Experimental investigation of the fine structure of the instability of nonlinear ferromagnetic resonance

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The fine structure of the instability of nonlinear ferromagnetic resonance (NFMR) under parallel pumping is investigated. Several singularities have been observed on the plot of the onset of the low-frequency oscillations accompanying the NFMR against the constant magnetic field. Some of these singularities are due to known physical causes, while the nature of the others is unclear. Peaks due to processes of merging between parametrically excited and thermal magnons are revealed. New properties of low-frequency oscillations are observed: In a certain range of magnetic fields and tempertatures, they are produced with a certain inertia and have different excitation and cessation thresholds. The dependence of the frequency of the low-frequency oscillations on the constant magnetic field is investigated, and a number of singularities is observed also on this dependence.

INTRODUCTION

To understand the processes that occur in a strongly excited ferromagnetic crystal (nonlinear ferromagnetic resonance, NFMR) it is necessary to study the multiple-wavelength character of the spin system of the ferromagnet, whereby microwave pumping excites in a ferrite an ensemble of many spin waves (magnons). Two groups of processes should take place in such an ensemble, one due to magnon-magnon and magnon-phonon interactions (interactions of parametrically excited magnons with one another, with thermal magnons, and with crystal-lattice vibrations)[1-3], and the other due to the instability of the NFMR (the instability of the ensemble of parametrically excited magnons as a result of the large number of degrees of freedom [4-6]).

At the present time, the only processes observed among the appreciable number of theoretically predicted magnon-magnon and magnon-phonon interactions have been the coalescence of two parametrically excited magnons into a single magnon or phonon [7-9] and a two-step process whereby one spin wave parametrically excites another at the same frequency but with a different propagation direction and the latter splits into two [9]. In addition, it was found that the research on NFMR instability had been incomplete, as is evidenced by a number of accidentally discovered subtle effects [10-12].

All this makes it urgent to organize experiments aimed at systematic disclosure of the subtle effects connected with both magnon-magnon interaction and with NFMR instability, as has indeed been the purpose of the present paper. An experimental study was made of NFMR instability in wide ranges of magnetic fields, microwave pump powers, and temperatures.

EXPERIMENT

The desired results can be obtained only by choosing a sensitive measurement procedure, if high-grade materials are available, and if the accuracy of the measurements of the required quantitites is increased. There are two known procedures for the investigation of magnon-magnon and magnon-phonon interactions, namely determining how the constant magnetic field H affects either the imaginary part of the magnetic susceptibility [7-9] or the threshold of the appearance of low-frequency (compared with the microwaves) oscillations that accompany the NFMR and are due to its instability [5,11]. The latter procedure is sensitive enough [5] and, when supplemented with measurements of the spectral relationships, can yield much new information. We investigated the singularities of the dependences of the threshold and spectrum of the low-frequency oscillations and the threshold of parametric spin-wave excitation on the constant magnetic field H.

The experiments were performed with parallel pumping in the cw mode at 9380 MHz. The investigated material was single-crystal yttrium iron garnet, from which a spherical sample of 1.9 mm diameter was made (saturation magnetization $4\pi M_S = 1750$ G, width of resonance line $2\Delta H = 0.25$ Oe at 4.5 GHz). The sphere was magnetized along the [100] crystallographic axis and placed in the center of a rectangular H_{101}-mode resonator (natural Q ~ 5000, loaded Q ~ 2000). The threshold of the low-frequency oscillations and the frequencies of their spectral lines were measured with an S4-8 spectrum analyzer. The relative power at which the oscillations set in was measured accurate to ±0.15 dB. The measurements were performed in the magnetic-field intervals from 800 to 1800 Oe in steps of 3–5 Oe (the absolute accuracy of the magnetic-field measurement was ±1.5 Oe) and in the temperature range from 130 to 300°K (the absolute temperature measurement accuracy was ±2°K).

RESULTS AND DISCUSSION

1. Spectrum and Properties of Low-Frequency Oscillations

It was observed that the low-frequency oscillation spectrum depends strongly on the constant magnetic field and on the pump power. In particular, at a given field intensity, the oscillation spectrum can vary strongly in a wide range of magnetic fields, depending on the pump power, when the latter exceeds a certain value by only 0.1–0.4 dB (depending on the magnetic field). This allows us to speak of two types of low-frequency oscillations with different thresholds, namely one with a discrete (line) spectrum (consisting, as a rule, of one line) and one with a spectrum that is continuous in a certain frequency interval. In a certain interval of magnetic fields, only one type of oscillation (with the continuous spectrum) exists. The properties of both types of oscillations depend strongly on the magnetic field, on the temperature, and on the ratio of the thresholds at which they occur. Throughout the investigated interval of magnetic fields, with the exception

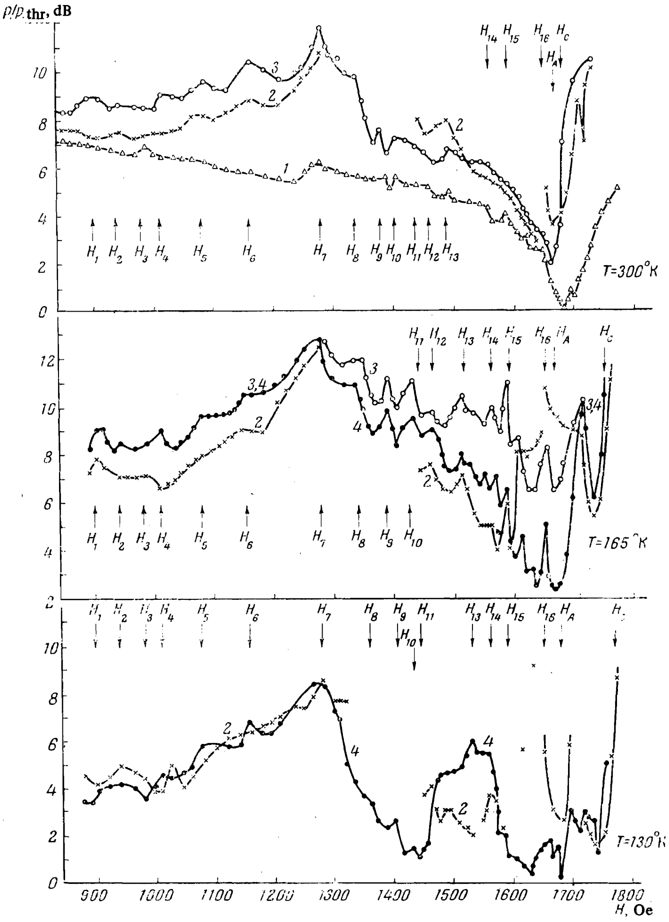

FIG. 1. Spin-wave parametric-excitation thresholds and low-frequency-oscillation thresholds vs. the constant magnetic field at different temperatures. P—power, P_{thr}—minimum power necessary for parametric excitation of the spin waves; △ (curve 1)—threshold of parametric excitation of the spin waves, X (curve 2)—threshold for the excitation of low-frequency oscillations with discrete spectrum, ○ (curve 3)—threshold of low-frequency oscillations with continuous spectrum, ● (curve 4)—threshold for termination of low-frequency oscillations with continuous spectrum.

of the range from 1442 to 1651 Oe, we noted the following regularity: if the threshold of the oscillations with the continuous spectrum is lower than the threshold of the oscillations with the discrete spectrum, then the former have two properties, namely hysteresis with respect to pump power and a time lag in their occurrence (inertia).

The pump-power hysteresis is manifest in the fact that the oscillations have different thresholds for their onset and for their termination: they are produced at one value of the pump power, and terminate at another value lower by q dB.

The inertia of the oscillations is manifest in the fact that they do not occur immediately, but at a certain time t_{in} after the microwave pump is turned on. At room temperature (T = 300°K) no inertia is observed (or at least t_{in} is less than 0.01 sec), and then the inertia increases rapidly with decreasing temperature ($t_{in} \sim 0.5$ sec), indicating that it is of nonthermal origin. Information on the dependence of the properties of the low-frequency oscillations on the magnetic field and on the pump power can be obtained from an analysis of the corresponding dependences of the threshold curves.

Figure 1 shows plots of the threshold of parametrically excited spin waves (curve 1), the threshold of the low-frequency oscillations with discrete spectrum (curve 2), and the threshold of the low-frequency oscillations with continuous spectrum, measured both for the excitation of the oscillations (curve 3) and for their termination (curve 4), against the constant magnetic field at different temperatures.

In the magnetic-field interval from 840 to 1281 Oe (H_7 of Fig. 1), low-frequency oscillations with a discrete spectrum are produced first, and then, at a power higher by l db, oscillations with the continuous spectrum appear. The dependence of l on the field and on the temperature can be determined from the data of Fig. 1. At T = 130°K, in those magnetic fields where the threshold of the low-frequency oscillations with continuous spectrum is lower than that of the oscillations with the discrete spectrum ($l < 0$) the oscillations are produced with a time lag (t_{in} on the order of a tenth of a second) and exhibit a power hysteresis (q on the order of 1–1.5 dB).

In the magnetic-field interval from $H_7 = 1281$ Oe to $H_{11} = 1442$ Oe there exist only oscillations with a continuous spectrum (the oscillations with discrete spectrum vanish jumpwise in a field H_7 and appear again in a field H_{11}), which at room temperature have a small power hysteresis (about 0.5 dB), which is difficult to observe against the background of the possible error (±0.15 dB). With decreasing temperature, however, the pump-power hysteresis increases and inertia appears in the onset of the oscillations. At 165°K, the time is $t_{in} \sim 0.1$ sec and $q \sim 3-5$ dB (see Fig. 1); at T = 130°K we have $t_{in} \sim 1$ sec and $q \sim 10-13$ dB.

In the magnetic field interval from $H_{11} = 1442$ Oe to $H_{16} = 1651$ Oe, the picture is particularly complicated (see Fig. 1). At T = 300°K in fields 1442–1525 Oe, oscillations with continuous spectrum appear first. Then, at a power p dB higher, oscillations with discrete spectrum appear. In fields 1525–1651 Oe, to the contrary, the latter oscillations appear before the former. At T = 165°K in fields 1442–1595 Oe, oscillations are first produced with a discrete spectrum, and then, at higher power, with a continuous spectrum; in fields 1595–1651 Oe, the inverse picture is observed. At T = 130°K in fields 1442–1465 and 1580–1651 Oe, oscillations with a continuous spectrum first appear, and then at higher power those with the discrete spectrum; the picture is reversed in fields 1465–1580 Oe. At T = 300°K, the oscillations have a power hysteresis ($q \sim 0.5$ dB) only in fields 1442–1525 Oe, while at 165 and 130°K the oscillations are produced with a time lag in the entire field interval and have a pump-power hysteresis of the same order as in the magnetic-field interval H_7-H_{11}. The aforementioned regularity of the appearance of the hysteresis and inertia does not hold in this field interval. It should be noted that when the temperature is lowered the oscillations with the discrete spectrum are not observed in some magnetic fields, namely at T = 130°K seven sections of 5–20 Oe each in which no such oscillations were produced were observed.

In the magnetic-field interval from $H_{16} = 1651$ Oe to H_c (field where the spin-wave parametric-excitation threshold has a minimum), low-frequency oscillations with continuous spectrum are first produced, followed, at large pump power, by the oscillations with the discrete spectrum; $q \sim 0.2-0.3$ dB at T = 300°K, and $q \sim 2-10$ dB, $t_{in} \sim 0.5$ sec at T = 130°K.

In magnetic fields stronger than H_C, the low-frequency oscillations have exactly the same character as in the magnetic-field interval from 840 to 1281 Oe, but in fields 1711–1730 Oe (at $T = 300°K$), the resultant oscillations with the discrete spectrum have not one but two lines. These oscillations are unstable: they alternately go over to oscillations with one spectral line and resume their form. In fields of 1711 and 1730 Oe, their threshold is very close to the excitation threshold of the oscillations described in [12]. In the field interval between 1711 and 1730 Oe, the threshold of oscillations with two spectral lines is much lower than that of oscillations having one line in the spectrum, so that a minimum appears on the threshold curve of the oscillations with discrete spectrum ($H = 1720$ Oe).

Thus, the oscillation spectrum depends strongly on the magnetic field and is much richer than had been previously assumed [5]. Some of the magnetic fields at which the spectrum changes coincide with the upper limits of the existence of the corresponding magnon-magnon and magnon-photon interactions (see below); this possibly indicates that these processes influence the oscillation spectrum. With decreasing temperature, the properties of the low-frequency oscillations observed by us become more pronounced (inertia and pump-power hysteresis). At $T = 130°K$, inertialess oscillations having no hysteresis can be observed only in very small sections of the investigated magnetic-field interval, although the situation is reversed at $T = 300°K$. The two mentioned properties of the oscillations appear to be mutually related; thus, inertialess oscillations (at $T = 300°K$) have likewise a negligible hysteresis, while the hysteresis of the inertial oscillations is tremendous ($q > 10$ dB in certain fields at $T = 130°K$). It appears that each change of the oscillation spectrum offers evidence of considerable changes in the properties of the spin system of the crystal, and, depending on the pump power, this occurs at much lower powers than reported in [13]. The observed pump-power hysteresis was not accompanied by magnetic-field hysteresis (within the ±1.5 Oe accuracy limit of the latter), so that it is impossible to establish a connection between this phenomenon and the hysteresis of the low-frequency oscillations under transverse pumping [10].

2. Magnon-Magnon and Magnon-Phonon Interactions in NFMR

The plots of the spin-wave excitation thresholds and of the low-frequency oscillation thresholds against the constant magnetic field reveal a number of singularities, of which 14 have been observed for the first time, in the following magnetic fields (in Oe) (at $T = 300°K$): $H_1 = 900$, $H_2 = 940$, $H_3 = 984$, $H_4 = 1012$, $H_5 = 1080$, $H_6 = 1159$, $H_7 = 1281$, $H_8 = 1340$, $H_9 = 1380$, $H_{10} = 1407$, $H_{11} = 1442$, $H_{12} = 1462$, $H_{13} = 1490$, $H_{14} = 1557$, $H_{15} = 1590$, $H_{16} = 1651$, $H_A = 1668$, $H_C = 1685$.

The most badly chopped-up curve was that for the threshold of low-frequency oscillations with continuous spectrum.

It was natural to attempt to relate the observed singularities with the upper limits of the processes of coalescence of n magnons into m magnons, the splitting of n magnons into m magnons (the corresponding values of the magnetic field are designated H_{nMmM}), and also with the coalescence of n magnons into m longitudinal or transverse phonons (the corresponding fields are marked $H_{nMmph\parallel,\perp}$). We calculated the upper limits of all the possible magnon-magnon and magnon-phonon coalescence and splitting processes, the upper limits for the coalescence and splitting of parametrically-excited magnons being (generalized formulas from [3]):

$$H_{n_M m_M} = H_c - \frac{m^2}{2(n^2-m^2)|\gamma|} \times$$

$$\times \left\{ \left[\omega_M^2 \sin^4\theta_k + \frac{n^2}{m^2}\omega_0^2 \right]^{1/2} - [\omega_M^2 \sin^4\theta_k + \omega_0^2]^{1/2} \right\},$$

where

$$|\gamma| H_c = \frac{1}{2}\{-\omega_M \sin^2\theta_k + [\omega_M^2 \sin^4\theta_k + \omega_0^2]^{1/2}\}, \quad \omega_M = 4\pi|\gamma| M_s,$$

γ is the gyromagnetic ratio for the electron, ω_0 is the microwave-pump frequency, M_S is the saturation magnetization, and θ_k is the angle between the directions of the constant magnetic field and the propagation of the spin wave.

As a result, comparison of the calculated and experimental data has shown that the magnetic fields at which singularities are observed do not, as a rule, coincide with the calculated values of the upper limits of the magnon-magnon coalescence and splitting processes at $\theta_k = \pi/2$, with the exception of $H_6 = H_{2M1M}$ and $H_7 = H_{3M1M}$. In addition, the points $H_4 = H_{4M3M} - 7$ Oe, $H_5 = H_{5M3M} - 12$ Oe, and $H_{16} = H_{2M1ph,\perp} - 5$ Oe are quite close. The fact that the calculated values of the magnetic field H_{nMmM} do not agree with the experimental values for spin waves with $\theta_k = \pi/2$ seems to favor the hypothesis advanced in [9], that spin waves with $\theta_k = \pi/2$ can excite spin waves with another propagation direction (in particular, with $\theta_k = 0$). According to [2], an interaction (3-magnon coalescence process) can occur between parametrically-excited and thermal magnons, and can lead to the existence of a nonlinear negative damping. One can expect in this case singularities in fields close to H_2, H_6, H_7, and H_{13}.

The treatment of the results is complicated. In our opinion, the peaks in the fields H_1, H_2, H_4, H_5, and H_6 are due to magnon-magnon coalescence processes, in spite of the fact that the fields H_1 and H_2 cannot be identified with the upper limit of any coalescence process, the probability of the coalescence of 4 and 5 magnons into 3 is low. Favoring this opinion is the fact that the peaks in the fields H_1, H_2, H_4, and H_5 behave, when the pump power and temperature are varied, in the same manner as the peak in the field $H_6 = H_{2M1M}$, which is attributed to 3-magnon coalescence [7,9].

An interesting peak occurs at $H_7 = 1281$ Oe ($\sim H_{3M1M}$ or H_1 from [9]), and has been reported earlier [9,11]. The field H_7 is the upper limit of the existence of at least two processes, the coalescence of three parametrically-excited magnons into one (H_{3M1M}), and three-magnon splitting of the spin wave with $\theta_k = 0$ excited by a spin wave with $\theta_k = \pi/2$ [9]. It seemed natural to attribute the peak at this field to one of these processes. (The attempt to attribute it to three-magnon coalescence [11], based on the erroneous identification of the field H_7 with the field H_{2M1M}, is incorrect [9].) In our opinion, however, this peak cannot be explained by either of these processes. The point is that, as observed by us, in the field H_7 there takes place an abrupt change in the spectrum of the low-frequency oscillations (oscillations with the discrete spectrum cease to exist, and pump-power hysteresis and inertia appear in the oscillations with the continuous spectrum), and the spin-wave parametric-excitation threshold increases by 0.8 dB (see Fig. 1). These are not features of magnon-magnon coalescence or splitting processes, and are not observed, for example, in the field H_6 in the case of three-magnon coalescence. It is possible, however, that some third process, as yet unknown (or else a process connected, say, with the processes described in [2]) occurs at this value of the field and masks the nature of the coalescence and splitting of the parametrically excited magnons. This assumption is favored by the possible absence of a connection between the abrupt changes of the spectrum of the low-frequency oscillations and the singularities on the threshold curves (see Fig. 1, field H_{11}). The field position of the described peak changes little with temperature.

The first to appear on all the threshold curves were the peaks in the fields H_8, H_9, H_{10}, and H_{13}. According to [2,14], the field H_{13} is the upper limit of the existence of nonlinear negative damping due to rapid depletion of the reservoir of thermal magnons that participate in three-magnon processes of coalescence with parametrically-excited magnons. The characteristics of the peaks in the fields H_8, H_9, H_{10}, and H_{13} with changing pump power and temperature are similar to one another, and this may indicate that thermal magnons play an important role in processes that take place in these fields [1-3,5,14]. With decreasing temperature, the field position of these peaks shifts towards stronger fields.

Peaks whose nature is not clear were observed in the fields H_3, H_{12}, H_{14}, and H_{15}. These peaks are more readily revealed by the spin-wave excitation threshold than by the low-frequency oscillation thresholds.

In the field $H_{16} \sim H_{2\mathrm{M1ph},\perp}$, peaks are observed at the excitation thresholds of the spin waves and of the low-frequency oscillations with continuous spectrum, while an abrupt jump in the value of the threshold is observed at the threshold of the low-frequency oscillations with the discrete spectrum. These singularities are undoubtedly connected with coalescence of two magnons into one transverse phonon [3,8].

The minimal thresholds for the low-frequency oscillations and spin waves were observed in the fields H_A and H_C, respectively.

Figure 2 shows a plot of the spectral-line frequency of the low-frequency oscillations with discrete spectrum against the constant magnetic field. In some magnetic-field ranges this frequency does not depend on the constant magnetic field. Singularities on this plot are observed in the fields H_1, H_6, H_7, H_{11}, H_{15}, H_{16}, H_A, H_C.

Thus, it has been established experimentally that the character of the low-frequency oscillations produced in nonlinear ferromagnetic resonance depends strongly on the constant magnetic field, on the temperature, and on the microwave pump power. New properties of the low-frequency oscillations, which become strongly pronounced at low temperatures, were observed, namely a lag in the appearance of the oscillations and a difference between the excitation and termination thresholds (power hysteresis). The following regularity was observed in the manifestation of these properties: the inertia and the power hysteresis are typical, as a rule, of low-frequency oscillations with continuous spectrum when their threshold is lower than the threshold for the low-frequency oscillations with discrete spectrum or when there are no low-frequency oscillations with discrete spectrum. The threshold of the high-frequency oscillations is sensitive to the interaction of parametrically-excited spin waves with one another and with thermal spin waves, a fact reflected in the dependence of the threshold of the low-frequency oscillations on the magnetic field, where 18 singularities have been observed. The results offer evidence of an appreciable change in the properties of the spin system of a ferromagnet when the magnetic field, pump power, and temperature are varied. The investigations illustrate the need for careful quantitative analysis of the low-frequency oscillation spectra and for quantitative study of the inertia effect, which will apparently make it possible to observe new subtle effects accompanying the NFMR and to understand better the nature of the processes occurring in a strongly excited ferromagnet.

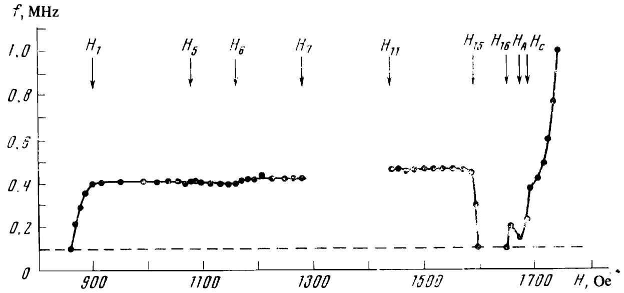

FIG. 2. Spectral-line frequency of low-frequency oscillations with discrete spectrum vs. constant magnetic field. f = 0.1 MHz—lowest frequency that can be measured with the S4-8 instrument.

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