rangle+\varphi_0^2\langle\Delta b^2\rangle)\cos NX\times\frac{(N+1)\sin(N-1)X-(N-1)\sin(N+1)X}{\sin^3X}\right]+\varphi^2\langle\Delta a^2\rangle\left[N\cos\varphi\bar a-\frac{\sin\varphi\bar a}{\sin X}\left(N\cos X-\frac{\sin2NX}{2\sin X}\right)\right], \tag{17}$$

where $X = (\varphi\bar a + \varphi_0\bar b)/2$. Substituting (17) in (14), we obtain formula (4) of our earlier paper[5].

4. ALLOWANCE FOR ABSORPTION

The results obtained in the preceding section can be easily generalized to include the case when it is necessary to take into account absorption in the plates; this absorption is described by the imaginary part of the dielectric constant ϵ. We note to this end that, say at $n < m$, we have

$$i(A_m-A_n^*)=-2\mu\sum_{k=n+2}^{N}a_k-i\varphi\sum_{k=m+2}^{n+1}a_k-i\varphi_0\sum_{k=m+2}^{n+1}b_{k-1}, \tag{18}$$

where $\mu = \omega\epsilon''/2c \approx -\operatorname{Im}\varphi$ is the linear coefficient of expansion in the field amplitude. An analogous expression holds also at $n < m$. Proceeding as in the preceding section, we obtain

$$I=2\frac{1-p^N}{1-p}\operatorname{Re}\frac{[(1+p)/2-h_a]-[p-h_a(1+p)/2]h_b}{1-h_ah_b}$$

$$+2\,\mathrm{Re}\frac{(1-h_a)(p-h_a)h_b[p^N-h_a{}^Nh_b{}^N]}{(1-h_ah_b)(p-h_ah_b)} \qquad (19)$$

where $p = \langle \exp(-2\mu a_i)\rangle$. We note that at $|1 - p^N| \ll 1$, or, equivalently, $N\mu\bar{a} \ll 1$, formula (19) goes over into (13). Substituting (19) in (14) we obtain the average number of transition-radiation quanta with allowance for absorption.

If the inequality $|1 - p| \ll 1$ holds (i.e., $\bar{\mu} a \ll 1$), but $1 - p^N$ cannot be neglected in comparison with unity, then we obtain approximately from (19)

$$I\approx 2\frac{1-p^N}{1-p}\mathrm{Re}\frac{(1-h_a)(1-h_b)}{1-h_ah_b}+2\,\mathrm{Re}\frac{(1-h_a)^2h_b[p^N-h_a{}^Nh_b{}^N]}{(1-h_ah_b)^2}. \qquad (20)$$

5. DISCUSSION

In the limiting case when the medium is ideally regular (i.e., when all $\Delta a_i = a_i - \bar{a}$ and $\Delta b_i = b_i - \bar{b}$ are equal to zero) and is non-absorbing, we obtain from (17)

$$I=I_{\mathrm{reg}}=4\sin^2\frac{\varphi\bar{a}}{2}\frac{\sin^2 NX}{\sin^2 X}. \qquad (21)$$

By substituting (21) in (14) we obtain the well known formula for transition x-radiation produced in a stack of regularly arranged plates (see[2]). The factor $4\sin^2(\varphi\bar{a}/2)$ in (21) then gives the radiation produced by one plate, and the factor $\sin^2 NX/\sin^2 X$ is due to the presence of N regularly disposed plates. As is well known, at sufficiently large N the factor $\sin^2 NX/\sin^2 X$ takes the form of a δ-function. This factor reaches its maximum value N^2 at $X = n\pi$, where n is an integer.

This δ-function character of I_{reg} is due to the fact that the denominator $\sin^2 X$, or equivalently $|1 - \exp(-i\varphi\bar{a} - i\varphi_0\bar{b})|^2/4$, can vanish. This δ-function character of I is gradually lost the more irregular the medium becomes. Indeed, in the presence of irregularity the denominator of (13) takes the form

$$|1-h_ah_b|=|1-\exp(-2iX)q_aq_b|;$$
$$q_a=\langle\cos\varphi\Delta a-i\sin\varphi\Delta a\rangle, \qquad (22)$$
$$q_b=\langle\cos\varphi_0\Delta b-i\sin\varphi_0\Delta b\rangle.$$

It is easy to show that $|q_a|^2 \le 1$ and $|q_b|^2 \le 1$. Equality is attained, generally speaking, only in the case of a regular medium. In addition, the less regular the medium the larger the difference between $|q_a|^2$ or $|q_b|^2$ (or both) from unity. The minimum value of the denominator (2) is then different from zero and becomes increasingly larger, and consequently I becomes less similar to a δ-function. A similar conclusion can be drawn also in the case when absorption is taken into account.

Let us examine another limiting case, when the medium is extremely irregular. We then distinguish between two modifications:

1. All the plates are of equal thickness, but they are arranged in extreme disorder, i.e., $\Delta a_i = 0$, but $\langle\cos\varphi_0\Delta b_i\rangle = \langle\sin\varphi_0\Delta b_i\rangle = 0$.

In other words $h_a = \exp(-i\varphi_a)$ and $h_b = 0$. We then obtain from (13)

$$I=4N\sin^2(\varphi\bar{a}/2).$$

Substituting the last formula in (14) we find that in this case the number of transition-radiation quanta is simply the sum of the quanta from all the plates. This result is natural. Indeed, as noted at the end of Sec. 2, the quantities a_i and b_i are positive. This means that a large scatter in these quantities makes their averages large. In particular, if the scatter of the distances between the plates is much larger than the zone of formation in vacuum, then the average values of these distances are also much larger than the latter. It follows therefore that the interference between the radiation from the different plates can vanish, so that additive summation of the intensities (meaning also of the numbers of the quanta) of the radiation from all the plates occurs.

A similar conclusion is obtained also when absorption is taken into account. Indeed, at $\Delta a_i = 0$ we have $p = \exp(-2\mu a)$, where $a = a_i$, and we obtain from (19)

$$I=\frac{1-\exp(-2\mu Na)}{1-\exp(-2\mu a)}[1+\exp(-2\mu a)-2\cos\varphi a]. \qquad (23)$$

After substituting (23) in (14), the quantity in the square brackets is the number of transition-radiation quanta produced on one plate with allowance for the absorption in the plate, and the factor

$$[1-\exp(-2\mu Na)]/[1-\exp(-2\mu a)]$$

appears if one sums additively the radiation quanta from different plates with allowance for absorption in the plates.

2. Both the plate thicknesses and the distances between the plates are extremely irregular, i.e.,

$$\langle\cos\varphi\Delta a\rangle=\langle\sin\varphi\Delta a\rangle=\langle\cos\varphi_0\Delta b\rangle=\langle\sin\varphi_0\Delta b\rangle=0.$$

In other words, $h_a = h_b = 0$. We then obtain from (13)

$$I=2N, \qquad (24)$$

and from (19)

$$I=\frac{1-p^N}{1-p}(1+p). \qquad (25)$$

Expressions (24) and (25) have a simple physical meaning: when the scatter of the plate thicknesses and of the distances between the plates (and consequently also the mean values of these thicknesses and the distances) is much larger than the formation zone in the medium and in the vacuum, respectively, the number of quanta of the resultant radiation is simply the additive sum of the numbers of the radiation quanta of the 2N boundaries. Formula (24) does not take absorption into account, while formula (25) takes into account the absorption of the radiation and the plates.

6. CASE OF GAMMA DISTRIBUTION

To illustrate the applications of the derived formulas and conclusions, let us consider an example with concrete distributions for a_i and b_i. It is most convenient to choose for this purpose the so-called gamma distribution

$$f(y)=\beta_0^{\alpha+1}y^\alpha\exp(-\beta_0 y)/\Gamma(\alpha+1), \qquad (26)$$

where α and β_0 are real parameters, $\alpha > -1$, $\beta_0 > 0$, and $\Gamma(x)$ is the gamma function.

The mean value (mathematical expectation) $\bar{y} \equiv \langle y\rangle$ and the mean-squared deviation (variance) $\langle\Delta y^2\rangle = \langle(\bar{y} - y)^2\rangle$ are expressed in terms of the parameters α and β_0 by the respective formulas

$$\bar{y}=(\alpha+1)/\beta_0,\quad \langle\Delta y^2\rangle=(\alpha+1)/\beta_0^2=\bar{y}^2/(\alpha+1). \qquad (27)$$

We introduce also the degree of irregularity, defined by

$$\zeta=\langle\Delta y^2\rangle^{1/2}/\bar{y}=(\alpha+1)^{-1/2}. \qquad (28)$$

Plots of the gamma-distribution functions (26) are shown in Fig. 1 for a number of values of α.

With the aid of the distribution (26) we can easily calculate the values of h_a and h_b. We obtain from (10) and (26)

$$h_a=\left(1+\frac{i\varphi\bar{a}}{\alpha_a+1}\right)^{-\alpha_a-1}, \quad h_b=\left(1+\frac{i\varphi_0\bar{b}}{\alpha_b+1}\right)^{-\alpha_b-1}, \tag{29}$$

where α_a and α_b are the parameters of the gamma distributions for the plate thicknesses a_i and for the distances b_i between the plates, respectively.

We note that, for example, we have $h_a \to \exp(-i\varphi\bar{a})$ as $\alpha_a \to \infty$ and at a = const and $h_a \to 0$ as $\alpha_a \to \infty$ and at $\bar{a}/\alpha_a$ = const. These are respectively the cases of identical thicknesses and of an extremely large scatter of the thicknesses. Analogous limiting values hold also for h_b.

For convenience we present also the formulas (at $\mu = 0$)

$$|h_a|=\left[1+\left(\frac{\varphi\bar{a}}{\alpha_a+1}\right)^2\right]^{-(\alpha_a+1)/2}$$

$$\operatorname{Re} h_a=|h_a|\cos\psi_a, \quad \operatorname{Im} h_a=|h_a|\sin\psi_a; \tag{30}$$

$$\psi_a=-(\alpha_a+1)\operatorname{arctg}\frac{\varphi\bar{a}}{\alpha_a+1}.$$

Similar formulas hold also for the quantity h_b. They can be obtained from (30) by replacing a with b and φ with φ_0.

With the aid of (30) and the analogous formulas for h_b, together with formula (13) or ((19)) and (14), we can calculate the angle and frequency spectra of the transition x-radiation in the considered model of an irregular medium.

Figures 2–6 show the angle spectra calculated by this method for media with different values of the parameters $\bar{a}$ and $\bar{b}$ and different degrees of irregularity ζ_a and ζ_b, different quantum energies and different charged-particle factors $\gamma = (1-\beta^2)^{-1/2}$.

All the curves were calculated for N = 50 and ω_0 = 20 eV. Figure 7 shows also the frequency spectra for the corresponding regular medium (solid curves marked by the letters c). The angle spectra for the regular media, at the considered frequencies and particle energies, have a strong oscillatory character and are therefore not presented. Figure 6 does not show the frequency spectra for the irregular medium, since they almost coincide with the corresponding spectra at $\zeta_a = \zeta_b$ = 4.47% (dotted curves). For comparison, Figs. 6 and 7 show also the radiation spectra obtained by additive addition of the numbers of the quanta from all the plates and interfaces (solid curves labeled a and b).

From the results of the calculations we see clearly that the irregularity leads mainly to a vanishing of the interference between the radiation produced on different boundaries of the plates of the irregular stack.

When the irregularity is weak, i.e., when condition (16) is satisfied, the spectrum of the irregular medium is quite close in character to the corresponding spectrum of the regular medium. In particular, in the angle spectrum of a weakly irregular medium one observes clearly pronounced interference maxima, which are characteristic of the regular medium, and whose positions are determined by the equation

$$X=n\pi \ (n \text{ are integers}). \tag{31}$$

On the other hand, in cases when the condition (16) is not satisfied, the interference vanishes and the spectrum (either frequency or angle) becomes monotonic.

For a regular medium with a = 7 μ and b = 410 μ, the frequency spectrum from 4 to 100 keV has no maxima or minima. In this case the frequency spectra of different irregular media with the same $\bar{a}$ and $\bar{b}$ differ little from the frequency spectrum of a regular medium. However, in the case a = 250 μ and b = 200 μ at γ = 7500 the frequency spectrum of a regular medium (see[7]) has a minimum in the region of 25 keV followed by a maximum near 30 keV, both undoubtedly of interference origin. A similar situation holds also at γ = 75,000 for the same values of a and b. In these cases, for a medium with the same $\bar{a}$ and $\bar{b}$ but one that

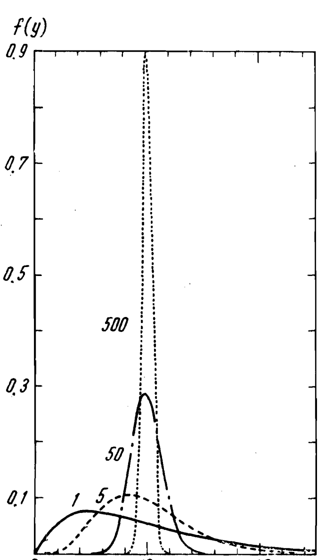

FIG. 1. Plots of the gamma distribution functions f(y) for the values a = 1, 5, 50, and 500 (indicated next to the curves) at y = 10. The corresponding values of ζ are 7.7, 40.8, 14.0, and 4.47%.

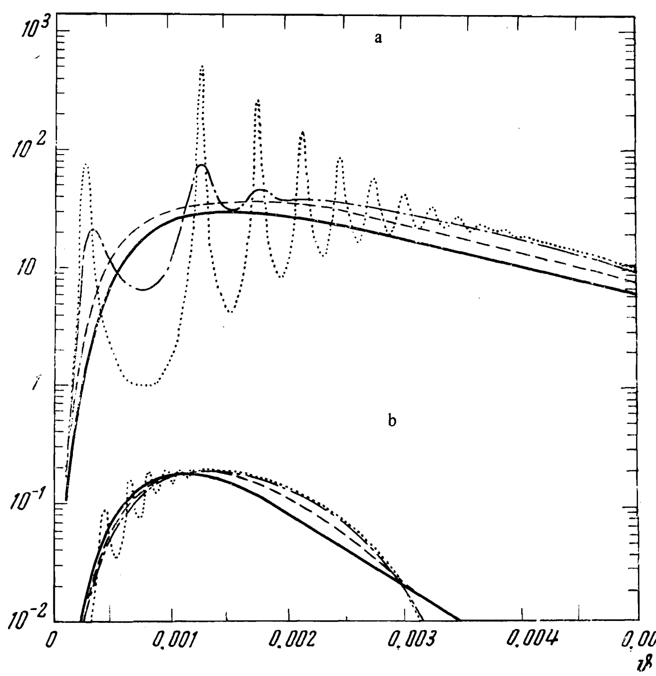

FIG. 2. Angle spectrum $d^2N_{qu}/d\omega d\vartheta$ (quanta/keV-rad-electron) for an irregular stack with $\bar{a} = 7\mu$ and $\bar{b} = 410\mu$ at $\gamma = 10^3$ and ω = 4 keV (a) or 25 keV (b). The dotted curves correspond to the case $\zeta_a = \zeta_b$ = 4.47%; the dash-dot curves to $\zeta_a = \zeta_b$ = 14.0%; the dashed curves to $\zeta_a = \zeta_a$ 40.8%; the solid curves to $\zeta_a = \zeta_b$ 70.7%.

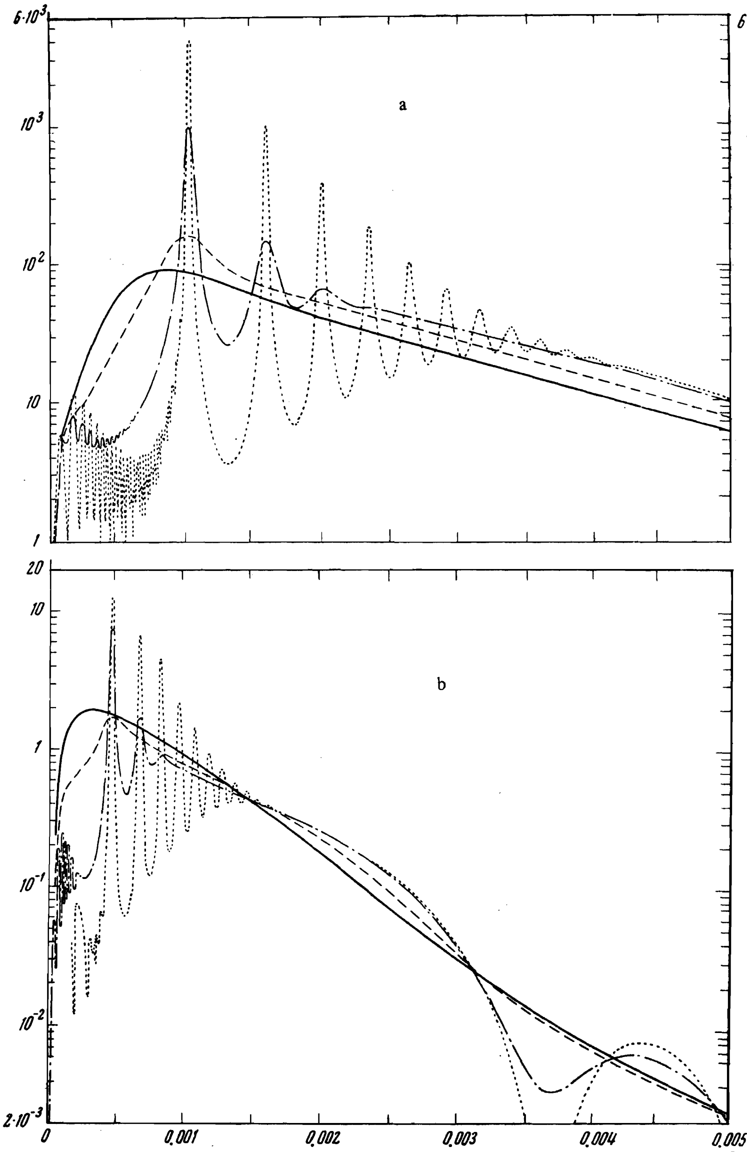

FIG. 3. The same as Fig. 2, for $\gamma = 10^4$.

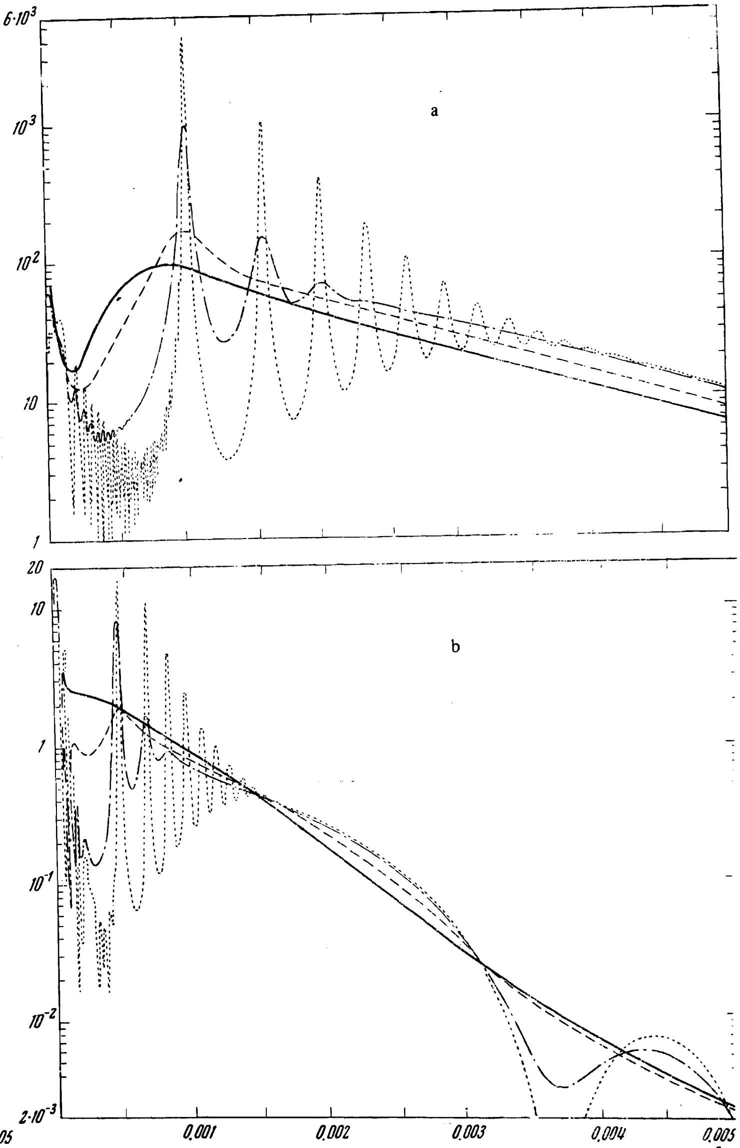

FIG. 4. The same as Fig. 2, for $\gamma = 10^5$.

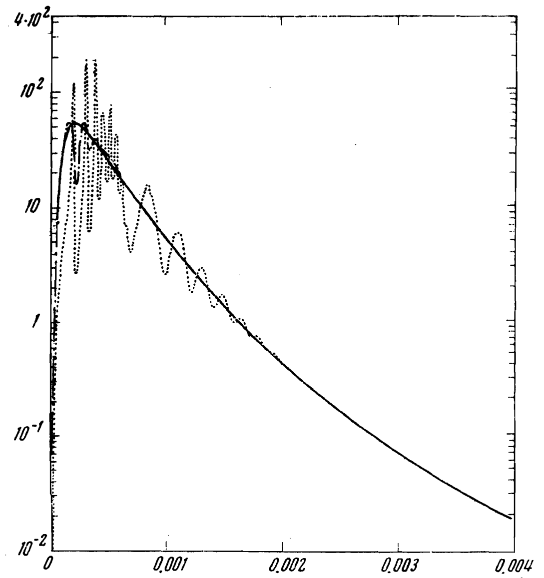

FIG. 5. Angle spectrum $d^2N_{qu}/d\omega d\vartheta$ for an irregular stack with a = 250 μ and b = 2000 μ at $\gamma = 7500$ and ω = 20 keV. The dotted curve corresponds to the case $\zeta_a = \zeta_b = 4.47\%$; the dash-dot curve to $\zeta_a = \zeta_b = 14\%$; the solid curve to $\zeta_a = \zeta_b = 70.7\%$.

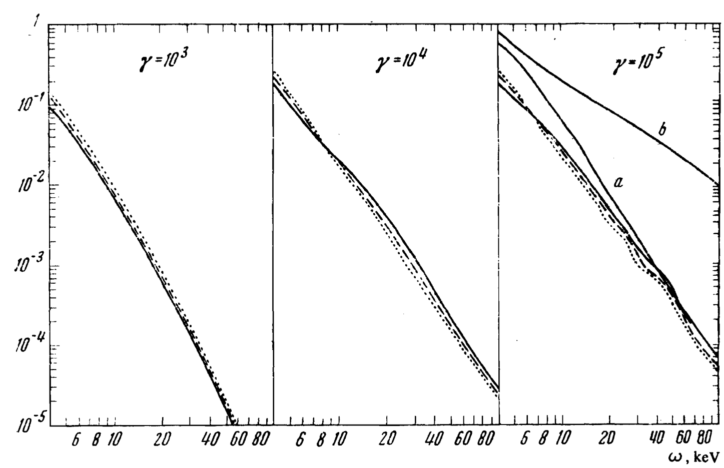

FIG. 6. Frequency spectrum $dN_{qu}/d\omega$ (quanta/keV-electron) for an irregular stack with $\bar{a} = 7\ \mu$ and $\bar{b} = 410\ \mu$. The dotted curves correspond to $\zeta_a = \zeta_b = 4.47\%$; the dashed curves to $\zeta_a = \zeta_b = 40.8\%$; the solid curves without letters to $\zeta_a = \zeta_b = 70.7\%$; the solid curve with letter a corresponds to the spectrum obtained by additive addition of the numbers of the quanta from all plates; the solid curve labeled b corresponds to additive addition of the numbers of quanta from all the boundaries.

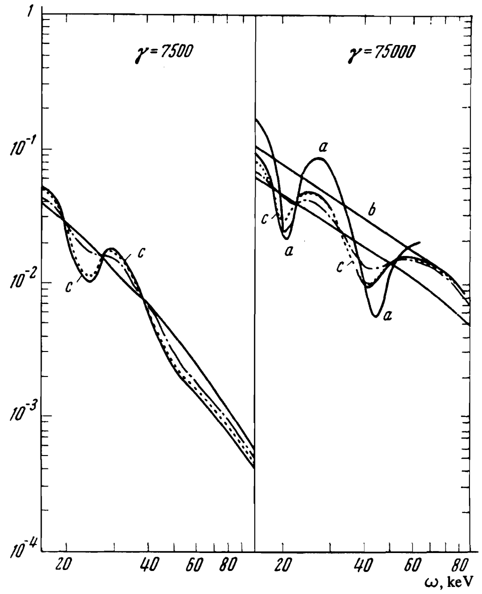

FIG. 7. The same as in Fig. 6 for $\bar{a}$ = 250 and $\bar{b}$ = 200. The dash-dot curves correspond to the case $\zeta_a = \zeta_b$ = 14.0%; the solid curves with letter c correspond to the case of aregular stack.

is sufficiently irregular (for example, at ζ_a, ζ_b > 40%), the frequency spectrum becomes equalized and in the region 20–50 keV it differs very appreciably from the spectrum of the regular medium (see Fig. 7).

All the foregoing, of course, is in full agreement with the general analysis presented in the preceding section.

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