# Photoeffect in strong magnetic fields and x-ray emission from neutron stars

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The cross section is obtained for the photoabsorption of x rays of arbitrary polarization incident at an arbitrary angle on a hydrogenlike atom in a strong magnetic field ($B \sim 10^{12}$–$10^{13}$ G, $\omega_B = eB/m_e c > \omega$). Over an energy range of several keV the cross section for the photoeffect is of the order of $10^{-22}$–$10^{-20}$ cm$^2$, and this significantly exceeds both the cross section for the photoeffect in the absence of a magnetic field, and the cross sections for scattering and for bremsstrahlung absorption by electrons of a thermal plasma ($T \sim 10^6$–$10^7$ °K) in a magnetic field. An equation has been obtained for the ionization equilibrium in a magnetized plasma and it is shown that the magnetic field can both increase and decrease the degree of ionization depending on the value of the temperature. Owing to the high value for the photoionization cross section in a magnetic field, photorecombination can be the principal source of release of thermal energy by the plasma of neutron stars at a temperature of $T \sim 10^6$ –$10^7$ °K. Its intensity in the soft x-ray region exceeds the intensity of bremsstrahlung from electrons by severalfold in the case of transverse propagation and by more than an order of magnitude in the case of longitudinal propagation. The characteristic features of structure of atoms in a strong magnetic field can lead to the appearance of discontinuities in the spectrum of a neutron star lying in the region of soft x-ray radiation. For quanta of energy $\hbar\omega < 3$keV a situation is possible when the characteristics of the radiation from a neutron star are determined by the processes of photoionization and photorecombination. In this case the x radiation does not have directional properties.

## 1. INTRODUCTION

According to present concepts magnetic fields at the surface of neutron stars can attain values of $10^{12}$–$10^{13}$ G. These fields significantly affect the structure of matter[1] and the processes of interaction between radiation and matter. At the present time investigations have been carried out of processes of scattering of radiation by electrons in such strong fields[2-5], and also of bremsstrahlung and magnetic bremsstrahlung absorption and emission[6].

In magnetic fields B = $10^{12}$–$10^{13}$ G the ionization energy of atoms increases greatly and falls into the soft x-ray region. Therefore the processes of photoionization of atoms and ions can turn out to be the determining ones for the x-ray emission from the atmosphere of a neutron star and, in particular, of an x-ray pulsar. In Sec. 2 the cross section is derived for the photoabsorption by a hydrogenlike atom in a strong magnetic field ($\omega_B = eB/m_e c > \omega$) for radiation incident on an atom at an arbitrary angle and with arbitrary polarization. The formulas obtained are valid in a range of energies of the quanta of the order of several keV. In this range the cross section for the photoeffect is $\sim 10^{-22}$–$10^{-20}$ cm$^2$, and this significantly exceeds both the cross section for the photoeffect in the absence of a magnetic field and also the cross sections for the scattering and the bremsstrahlung absorption by electrons in a strong magnetic field. The spectral, angular and polarization dependence of the cross section for the photoeffect differ from the same quantities for scattering by electrons and for bremsstrahlung absorption. In particular, the cross section for the photoeffect from the ground state along the magnetic field is lower than the cross section in the perpendicular direction by a factor of $\omega_B/\omega$.

In order for the processes of photoabsorption by atoms to affect the radiation from a neutron star it is necessary for a sufficiently large number of atoms to be present in its atmosphere. For a plasma of temperature T the relative number of ions of different stages of ionization can be obtained from the Saha formula. In Sec. 3 the question is investigated concerning the effect of a magnetic field on the equation for ionization equilibrium and a generalization of the Saha formula is obtained for a magnetized plasma. From the formula obtained it follows that the magnetic field can both increase and diminish the degree of ionization depending on the value of the temperature T.

In Sec. 4 the photoeffect in the atmosphere of a neutron star is investigated. Since the cross sections for photoionization in a strong magnetic field B ~ $10^{12}$–$10^{13}$ G are large, then even a small admixture of atoms and ions leads to appreciable recombination radiation from the plasma. Photorecombination can serve as the principal source of release of thermal energy by a plasma of temperature T ~ $10^6$–$10^7$ °K in the soft x-ray region exceeding the intensity of bremsstrahlung by electrons by severalfold in the case of transverse propagation and by more than an order of magnitude in the case of longitudinal propagation. The spectrum of recombination radiation from an optically thin plasma differs appreciably from the bremsstrahlung spectrum, and in the Rayleigh-Jeans region the formation of an inverted spectrum of the form $I(\omega) \propto \omega^{-1/2}$ is possible. We note that special features of the structure of atoms in a strong magnetic field can lead to the appearance in the neutron-star spectrum of discontinuities lying in the x-ray domain.

For quanta of energy $\hbar\omega \lesssim 3$ keV the characteristics of the radiation from a neutron star are determined by the processes of photoionization and photorecombination. This radiation does not have directional properties in contrast, for example, to radiation scattered by electrons.

## 2. THE CROSS SECTION FOR PHOTOABSORPTION BY A HYDROGENLIKE ATOM IN A STRONG MAGNETIC FIELD

If the magnetic field is $B \gg B_0 = Z^2 m_e^2 e^3 c/\hbar^3 = 2.35 Z^2 \times 10^9$ G, then the forces acting on an electron of a hydrogenlike atom due to its interaction with a magnetic field predominate over the Coulomb forces, the transverse dimension of the atom becomes smaller than the Bohr radius, while the velocity of the electron in the transverse direction becomes greater than in the longitudinal direction. In this case [7] the wave function for the electron can be approximately represented in the form of a product of functions describing the transverse and the longitudinal motion, and the function describing the transverse motion can be used without taking the Coulomb field into account, while the function describing the longitudinal motion is determined from the solution of the one-dimensional Schrödinger equation the potential in which is the Coulomb potential averaged over the transverse motion. The total energy of the electron in this approximation is given by the sum of the energies of the transverse and the longitudinal motions:

$$E=\hbar\omega_B(N+s_z+1/2)+\varepsilon, \tag{1}$$

where $\omega_B = eB/m_ec$ is the electron cyclotron frequency, N is the number of the Landau level (N = 0, 1, 2, ...), $s_z$ is the component of the electron spin along the direction of the magnetic field, $\epsilon$ is the energy of the longitudinal motion. In the case of $B \gg B_0$ the energy is $\epsilon \ll \hbar\omega_B$.

We shall be interested in the cross section for the absorption of photons of frequency $\omega < \omega_B$ for a temperature of the gas (plasma) $kT \ll \hbar\omega_B$. In this case a contribution to absorption is given only by the ground energy level for the transverse motion, i.e., N = 0, $s_z = -1/2$, $E = \epsilon$. The wave function for the transverse motion for N = 0[8] is

$$\Phi_s(\rho\varphi)=\lambda^{-1}(2^s s!2\pi)^{-1/2}(\rho/\lambda)^s\exp(-\rho^2/4\lambda^2-is\varphi), \tag{2}$$

where $\rho$ and $\varphi$ are cylindrical coordinates, $\lambda = (c\hbar/eB)^{1/2}$ is a "magnetic length," s is the negative of the component along the direction of the magnetic field of the moment of the electron momentum (s = 0, 1, 2, ...). The wave function for the longitudinal motion satisfies the equation

$$\left[-\frac{\hbar^2}{2m_e}\frac{d^2}{dz^2}+V_s(z)\right]f_{sn}(z)=\varepsilon_{sn}f_{sn}(z), \tag{3}$$

where n is the quantum number determining the energy of the longitudinal motion for a given s,

$$V_s(z)=-\frac{Ze^2}{s!}\int_0^\infty\frac{u^s e^{-u}\,du}{\sqrt{z^2+2\lambda^2u}}. \tag{4}$$

In the case $z \gg \lambda$ the potential $V_s(z)$ coincides with the one-dimensional Coulomb potential: $V_s(z) = -(Ze^2/z)[1 + O(\lambda^3/z^3)]$. In the case $z \ll \lambda$

$$V_0(z)\approx-\frac{Ze^2}{\lambda}\left(\sqrt{\frac{\pi}{2}}-\frac{|z|}{\lambda}\right),$$
$$V_s(z)\approx\frac{-Ze^2}{\lambda}\frac{1}{s!\sqrt{2}}\Gamma\left(s-\frac{1}{2}\right)\left(s-\frac{1}{2}-\frac{z^2}{2\lambda^2}\right)\quad(s\geq1). \tag{5}$$

The spectrum $\epsilon_{sn}$ and the wave functions $f_{sn}(z)$ have been investigated by many authors in connection with the problem of the exciton absorption of light in a magnetic field. Variational methods were utilized in [9,10], approximate analytic expressions were obtained in [11,12], and the spectrum was calculated in [13] by a numerical solution of Eq. (3).

For the discrete spectrum the quantum number n in (3) can be so chosen that it is equal to the number of nodes of the function $f_{sn}(z)$ in the region $0 \leq z < \infty$. Then levels with n = 0 and with n = 1, 2, ... behave significantly differently as the magnetic field is increased [11-13]. The energies of the first group of levels are proportional to $-\ln^2(B/B_0)$ for very high fields[1]. Approximately they can be found in accordance with the formula

$$\varepsilon_{s0}=16\pi^{-1}Z^2\,\mathrm{Ry}\,\alpha_s(1+\alpha_s/2), \tag{6}$$

where Ry = 13.6 eV, $\alpha_s$ is a root of the equation

$$2\alpha_s=\ln(\pi B/8\alpha_s B_0)-2-A_s;\quad A_0=0,\quad A_s=\sum_{k=1}^{s}k^{-1}.$$

The corresponding wave functions $f_{s0}(z)$ are even functions of z.

The energies of levels with $n \geq 1$ (hydrogenlike levels) tend to $-Z^2\mathrm{Ry}/n^2$ as B is increased, and each n corresponds to a pair of states of opposite parity, slightly differing in energy. For $B \gg B_0$ below the first hydrogenlike level $\epsilon_{01}$ there lies a large number of tightly bound levels $\epsilon_{s0}$. The results of a numerical calculation [13] for hydrogen in the case of $B = 2 \times 10^{12}$ G ($\lambda = 1.8 \times 10^{-10}$ cm, $\hbar\omega_B$ = 24 keV) yield for the energy of the ground state $\epsilon_{00} = -190$ eV, for the energy of the first excited state $\epsilon_{10} = -145$ eV, and of the first hydrogenlike level $\epsilon_{01} = -13.5$ eV. The corresponding values for the He II ion are: $\epsilon_0 = -4.80$ eV, $\epsilon_{10} = -336$ eV, $\epsilon_{01} = -54$ eV. In the case of the field $B = 10^{13}$ G the energy of the ground level of hydrogen is $\epsilon_{00} = -280$ eV.

If the temperature is not too high, then primarily the levels with n = 0 are populated. We obtain the cross section for the absorption of arbitrarily polarized photons by a one-electron ion, occupying one of such levels. It is given by the formula

$$\sigma_j^{(s)}(\mathbf{n}\omega)=\frac{2\pi}{\hbar c}\sum_f|V_j^{fs}(\mathbf{n}\omega)|^2\delta(\varepsilon_f-\varepsilon_s-\hbar\omega). \tag{7}$$

Here f are quantum numbers describing the final state of the electron; **n**, $\omega$ and j are the direction of propagation, the frequency and the polarization index of the radiation; $\epsilon_s \equiv \epsilon_{s0}$; $V_j^{fs}(\mathbf{n}\omega)$ is the matrix element of the potential describing the interaction of the electron with the radiation field corresponding to the transition from the state $|s0\rangle$ into the state $|f\rangle$. In the dipole approximation[2]

$$V_j^{fs}(\mathbf{n}\omega)=-ie(2\pi\hbar\omega)^{1/2}(\mathbf{e}_{nj}\mathbf{r}^{fs}), \tag{8}$$

where $\mathbf{e}_{nj}$ is the polarization unit vector, $\mathbf{r}^{fs}$ is the matrix element of the coordinate. By decomposing the unit vector of arbitrary polarization $\mathbf{e}_{nj}$ in terms of the unit vectors of circular polarization $\mathbf{e}_{n\alpha}$ ($\alpha = \pm 1$ corresponds to the right- and left-handed circular polarization), the cross section (7) can be easily expressed in terms of

$$\sigma_{\alpha\beta}^{(s)}(\mathbf{n}\omega)=\frac{2\pi}{\hbar c}\sum_f V_\alpha^{fs}(\mathbf{n}\omega)[V_\beta^{fs}(\mathbf{n}\omega)]^*\delta(\varepsilon_f-\varepsilon_s-\hbar\omega), \tag{9}$$

where

$$V_\alpha^{fs}(\mathbf{n}\omega)=-ie(2\pi\hbar\omega)^{1/2}\sum_{\mu=-1}^{1}D_{\mu\alpha}^{(1)}(\varphi\theta 0)r_\mu^{fs}. \tag{10}$$

In (10) $r_\mu^{fs}$ is the matrix element of the cyclic component of the radius-vector $r_0 = z$, $r_{\pm1} = \pm 2^{-1/2}\rho\exp(\pm i\varphi)$, $\theta$ and $\varphi$ are the polar and the azimuthal angles of the vector **n** in the system of coordinates whose polar axis is directed along the magnetic field, $D_{\mu\alpha}^{(1)}(\varphi\theta\gamma)$ are the matrices for finite rotations [15]:

$$D_{\mu\alpha}^{(1)}(\varphi\theta\gamma)=e^{-i\mu\varphi}\,d_{\mu\alpha}^{(1)}(\theta)\,e^{-i\gamma\alpha},$$
$$d_{00}^{(1)}(\theta)=\cos\theta,\quad d_{0\alpha}^{(1)}(\theta)=-d_{\alpha0}^{(1)}(\theta)=2^{-1/2}\alpha\sin\theta, \tag{11}$$
$$d_{\alpha\alpha'}^{(1)}(\theta)=1/2(1+\alpha\alpha'\cos\theta).$$

If $\hbar\omega > |\epsilon_0|$, then the final state belongs in the continuous spectrum and absorption of radiation occurs as a result of photoionization (bound-free transitions). In the special case $\theta = 0$, $s = 0$ this process has been studied in[10,12]. But even in this case the results obtained are valid only near the threshold of the photoeffect. We shall be interested in photoionization by quanta of high frequency $\hbar\omega > |V_s(0)| > |\epsilon_s|$. Then in the continuous spectrum one can neglect the effect of the potential $V_s(z)$ and treat longitudinal motion as if it were free:

$$\langle z|f\rangle = f_{sq}(z) = e^{iqz}, \quad \varepsilon_f = \varepsilon_q = \hbar^2 q^2/2m_e, \quad -\infty < q < \infty.$$

This approximation is certainly valid for $\hbar\omega > Ze^2/\lambda$ ($\hbar\omega > 1.6$ keV for $B = 2\times 10^{12}$ G, $Z = 2$). Taking (2) and (10) into account and going over in the usual manner from summing over the quantum numbers of the final state to integrating over q we obtain

$$\sigma_{\alpha\beta}^{(s)}(\theta\omega) = \frac{2\pi e^2\omega m_e}{\hbar^2 cq}\sum_\mu d_{\mu\alpha}^{(1)}(\theta) d_{\mu\beta}^{(1)}(\theta)\frac{1}{s!(s-\mu)!}\left[\left(s+\frac{|\mu|-\mu}{2}\right)!\right]^2 \times \lambda^{2|\mu|}\{|J_\mu^s(q)|^2 + |J_\mu^s(-q)|^2\}, \tag{12}$$

where $q = (2m_e(\hbar\omega + \epsilon_s))^{1/2}/\hbar \approx (2m_e\omega/\hbar)^{1/2}$ and

$$J_\mu^s(q) = \int_{-\infty}^{\infty} e^{-iqz} z^{1-|\mu|} f_s(z)\,dz. \tag{13}$$

In the domain $|z| \gg \lambda$ the solution of Eq. (3) is given by[11] the Whittaker function $W_{\beta_s 1/2}(2|z|/\beta_s a_B)$, where $a_B = \hbar^2/Zm_e e^2$ is the Bohr radius, $\beta_s^2 = -Z^2 Ry/\epsilon_s$. Comparing the asymptotic behavior of the Whittaker function for small values of the argument and the asymptotic behavior of the solution of equation (3) for $|z| \ll \lambda$, one can easily verify that the replacement in (13) of the exact solution by $W_{\beta_s 1/2}(2|z|/\beta_s a_B)$ gives an error $\sim (\omega/\omega_B)^{1/2}\beta_s \exp(-1/2\,\beta_s - 0.58) \ll 1$, since $\omega < \omega_B$ and $\beta_s \ll 1$ for not too great values of s. The integral so obtained can be approximately evaluated by utilizing the integral representation of the Whittaker function and the smallness of the parameter $\beta_s$. Calculation yields

$$J_{\pm 1}^s(q) = \frac{2(\beta_s a_B)^{1/2}}{1+(q\beta_s a_B)^2}\left\{1 + O\left[2\beta_s \ln\frac{1+(q\beta_s a_B)^2}{4}\right]\right\} \approx 2(\beta_s a_B)^{1/2}|\varepsilon_s|/\hbar\omega,$$
$$J_0(q) = i\frac{d}{dq}J_{\pm 1}^s(q) \approx -i4(\beta_s a_B)^{1/2}\frac{|\varepsilon_s|}{\hbar\omega}\left(\frac{\hbar}{2m\omega}\right)^{1/2}. \tag{14}$$

Formulas (14) are valid for $2\beta_s \ln(\hbar\omega/4|\epsilon_s|) < 1$. Substituting (14) in (12) and utilizing (11) we obtain

$$\sigma_{\alpha\beta}^{(s)}(\theta\omega) = \sigma_0(\omega)|\varepsilon_s/\varepsilon_0|^{3/2}\{\alpha\beta\sin^2\theta + (\omega/4\omega_B)[(2s+1)\times(1+\alpha\beta\cos^2\theta) - (\alpha+\beta)\cos\theta]\}, \tag{15}$$

where

$$\sigma_0(\omega) = 8\pi\frac{e^2}{\hbar c}\left(\frac{|\varepsilon_0|}{\hbar\omega}\right)^{1/2}\frac{\hbar}{m_e\omega} = 1{,}4\cdot 10^{-19}\left(\frac{|\varepsilon_0|}{\hbar\omega}\right)^{1/2}\frac{1[\mathrm{keV}]}{\hbar\omega}[\mathrm{cm}^2]. \tag{16}$$

From this we obtain for circularly polarized radiation

$$\sigma_{\pm 1}^{(s)}(\theta\omega) = \sigma_0(\omega)\left|\frac{\varepsilon_s}{\varepsilon_0}\right|^{3/2}\left\{\sin^2\theta + \frac{\omega}{4\omega_B}\left[2s(1+\cos^2\theta) + (1\mp\cos\theta)^2\right]\right\}; \tag{17}$$

for linearly polarized radiation, the electric vector of which oscillates in the plane containing **n** and **B**, we get

$$\sigma_2^{(s)}(\theta\omega) = \sigma_0(\omega)\left|\frac{\varepsilon_s}{\varepsilon_0}\right|^{3/2}\left\{2\sin^2\theta + \frac{\omega}{2\omega_B}(2s+1)\cos^2\theta\right\}; \tag{18}$$

for radiation linearly polarized in the perpendicular direction we have

$$\sigma_1^{(s)}(\theta\omega) = \sigma_0(\omega)\left|\frac{\varepsilon_s}{\varepsilon_0}\right|^{3/2}\frac{\omega}{\omega_B}\left(s+\frac{1}{2}\right) = 4\pi\frac{e^2}{\hbar c}\left(\frac{|\varepsilon_s|}{\hbar\omega}\right)^{3/2}(2s+1)\lambda^2; \tag{19}$$

and for unpolarized radiation

$$\sigma^{(s)}(\theta\omega) = \sigma_0(\omega)|\varepsilon_s/\varepsilon_0|^{3/2}\left\{\sin^2\theta + \frac{\omega}{4\omega_B}(2s+1)(1+\cos^2\theta)\right\}. \tag{20}$$

Formulas (15)–(20) are valid for $Ze^2/\lambda < \hbar\omega < \hbar\omega_B$ and $2\beta_s \ln(\hbar\omega/4|\epsilon_s|) < 1$. In the field $B = 2\times 10^{12}$ G this corresponds to $\hbar\omega \approx 0.5$–$5$ keV for HeI and $\hbar\omega \sim 1$–$10$ keV for He II. We note that in this range of frequencies the cross section obtained significantly exceeds the cross section for the photoeffect in the absence of a magnetic field. For example, for $\theta = \pi/2$ and unpolarized radiation the cross section for the photoeffect from the ground level of He II for $\hbar\omega = 5$ keV is equal to $8.7\times 10^{-22}$ cm$^2$, while in the absence of the magnetic field[16] we have

$$\sigma(B=0) = \frac{64\pi}{3}\frac{e^2}{\hbar c}\left(\frac{I}{\hbar\omega}\right)^{5/2}\frac{\hbar}{m_e\omega} = 8.7\cdot 10^{-25}\,\mathrm{cm}^2.$$

An increase of the cross section in a magnetic field occurs, first of all, due to the increase in the ionization energy in a magnetic field, and secondly due to the weaker frequency dependence of the cross section (instead of $\omega^{-7/2}$ one obtains $\omega^{-5/2}$ for $\theta = \pi/2$ or $\omega^{-3/2}$ for $\theta = 0$), which appears as a result of the transverse motion of the electron being quantized. The presence of a magnetic field leads to the fact that, on the one hand, the density of the number of final states becomes proportional to $\omega^{-1/2}$ instead of $\omega^{1/2}$, while, on the other hand, the matrix element of the dipole moment is proportional to $\omega^{-3/2}$ for $\theta = \pi/2$ or to $\omega^{-1}$ for $\theta = 0$ instead of to $\omega^{-5/2}$ in the absence of a magnetic field.

The frequency dependence of the cross section for the photoeffect depends also on the level from which the electron is ejected. If s is not great:

$$(2s+1)(1+\cos^2\theta)(\omega/4\omega_B) < \sin^2\theta,$$

then $\sigma(\theta\omega) \propto \omega^{-5/2}$. For large values of s and small angles $\theta^2 < (2s+1)\omega/4\omega_B$ the cross section $\sigma(\theta\omega)\alpha\omega^{-3/2}$, since in this case the principal role in absorption of radiation is played by the transverse dimension of the atom.

The angular and the polarization dependence of the cross section also depends on the number s. Thus, for s = 0 primarily that radiation is absorbed which is incident at right angles to the field; in the case of longitudinal incidence only one circular polarization (left-handed) is absorbed, the right-handed polarization is transmitted without being absorbed. In the case of large s ($s > 2\omega_B/\omega$) the cross section depends weakly on the angle and on the polarization.

From formulas (15)–(20) it follows that the cross section for the photoeffect is considerably greater than the cross section for scattering by a free electron in a strong magnetic field $\omega_B > \omega$. For $\theta = \pi/2$ the cross section for the photoeffect is $\sigma_a \approx \sigma_0(\omega) = (460$–$1.5) \times 10^{-22}$ cm$^2$ (for He II and $B = 2\times 10^{12}$ G) while the cross section for scattering by an electron $\sigma_e$ is of the order of the Thompson cross section $\sigma_e \approx \sigma_T = 6.65\times 10^{-25}$ cm$^2$. For $\theta = 0$ the cross section for the photoeffect is diminished by a factor $\omega_B/\omega$, while the cross section for scattering by an electron is diminished by a factor $(\omega_B/\omega)^2$[2,4], i.e., the ratio $\sigma_a/\sigma_e$ becomes even greater.

## 3. IONIZATION EQUILIBRIUM IN A MAGNETIC FIELD (THE SAHA FORMULA)

In order to determine the atomic and the electron coefficients of absorption of radiation in a plasma it is necessary to know the densities of electrons, atoms and ions in the presence of a magnetic field.

We consider the thermal ionization of a gas in a magnetic field. The gas temperature is assumed to be sufficiently high, so that the gas can be regarded as monatomic. Let $\mu_i$ be the chemical potential for a gas of i-fold ionized atoms (i = 0, 1, 2, ..., Z), $\mu_e$ be the chemical potential for the electron gas. Then the ionization equilibrium is described by a system of Z equations[17]

$$\mu_{i-1}-\mu_i-\mu_e=0. \tag{21}$$

For a sufficiently rare and hot plasma the free energy is the sum of the free energies of the individual components:

$$F=F_e+\sum_{i=0}^{Z} F_i \quad \text{and} \quad \mu_{e,i}=(\partial F_{e,i}/\partial N_{e,i})_{V,T}.$$

We take into account the fact that in a magnetic field the longitudinal motion of the electron is quasiclassical, while the transverse motion is quantized (cf., (1)); the degree of degeneracy of an energy level[8] is $g = V^{2/3}eB/2\pi c\hbar$. Then, utilizing the formula for the free energy of an ideal gas we obtain

$$\begin{gathered} F_e=-N_e kT\left\{1+\ln\frac{1}{n_e}\left(\frac{m_e kT}{2\pi\hbar^2}\right)^{1/2} 2\eta \operatorname{cth}\eta\right\}, \\ \mu_e=kT\ln\left[n_e\left(\frac{2\pi\hbar^2}{m_e kT}\right)^{1/2}\left(\frac{\operatorname{th}\eta}{2\eta}\right)\right], \end{gathered} \tag{22}$$

where $n_e = N_e/V$ is the electron density, $\eta = \hbar\omega_B/2kT$. The chemical potential for a gas of i-fold ionized atoms is

$$\mu_i=kT\ln\left[n_i(2\pi\hbar^2/m_i kT)^{3/2} z_i^{-1}\right], \tag{23}$$

where $m_i$ is the ion mass, $n_i$ is the density of i-fold ionized atoms

$$z_i=\sum_k g_{i,k}\exp(-E_{ik}/kT)$$

is the distribution function for the ions ($E_{ik}$ and $g_{ik}$ are the energy and the degree of degeneracy of the k-th level). For a completely ionized atom $z_i = 1$. Formula (23) is valid if $kT \gg \hbar\omega_{B_i}$, where $\omega_{B_i} = z_i eB/m_i c$ is the ion cyclotron frequency. For the He II ion for $B = 2 \times 10^{12}$ G this corresponds to $T \gg 10^4$ °K.

Substituting (22) and (23) into (21) we obtain a system of equations describing ionization equilibrium in a magnetic field (generalization of the Saha formula):

$$\frac{n_i}{n_{i-1}}n_e=\frac{z_i}{z_{i-1}}\left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2}\frac{\hbar\omega_B}{kT}\operatorname{cth}\frac{\hbar\omega_B}{2kT}. \tag{24}$$

If instead of $n_i$ we make use of the density of ions occupying the k-th quantum level $n_{i,k}$, then instead of (24) we have

$$\frac{n_{i,k}}{n_{i-1,l}}n_e=\frac{g_{i,k}}{g_{i-1,l}}\exp\left[\frac{-(E_{i,k}-E_{i-1,l})}{kT}\right]\left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2}\frac{\hbar\omega_B}{kT}\operatorname{cth}\frac{\hbar\omega_B}{2kT}. \tag{25}$$

If i = Z (the last ionization), then in (25) $g_{i,k} = 1$, $E_{i,k} = 0$. For $\hbar\omega_B \ll kT$ this formula goes over into the usual Saha formula, for $\hbar\omega_B \gg kT$ we have

$$\frac{n_{i,k}}{n_{i-1,l}}n_e=\frac{g_{i,k}}{g_{i-1,l}}\exp\left[\frac{-(E_{i,k}-E_{i-1,l})}{kT}\right]\left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2}\frac{\hbar\omega_B}{kT}. \tag{26}$$

Thus, the magnetic field affects the degree of ionization in two ways. On the one hand, it increases the ionization energy and thereby diminishes the degree of ionization. On the other hand, the magnetic field decreases the phase space corresponding to one quantum state, and as a result of this the degree of ionization is increased by a factor of $2\eta\coth\eta$. For example, for $B = 2 \times 10^{12}$ G the ionization energy of He II increases from 54 eV to 0.48 keV, and this at $T = 3 \times 10^6$ °K diminishes the degree of ionization by a factor of 4.8. But at the same values of the field and of the temperature $\eta = 44$. As a result the degree of ionization increases by a factor of 17.3. A field of the same magnitude at $T = 9 \times 10^5$ °K diminishes the degree of ionization by a factor of 1.6.

## 4. PHOTOEFFECT IN THE ATMOSPHERE OF A NEUTRON STAR

Since the cross section for photoionization in a strong magnetic field $B \sim 10^{12}-10^{13}$ G is large, then even a small number of incompletely ionized atoms can significantly alter the conditions for the production and transport of radiation and, in particular, alter processes leading to the formation of an x-ray pulsar.

As a result of different mechanisms of release of energy hot spots can arise at the magnetic poles of a neutron star. If the neutron star is a member of a binary system, then the spots arise as a result of the fact that accreting matter of the principal component is chanelled by the magnetic field of the neutron star and arrives at the poles[18]. For a star which is not a member of a binary system the energy release can occur either in the process of internal friction between the neutron and the charged components as a result of differential rotation[19], or at the expense of nuclear reactions[20]. In this case the anisotropic process of transport of heat energy in a strong magnetic field[21] is responsible for the formation of hot spots.

However, isotropically radiating hot spots at the magnetic poles of an inclined rotator can not lead to a sharp picture of an x-ray pulsar. Directional properties of radiation are necessary. They can arise, for example, as a result of higher transparency of surface layers of the star in the direction of the magnetic field[22] which holds for bremsstrahlung absorption and scattering by electrons for $\omega_B \gg \omega$.

It will be shown below, that, firstly, in a hot plasma of temperature $T = 10^6-10^7$ °K for x-ray quanta of energy $\hbar\omega = 0.5-10$ keV the recombination radiation from hydrogen atoms, ions of helium and of heavy elements in a strong magnetic field can be more effective than bremsstrahlung from electrons. Secondly, a situation is possible in which the coefficient of photoionization is greater than the coefficient for scattering by electrons in a magnetic field. In this case radiation from a plasma will not have directional properties.

### A. Energy Release Accompanying Recombination Radiation; Comparison with the Bremsstrahlung Process

The cross section for the photoionization of a hydrogenlike ion in a strong magnetic field increases with increasing atomic number of the element. The ionization energy of a hydrogen atom in a strong magnetic field is equal to 0.19 keV, and of a singly ionized helium (He II) is equal to 0.48 keV.

According to calculations[23] a large amount of helium is present in the surface layer of a neutron star. For it the conditions of ionization equilibrium (26) for $\hbar\omega_B \gg kT$ have the form

$$\begin{gathered} \frac{n_2}{n_{1,0}}n_e=2.4\cdot10^{15}T^{3/2}\frac{\hbar\omega_B}{kT}\exp\left(-\frac{I_1}{kT}\right), \\ \frac{n_{1,0}}{n_{0,0}}n_e=2.4\cdot10^{15}T^{3/2}\frac{\hbar\omega_B}{kT}\exp\left(-\frac{I_0}{kT}\right). \end{gathered} \tag{27}$$

Here $I_1$ and $I_0$ are the ionization potentials respectively of He II and He I, while $n_{i,0}$ is the density of ions in the ground level.

If

$$n_e \ll f_B(T) = 4.8\cdot 10^{15} T^{1/2} \frac{\hbar\omega_B}{kT} \exp\left(-\frac{I_1}{kT}\right) = 10^{25} - 3\cdot 10^{27}\ \mathrm{cm}^{-3}$$

for $T = 10^6 - 10^7$ °K and $B = 2\times 10^{12}$ G, then the degree of ionization is high: $n_2 \gg n_1 \gg n_0$ and $n_e = 2n_2$. Then

$$n_e^2 = f_B(T) n_{1,0}. \tag{28}$$

The function $f_B(T)$ depends only on the temperature and on the value of the magnetic field.

The coefficients for photoabsorption from the ground level $k_{bf}$ and for bremsstrahlung absorption $k_{ff}$ in the case of transverse propagation of radiation $(\theta \gg \omega/\omega_B)^{1/2}$) can be written in the form

$$\begin{aligned} k_{bf} &= n_{1,0} \frac{8\pi e^2}{\hbar c}\left(\frac{I_1}{\hbar\omega}\right)^{1/2} \frac{\hbar}{m_e\omega}\sin^2\theta\,(1-e^{-\hbar\omega/kT}) \\ &\approx 5.5\cdot 10^{-6} n_e^2 \left(\frac{I_1}{\hbar\omega}\right)^{1/2} \frac{\sin^2\theta}{\omega\omega_B\sqrt{T}} e^{I_1/kT}(1-e^{-\hbar\omega/kT}), \\ k_{ff} &= 9\cdot 10^{10} n_e^2 \frac{Z^2 g(\omega)}{\omega^3\sqrt{T}} \sin^2\theta\,(1-e^{-\hbar\omega/kT}), \end{aligned} \tag{29}$$

where $g(\omega)$ is the Gaunt factor. For $\hbar\omega \gg kT$ it is equal to unity. For $\hbar\omega \ll kT$ and $B = 0$ the factor is $g(\omega) = (\sqrt{3}/\pi)\ln(2.35kT/\hbar\omega)$. In a strong magnetic field when the gyroradius of the electron is much smaller than the Debye radius, i.e., $(B^2/4\pi) \gg n_e m_e c^2$, $g(\omega)$ already depends on the value of the magnetic field, but only logarithmically[24]. This can lead to changes in the value of $g(\omega)$ by a factor of severalfold compared to a plasma without a magnetic field.

On comparison we obtain

$$\frac{k_{bf}}{k_{ff}} \approx \frac{10^2}{Z^2 g(\omega)} \left(\frac{I_1}{\hbar\omega}\right)^{1/2} \left(\frac{\omega}{\omega_B}\right)\left(\frac{\hbar\omega}{1\,\mathrm{keV}}\right) e^{I_1/kT}, \tag{30}$$

i.e., the ratio of the absorption coefficients does not depend on the electron density and for $kT \gg I_1$ depends weakly on the temperature, but depends strongly on the value of the magnetic field (we recall that $I_1 = I_1(B)$).

From (30) it follows that in the case of a helium atmosphere for $T = 10^7$ °K and $B = 2\times 10^{12}$ G the ratio $k_{bf}/k_{ff} > 1$ for X-ray quanta of energy $\hbar\omega > 3$ keV. At a temperature $T = 3\times 10^6$ °K and for a photon energy $\hbar\omega = 4$ keV the ratio is $k_{bf}/k_{ff} = 4.0$.

If hydrogen predominates in the atmosphere of a neutron star then at $T = 3\times 10^6$ °K the ratio is $k_{bf}/k_{ff} > 1$ for $\hbar\omega > 1$ keV.

In the case of longitudinal propagation ($\theta \ll \sqrt{\omega/\omega_B}$) the ratio of the coefficient for recombination radiation to the coefficient for bremsstrahlung becomes even greater

$$\frac{k_{bf}(\theta=0)}{k_{ff}(\theta=0)} \sim \frac{\omega_B}{\omega} \frac{k_{bf}(\theta=\pi/2)}{k_{ff}(\theta=\pi/2)} \tag{31}$$

Taking into account recombination into excited levels ($s \geq 1$) can only increase the value of the ratio $k_{bf}/k_{ff}$, since the total cross section $\sigma_{bf}$ is determined by means of the formula

$$\sigma_{bf} = \sum_k \sigma_f^{(k)}(n\omega)\exp[(\varepsilon_k - \varepsilon_0)/kT]. \tag{32}$$

Its value depends in an essential manner on the temperature and on the structure of the levels of the He II ion. In Sec. 2 we have found the cross sections for photoionization from different levels of a stationary ion. In the case of thermal motion of an ion in a magnetic field an electric field $\mathbf{E} = c^{-1}[\mathbf{v}_i\mathbf{B}]$ acts on its electron. This field distorts the energy spectrum of the ion. The operator for the energy of interaction of the bound electron with the electric field has nonzero off-diagonal components: $V_{ss'} = ec^{-1}v_i B\lambda\sqrt{2s+1}\,\delta_{s,s'\pm 1}$. From this it is clear that the shift of the s-th level of the ion under the action of this field is small for small values of s and increases with increasing s. For example, for $B = 2\times 10^{12}$ G and $T = 3\times 10^6$ °K the value is $V_{ss'} \approx 30\sqrt{2s+1}$ eV, i.e., the shift becomes of the order of magnitude of the distance between levels already for s = 3. Therefore for an exact calculation of $\sigma_{bf}$ at high temperatures $T \gg 3\times 10^6$ °K when the excited levels are heavily populated it is necessary to calculate the energy spectrum of the ion and the cross section for the photoeffect taking thermal motion into account. The estimates given above give a lower bound on the value of the ratio $k_{bf}/k_{ff}$.

### B. The Spectrum of the Recombination Radiation

The coefficient for photoabsorption from the ground level (s = 0) behaves when the frequency is varied as $\omega^{-5/2}$ in the case of transverse propagation ($\theta \approx \pi/2$) and as $\omega^{-3/2}$ in the case of longitudinal propagation ($\theta \approx 0$). Taking into account the population of the excited states $s > \omega_B/\omega$ yields the dependence $k_{bf} \sim \omega^{-3/2}$ for all angles. As a result the spectrum of the recombination radiation from an optically thin plasma in the Rayleigh-Jeans region has the form

$$\begin{aligned} I(\omega) &\propto \omega^{-1/2} \quad \text{for} \quad 2\theta > [\omega(s+1/2)/\omega_B]^{1/2}, \\ I(\omega) &\propto \omega^{1/2} \quad \text{for} \quad 2\theta < [\omega(s+1/2)/\omega_B]^{1/2}. \end{aligned} \tag{33}$$

Thus, the recombination radiation from an optically thin plasma can lead to an inverted spectrum in the Rayleigh-Jeans domain.

Correspondingly in the Wien domain $\hbar\omega \gg kT$ we have the law

$$I(\omega) \propto \omega^{1/2} e^{-\hbar\omega/kT}, \quad I(\omega) \propto \omega^{3/2} e^{-\hbar\omega/kT}. \tag{34}$$

Near the values of energy corresponding to the ionization energies of atoms and of ions (soft x-ray quanta) appearance of discontinuities in the radiation spectrum becomes possible associated with the energy structure of atoms in a strong magnetic field (the analog of Balmer discontinuities).

### C. Directional Properties and Polarization of the Recombination Radiation

Directional properties and polarization of radiation from X-ray pulsars can arise as a result of scattering by electrons. In[22] it was shown that in a plasma which is optically thick with respect to Thompson scattering ($\tau_T \gg 1$, $\tau_T(\omega/\omega_B)^2 \ll 1$) in the case of scattering by electrons of radiation of frequencies $\omega \ll \omega_B$ a "pencil" directional diagram arises even in the case of an isothermal atmosphere. The directional diagram will be changed if the characteristics of the emerging radiation are determined by processes of photoionization and photorecombination. For this it is sufficient that in a plasma with $k_e(\theta = 0)L = \tau_T(\omega/\omega_B)^2 \ll 1$ the optical thickness with respect to photoabsorption along the field would exceed unity: $k_{bf}(\theta = 0)L \gtrsim 1$. From this, taking into account the relation (28) between $n_e$ and $n_1$, we obtain the condition

$$\left(\frac{\hbar\omega}{1\,\mathrm{keV}}\right)^{7/2} \ll 3\cdot 10^{-18} n_e T^{-1/2} \left(\frac{I_1}{1\,\mathrm{keV}}\right)^{3/2} \exp\left(\frac{I_1}{kT}\right). \tag{35}$$

When condition (35) is satisfied the atmosphere becomes optically thick in all directions. As a result of this radiation from an isothermal atmosphere will be nondirectional. If in the atmosphere there exists a temperature gradient then it is possible for a "pencil" diagram to arise which is broader (by approximately a factor of $\sqrt{\omega_B/\omega}$) than in the case of scattering by electrons. At high temperatures when the excited levels are heavily populated the cross section for photoabsorption is practically isotropic (cf., formula (20)) and the radiation is nondirectional. For a helium plasma at a temperature of $T = 10^7\,^\circ K$ of density $n_e = 10^{22}\ cm^{-3}$ for $B = 2 \times 10^{12}$ G the threshold energy of a quantum starting with which condition (35) is satisfied is given by $\hbar\omega \approx 3$ keV.

The polarization of the radiation from a neutron star of temperature $T = 10^6$—$10^7\,^\circ K$ will be circular in the case of a "pencil" directional diagram; its degree is $P_V \sim \omega/\omega_B$, i.e., of the same order of magnitude as in the case of scattering by electrons.

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1) These levels are called "tightly bound" in [13].
2) Formula (8) is valid both in the absence and also in the presence of a magnetic field contrary to the conclusion drawn in [14].


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