Electromagnetic excitation of sound in a metal located in a magnetic field parallel to the surface

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An exact solution is obtained to the problem of the conversion of an electromagnetic wave into an acoustic wave in a metal located in a magnetic field **H** parallel to the metal surface. It is shown that the sound amplitude is an oscillating function of H^{-1}, the oscillations being due to the deformation mechanism of the electron-lattice interaction. The period and phase of the oscillations do not depend on the nature of electron reflection from the boundary of the sample. The oscillation amplitude in the case of specular reflection of the electrons from the surface is $(qD_0)^{1/2} \gg 1$ times smaller than the oscillation amplitude in the case of diffuse scattering (q is the acoustic wave vector and D_0 is the maximum electron-orbit diameter).

1. The electromagnetic excitation of sound in metals in the absence of a magnetic field has been experimentally and theoretically well investigated. The excitation in the presence of a constant magnetic field **H** has been investigated mainly for the case when the vector **H** is parallel to the direction of propagation of the wave[1-3]. Wallace, Gaerttner, and Maxfield[2] found that in potassium single crystals at low temperatures the amplitude u of the excited transverse wave nonmonotonically depends on H in the region of field intensities where $qR \sim 1$ (R is the cyclotron electron radius and **q** is the acoustic wave vector). A theoretical explanation of this effect for the alkaline metals is given in [4], and consists in the following. The generation of sound by an electromagnetic wave incident on the surface of a metal is due to forces exerted by the conduction electrons on the lattice. In a magnetic field there arise the competing forces: the Lorentz force and the deformation force connected with the interaction of the electrons with the acoustic vibrations. Allowance for only the Lorentz force leads to the linear dependence of the wave amplitude on the magnetic field H; the displacement vector **u** is then perpendicular to the external electric field $\mathbf{E}_0$. Such a picture obtains only in the region of strong magnetic fields when $qR \ll 1$. In weaker fields the deformation force is comparable to the Lorentz force and is described by a nonmonotonic function of H that attains its extremum values in the vicinity of $qR \sim 1$. Owing to the deformation mechanism of the electron-lattice interaction, waves of both polarizations—along and perpendicular to the vector $\mathbf{E}_0$—are excited.

The electromagnetic excitation of sound in a magnetic field parallel to the sample surface has been experimentally investigated by Gaerttner and Maxfield[5]. In this geometry, there is observed an oscillating dependence of the amplitude of the transverse sound on H in the region of magnetic field intensities where the inequalities

$$\omega,\ \nu \ll \Omega, \tag{1.1}$$

$$\Omega \ll qv, \quad qR \gg 1 \tag{1.2}$$

are fulfilled (ω is the frequency of the external wave, ν is the electron-scatterer collision rate, and v and Ω are the velocity and cyclotron frequency of the electrons). As in the case of geometrical resonance in sound absorption, the period of the oscillations is inversely proportional to the magnitude H of the field. This effect is qualitatively explained in Babkin and Kravchenko's paper [6]. It is of interest to solve the exact problem of the conversion of an electromagnetic wave into an acoustic wave with allowance for the nature of electron scattering by the metal surface.

Because of the high electrical conductivity, an electromagnetic wave in a metal is localized near a surface layer of thickness δ. In a magnetic field, the periodic motion of the electrons and their resonant interaction with the wave in the skin layer lead to an anomalous penetration [7]. In the low-frequency region (1.1), when the magnetic field is parallel to the surface, the amplitude of the bursts is small and their participation in the sound excitation is unimportant. The skin effect can be regarded as the result of the propagation in the metal of a large number of plane waves with different wave numbers k ($\Delta k \sim \delta^{-1}$). The conversion by electrons of an electromagnetic wave into an acoustic wave occurs only for that harmonic whose wave vector satisfies the dispersion relation $k = \omega/s \equiv q$ (s is the speed of sound). In the linear approximation, the amplitude of the excited acoustic wave is proportional to the amplitude of the incident electromagnetic wave. The conversion coefficient depends in a complicated manner on the magnetic field. If the excitation occurs through the induction mechanism, then the conversion coefficient is described by a monotonic function of H. The oscillations in the amplitude of the sound are connected with the deformation force, which is the oscillating function $qD_0(H)$ (D_0 is the maximum electron-orbit diameter). Like the field bursts, they are due to the resonant interaction with the wave of the distinct group of electrons which move almost parallel to the surface and belong to the extremum cross sections of the Fermi surface.

2. The complete system of equations describing the propagation of electromagnetic and acoustic waves in a metal consists of the Maxwell equations, the linearized kinetic equation for conduction electrons, and the equations for the lattice vibrations:

$$\frac{\partial^2 \mathbf{E}(z)}{\partial z^2} = -\frac{4\pi i\omega}{c^2}\mathbf{j}(z), \tag{2.1}$$

$$(\nu - i\omega)\chi + v_z\frac{\partial\chi}{\partial z} + \Omega\frac{\partial\chi}{\partial\tau} = e\left\{\mathbf{E}(z) + \frac{1}{c}[\dot{\mathbf{u}}\mathbf{H}]\right\}\mathbf{v} + \Lambda_{iz}(\mathbf{p})\,\dot{u}_{iz}(z) \equiv g(\mathbf{p}, z), \qquad (2.2)^*$$

$$\rho\ddot{u}_i = \lambda_{izzl}\frac{\partial^2 u_l}{\partial z^2} + f_i(z) + \frac{\partial}{\partial z}\varphi_i(z). \tag{2.3}$$

We choose the system of coordinates such that the z axis is directed along the normal to the surface of the metal, which fills the halfspace $z > 0$; the vector **q** is parallel to the z axis, and the vector **H** to the x axis. We introduce the following notation: $\mathbf{E}(z)$ is the electrical field in the metal; $\mathbf{j}(z)$ is the current density, which is determined by the nonequilibrium correction to the distribution function χ:

$$j_\alpha(z) = \frac{2e}{(2\pi\hbar)^3}\int m\,dp_x \oint d\tau\, v_\alpha(\tau)\chi(\mathbf{p}, z), \quad j_z = 0; \tag{2.4}$$

where τ is the dimensionless time of motion of an electron along its orbit in the magnetic field; e and m are the electron charge and mass; $\mathbf{v}$ and $\mathbf{p}$ are the electron velocity and momentum (in a magnetic field they are related by the equation of motion $d\mathbf{p}/dt = ec^{-1}[\mathbf{v}\times\mathbf{H}]$); $\Lambda_{ik}(\mathbf{p}) = \Lambda_{ik}(-\mathbf{p})$ is the deformation potential; ρ is the density of the metal; Λ_{iklm} is the elasticity tensor; $f_i(z)$ is the induction force:

$$f_i(z) = \frac{1}{c}[\mathbf{jH}]_i; \tag{2.5}$$

$\partial\varphi_i(z)/\partial z$ is the deformation force:

$$\frac{\partial}{\partial z}\varphi_i(z) = \frac{\partial}{\partial z}\frac{2}{(2\pi\hbar)^3}\int m\, dp_x \oint d\tau\, \Lambda_{iz}(\mathbf{p})\chi(\mathbf{p}, z). \tag{2.6}$$

The boundary conditions for Eqs. (2.1) and (2.3) are the conditions of continuity of the tangential components of the alternating electric and magnetic fields and the vanishing of the potentials at the free boundary $z = 0$. They are described by the relation

$$\lambda_{izzl}\partial u_l(0)/\partial z + \varphi_i(0) = 0. \tag{2.7}$$

The electromagnetic field tends to zero at large distances ($z \to \infty$). The boundary conditions for the kinetic equation are determined by the nature of electron reflection from the surface:

$$\chi|_{z=0,\, v_z>0} = \rho\chi|_{z=0,\, v_z<0}, \quad 0 \leqslant \rho \leqslant 1, \tag{2.8}$$

where ρ is the specularity coefficient. The role of the boundary condition involving τ is played by the periodicity of χ in τ with the period $\theta = m^{-1}\partial S/\partial\epsilon$ (S is the area of the intersection of the surface $\epsilon(\mathbf{p}) = \text{const}$ with the plane $p_x = \text{const}$): $\chi(\tau + \theta) = \chi(\tau)$.

In the expression for the change $g(\mathbf{p}, z)$ in the energy, only the term containing the electric field should be retained (i.e., $g = e\mathbf{v}\cdot\mathbf{E}(z)$). The remaining terms lead to small corrections proportional to $(m/M)^{1/2}$ (M is the ion mass) and describing the electronic renormalization of the velocity and damping of the sound. In such an approximation, the solution of the problem is carried out in two stages: the computation of the distribution of the electromagnetic field in the metal with the prescribed law of electron reflection from the boundary and the solution of the vibrational equations (2.3), in which the forces are functionals of the field $\mathbf{E}(z)$. Furthermore, we can consider the excitation of waves of the same polarization. For the longitudinal wave Eq. (2.3) has the form

$$\frac{d^2u_z}{dz^2} + q_z^2 u_z = -\lambda_{zzzz}^{-1}\left[f_z(z) + \frac{d}{dz}\varphi_z(z)\right], \tag{2.9}$$

while for the transverse wave

$$\frac{d^2u_t}{dz^2} + q_t^2 u_t = -\lambda_{tzzt}^{-1}\frac{d}{dz}\varphi_t(z). \tag{2.10}$$

Here $q_i = (\rho\omega^2\lambda_{izzi}^{-1})^{1/2}$ is the wave vector of the excited acoustic wave.

Deep inside the metal (i.e., as $z \to \infty$) only waves running from the boundary $z = 0$ exist. We are interested in the value of the function $u(z)$ at the sample surface $z = d$. If the thickness d is considerably greater than the sound-attenuation distance and the other parameters which have the dimensionality of length, then the problem reduces to that of computing the amplitude: $\lim_{z\to\infty} u_i(z) = U_i \exp(iq_i z)$. From Eqs. (2.9) and (2.10) we find

$$U_z = U_{z\,\text{def}} + U_{\text{ind}}, \quad U_t = U_{t\,\text{def}}, \tag{2.11}$$

$$U_{i\,\text{def}} = -\lambda_{izzi}^{-1}\int_0^\infty dz\, \varphi_i(z)\sin q_i z, \tag{2.12}$$

$$U_{\text{ind}} = cq\lambda_{zzzz}^{-1}H\int_0^\infty dz\, j_y(z)\cos q_z z. \tag{2.13}$$

These integrals can be evaluated asymptotically exactly for the cases of diffuse ($\rho = 0$) and specular ($\rho = 1$) reflection of electrons from the surface.

3. To solve the kinetic equation, it is convenient to introduce the functions

$$\Psi(z, \mathbf{p}) = \chi(z, \mathbf{p}) - \chi(z, -\mathbf{p}), \quad \Phi(z, \mathbf{p}) = \chi(z, \mathbf{p}) + \chi(z, -\mathbf{p}).$$

It follows from the parity properties of the velocity, energy, and deformation potential as functions of $\mathbf{p}$ that the current density is determined by an odd-parity function, while the function $\varphi_i(z)$ is the even-parity part of the distribution function χ:

$$j_\alpha(z) = \frac{e}{(2\pi\hbar)^3}\int m\, dp_x \oint d\tau\, v_\alpha(\mathbf{p})\Psi(z, \mathbf{p}), \tag{3.1}$$

$$\varphi_i(z) = (2\pi\hbar)^{-3}\int m\, dp_x \oint d\tau\, \Lambda_{iz}(\mathbf{p})\Phi(\mathbf{p}, z). \tag{3.2}$$

The functions $\Psi(z, \mathbf{p})$ and $\Phi(z, \mathbf{p})$ satisfy the following equations:

$$\partial^2\Psi/\partial z^2 - \hat{L}^2\Psi = -2\hat{L}(g/v_z), \tag{3.3}$$

$$\frac{\partial\Phi}{\partial z} = \frac{2g}{v_z} - \hat{L}\Psi, \quad \hat{L} = v_z^{-1}\left\{\nu - i\omega + \Omega\frac{\partial}{\partial\tau}\right\}. \tag{3.4}$$

The boundary condition (2.8) reduces to the form

$$\frac{\partial\Psi(0, \mathbf{v})}{\partial z} = (1-\rho^2)^{-1}\hat{L}\{\operatorname{sgn} v_z[(1+\rho^2)\Psi(0, p_x, v_z) - 2\rho\Psi(0, p_x, -v_z)]\}.$$

In finding the function $\Psi(z, \mathbf{p})$ in the case of the quadratic isotropic dispersion law, we used the method developed in Kaner's paper [8]. The generalization to the case of an arbitrary energy spectrum does not contain any fundamental changes. Let us give the results, noting that the equations for the functions $E_\alpha(z)$ and $\Psi(z)$ allow us to continue the functions in an even manner into the region $z < 0$ and to use the Fourier cosine transform:

$$\mathscr{E}_i(k) = 2\int_0^\infty dz\, E_i(z)\cos kz.$$

The solutions for the functions $\Psi(z, \mathbf{p})$ and $\Phi(z, \mathbf{p})$ are of the form

$$\Psi(\mathbf{p}, z) = \frac{2}{\pi}\int_0^\infty dk\cos kz\int_{-\infty}^{\tau}\frac{d\tau_1}{\Omega}\exp[\gamma(\tau_1 - \tau)] \times\cos\left[\frac{k}{\Omega}\int_\tau^{\tau_1} v_z d\tau_2\right]\{g(\tau_1, k) - T(\tau_1)\}, \tag{3.5}$$

$$\Phi(\mathbf{p}, z) = -\frac{2}{\pi}\int_0 dk\sin kz\int_{-\infty}^{\tau}\frac{d\tau_1}{\Omega}\exp[\gamma(\tau_1 - \tau)] \times\sin\left[\frac{k}{\Omega}\int_\tau^{\tau_1} v_z d\tau^2\right]\{g(\tau_1, k) - T(\tau_1)\}, \tag{3.6}$$

$$T(\tau_1) \equiv \frac{|v_z(\tau_1)|}{\pi\Omega}\left\{[e^{\gamma\tau_1} - \rho e^{\gamma\lambda(\tau_1)}]^{-1}\int_{\lambda(\tau_1)}^{\tau_1} d\tau_2 - \rho[e^{\gamma\lambda(\tau_1)} - \rho e^{\gamma(\tau_1-\theta)}]^{-1}\int_{\tau_1-\theta}^{\lambda(\tau_1)} d\tau_2\right\} \times e^{\gamma\tau_2}\int_0^\infty dk_1 g(\tau_2, k_1)\cos\left[\frac{k_1}{\Omega}\int_{\tau_1}^{\tau_2} v_z\, d\tau_3\right]$$

Here $\gamma = (\nu - i\omega)/\Omega$ and $\lambda(\tau)$ is the root of the equation

$$\int_\tau^{\lambda(\tau)} v_z\, d\tau_2 = 0; \quad \tau - \theta < \lambda(\tau) < \tau, \quad \lambda[\lambda(\tau)] = \tau - \theta.$$

The first terms in the curly brackets in (3.5) and (3.6) are the result for the infinite metal. The second terms are connected with the presence of the boundary. For simplicity, the case of a convex simply-connected Fermi surface is considered. The generalization to a more complicated surface presents no difficulty. Substituting the expressions (3.5) and (3.6) into the formulas (2.12) and (2.13), we obtain after some simple transformations the expressions

$$U_{\mathrm{ind}} = -2icH[4\pi\omega\lambda_{zzzz}q]^{-1}E_y'(0), \tag{3.7}$$

$$U_{i\,\mathrm{def}} = \lambda_{izzi}^{-1}\left\{\mathscr{E}_\beta(q)K_{i\beta}(q) - \int_0^\infty \frac{dk_1}{k_1}\mathscr{E}_\beta(k_1)Q_{i\beta}(q,k_1)\right\}. \tag{3.8}$$

In the frequency and magnetic-field regions (1.1) and (1.2) under consideration, the skin depth δ is considerably smaller than the wavelength λ of the sound. Therefore, in the expression for U_{ind} we have neglected the terms $q^2\mathscr{E}_y(q) \sim q^2\delta^2 E_y'(0) \ll E_y'(0)$. The amplitude U_{ind} is a linear function of the magnetic field H, and is not sensitive to the nature of electron reflection from the boundary.

According to the condition for resonant interaction, the generation of sound is due to the component of the electromagnetic field with the wave vector $k = q$. The Fourier component $\mathscr{E}_\alpha(k)$ of the field is the solution of the Maxwell equation (2.1) in the Fourier representation. The current density $j_\alpha(k)$ is related to the field $\mathscr{E}_\alpha(k)$ by the following formula:

$$j_\alpha(k) = \left\{\sigma_{\alpha\beta}^{(0)}(k)\mathscr{E}_\beta(k) - \int_0^\infty \frac{dk_1}{k_1}\mathscr{E}_\beta(k_1)\sigma_{\alpha\beta}^{(1)}(k,k_1)\right\}. \tag{3.9}$$

The tensor $K_{i\beta}(q)$ resulting from the distribution function of the infinite metal is of the form

$$K_{i\beta}(q) = \frac{2e}{(2\pi\hbar)^3}\int\frac{m\,dp_x}{\Omega}\oint d\tau\,\Lambda_{iz}(\tau,p_x) \times \int_{-\infty}^{\tau} d\tau_1\,v_\beta(\tau_1)\exp[\gamma(\tau_1-\tau)]\sin\left[\frac{q}{\Omega}\int_\tau^{\tau_1} v_z\,d\tau_2\right], \tag{3.10}$$

where $\sigma_{\alpha\beta}^{(0)}(k)$ is the Fourier transform of the conductivity operator of the infinite metal:

$$\sigma_{\alpha\beta}^{(0)}(k) = \frac{2e^2}{(2\pi\hbar)^3}\int\frac{m\,dp_x}{\Omega}\oint d\tau\,v_\alpha(\tau)\int_{-\infty}^{\tau} d\tau_1\,v_\beta(\tau_1) \times \exp[\gamma(\tau_1-\tau)]\cos\left[\frac{k}{\Omega}\int_\tau^{\tau_1} v_z\,d\tau_2\right]. \tag{3.11}$$

The form of the functions $\sigma_{\alpha\beta}^{(1)}(k, k_1)$ and $Q_{i\beta}(k, k_1)$ depends on the nature of electron reflection from the boundary. Let us consider the two limiting cases: a) $\rho = 0$, when all the electrons are diffusely scattered by the surface, and b) $\rho = 1$, when all the electrons are specularly reflected from the surface. The tensor $\sigma_{\alpha\beta}^{(1)}(k, k_1)$ can be written in the form of integrals:

$$\sigma_{\alpha\beta}^{(1)}(k,k_1) = \int\frac{m\,dp_x}{\Omega}\oint d\tau\,v_\alpha(\tau)e^{-\nu\tau}\int_{-\infty}^{\tau} d\tau_1\frac{k_1|v_z(\tau_1)|}{\pi\Omega}\cos\left[\frac{k}{\Omega}\int_\tau^{\tau_1} v_z\,d\tau_2\right] \times \int_{\lambda(\tau_1)}^{\tau_1} d\tau_2\,e^{\nu\tau_2}v_\beta(\tau_2)\cos\left[\frac{k_1}{\Omega}\int_{\tau_1}^{\tau_2} v_z\,d\tau_3\right], \quad \rho=0, \tag{3.12}$$

$$\sigma_{\alpha\beta}^{(1)}(k,k_1) = \int\frac{m\,dp_x}{\Omega}\oint d\tau\frac{k_1|v_z(\tau)|}{\pi\Omega}\int_\tau^\infty dy\,e^{-\nu(y-\tau)}v_\alpha(y) \times \cos\left[\frac{k}{\Omega}\int_y^\tau v_z\,d\tau_3\right]\left\{[e^{\nu\tau}-e^{\nu\lambda(\tau)}]^{-1}\int_\lambda^\tau d\tau_2 - [e^{\nu\lambda(\tau)}-e^{\nu(\tau-\theta)}]^{-1} \times \int_{\tau-\theta} d\tau_2\right\}e^{\nu\tau_2}v_\beta(\tau_2)\cos\left[\frac{k_1}{\Omega}\int_\tau^{\tau_2} v_z\,d\tau_3\right], \quad \rho=1. \tag{3.13}$$

Similar expressions can be derived for $Q_{i\beta}(q, k_1)$. We are interested in the values of the integrals (3.10)–(3.13) in the magnetic-field region (1.1)–(1.2). In the case of low frequencies ($\omega \lesssim \nu$), the criterion (1.1) corresponds to the situation in which the electrons have time to complete at least a few revolutions between collisions. The condition (1.2) allows us to use the stationary-phase method when evaluating the integrals over τ and τ_1. The stationary-phase points are determined by the equality

$$v_z(\tau=\eta_\mu)=0 \quad (\mu=1,2), \quad p_y(\eta_1)=p_{y\,\min}(p_x), \quad p_y(\eta_2)=p_{y\,\max}(p_x).$$

The dominant contribution to the interaction with the wave is made by the electrons which move parallel to the surface. The integration over p_x separates out the electrons with the maximum orbit diameters D_0 ($D(p_x) \equiv (c/eH)[p_{y\,\max}(p_x) - p_{y\,\min}(p_x)]$). The electrons of the neighboring nonextreme orbits experience the effect of the different phases, and their contributions cancel each other out.

In order not to encumber the exposition, let us give the results of the computations only for the functions $\sigma_{\alpha\beta}^{(0)}(q)$ and $K_{i\beta}(q)$:

$$\sigma_{\alpha\beta}^{(0)}(q) = \frac{A_{\alpha\beta}}{q} + \frac{B_{\alpha\beta}}{q}\frac{\sin(qD_0+\Delta)}{(qD_0)^{1/2}}, \tag{3.14}$$

$$K_{i\beta}(q) = \frac{M_{i\beta}}{q}\frac{\cos(qD_0+\Delta)}{(qD_0)^{1/2}}. \tag{3.15}$$

The tensors $A_{\alpha\beta}$, $B_{\alpha\beta}$ and $M_{i\beta}$, which depend on the specific form of the dispersion law $\epsilon(\mathbf{p})$, are described by the following formulas:

$$A_{\alpha\beta} \equiv \frac{2e^2}{(2\pi\hbar)^3}\int\frac{m\,dp_x}{\gamma}\sum_{\mu=1,2}\frac{v_\alpha(\eta_\mu)v_\beta(\eta_\mu)}{|v_z'(\eta_\mu)|}$$

$$B_{\alpha\beta} \equiv \frac{2e^2}{(2\pi\hbar)^3}\left\{\frac{m}{\gamma}\sum_{\substack{\mu,\nu=1,2\\ \mu\neq\nu}}\frac{v_\alpha(\eta_\mu,p_x)v_\beta(\eta_\nu,p_x)}{[|v_z'(\eta_1)v_z'(\eta_2)|]^{1/2}}\left(\frac{2\pi D_0}{|D''(p_x)|}\right)^{1/2}\right\}_{p_x=p_{x\,ext}}$$

$$M_{i\beta} = \frac{2e}{(2\pi\hbar)^3} \times \left\{\frac{m}{\gamma}\sum_{\substack{\mu,\nu=1,2\\ \mu\neq\nu}}\frac{\Lambda_{iz}(\eta_\mu,p_x)v_\beta(\eta_\nu,p_x)\,\mathrm{sgn}\,v_z'(\eta_\mu)}{|v_z'(\eta_1)v_z'(\eta_2)|^{1/2}}\left(\frac{2\pi D_0}{|D''(p_x)|}\right)^{1/2}\right\}_{p_x=p_{x\,ext}}, \tag{3.16}$$

$$\Delta \equiv {}^1/_4\pi\,\mathrm{sgn}\,D''(p_{x\,ext}).$$

The conductivity tensor $\sigma_{\alpha\beta}^{(0)}(q)$ is a sum of two terms, the first of which is a monotonic function, while the second is an oscillating function of the magnetic field H. The presence of the oscillating term leads to a periodic decrease of the conductivity, which, as is well known [7], is the cause of the field and current bursts that occur in the bulk of the metal. Since the oscillating term contains the factor $(qD_0)^{-1/2} \ll 1$, the amplitude of the bursts is small. The sum (3.14) can be regarded as the result of the expansion of the tensor in powers of the small parameter $(qD_0)^{-1/2}$. This means that the equation for the field can be solved by the method of successive approximations after representing $\mathscr{E}_\alpha(k)$ in the following form: $\mathscr{E}_\alpha(k) = \mathscr{E}_{1\alpha}(k) + \mathscr{E}_{2\alpha}(k)$. For the infinite metal the function $\mathscr{E}_{1\alpha}(k)$ of the first approximation is the solution of the equation in which the current density $j_\alpha(k) = A_{\alpha\beta}k^{-1}\mathscr{E}_\beta(k)$; in the second approximation $\mathscr{E}_{2\alpha}(k)$ is an oscillating function with a small amplitude: $|\mathscr{E}_{2\alpha}|/|\mathscr{E}_{1\alpha}| \sim (kD_0)^{-1/2} \ll 1$.

In contrast to the conductivity, there is in the expression for the tensor $K_{i\beta}$ no term that is a monotonic function of H [1)]. The leading term in the expansion of the function $K_{i\beta}(q, H)$ is a small term of the order of

$(qD_0)^{-1/2}$, and contains an oscillating factor. Therefore, for the purpose of the computation, it is sufficient to know the first approximation for the field: $\mathscr{E}_\alpha(k) \approx \mathscr{E}_{1\alpha}(k)$. The expansions of the functions $\sigma^{(1)}_{\alpha\beta}$ and $Q_{i\beta}$ are of similar nature, and, consequently, only the monotonic terms in the expression for the current density should be considered. In the case of diffuse reflection ($\rho = 0$) we obtain:

$$j_\alpha(k)=\frac{A_{\alpha\beta}}{2k}\left\{\mathscr{E}_\beta(k)-\frac{1}{\pi}\int_0^\infty \frac{dk_1}{k_1}\mathscr{E}_\beta(k_1)\psi\left(\frac{k}{k_1}\right)\right\}, \qquad (3.17)$$

$$U_{i\,\mathrm{def}}=\frac{1}{\lambda_{izzi}}\frac{M_{i\beta}}{2\pi q}\int_0^\infty \frac{dk_1}{k_1}\mathscr{E}_\beta(k_1)\psi\left(\frac{q}{k_1}\right)\left\{\frac{\cos(qD_0+\Delta)}{(qD_0)^{1/2}}-\frac{\cos(k_1D_0+\Delta)}{(k_1D_0)^{1/2}}-\frac{\sin[(q+k_1)D_0+\Delta]}{((q+k_1)D_0)^{1/2}}\right\}, \qquad (3.18)$$

where $\psi(x) \equiv x^{1/2}(1+x)^{-1}$ (here we have retained the dominant terms of the expansions in powers of the parameter $|\gamma| \ll 1$).

For specular reflection ($\rho = 1$) the current density $j_\alpha(k)$ and the sound amplitude $U_{i\,\mathrm{def}}$ have the following form:

$$j_\alpha(k)=\frac{A_{\alpha\beta}}{k}\mathscr{E}_\beta(k)+\frac{C_{\alpha\beta}}{k}(kD_0)^{1/2}\int_0^\infty \frac{dk_1}{(k_1)^{1/2}}\left[\frac{1}{(|k-k_1|)^{1/2}}-\frac{1}{(k+k_1)^{1/2}}\right]\mathscr{E}_\beta(k_1), \qquad (3.19)$$

$$U_{i\,\mathrm{def}}=\frac{1}{\lambda_{izzi_0}}\left\{\frac{M_{i\beta}}{q}\mathscr{E}_\beta(q)\frac{\cos(qD_0+\Delta)}{(qD_0)^{1/2}}+\frac{N_{i\beta}}{q}(qD_0)^{1/2}\int_0^\infty\frac{dk_1}{k_1^{1/2}}\left[\frac{1}{(q+k_1)^{1/2}}+\frac{\mathrm{sgn}(q-k_1)}{(|q-k_1|)^{1/2}}\right]\mathscr{E}_\beta(k_1)\right\}. \qquad (3.20)$$

In the formulas (3.19) and (3.20) the integral terms are due to the surface electrons. The tensors $C_{\alpha\beta}$ and $N_{i\beta}$ are determined in the following fashion:

$$C_{\alpha\beta}=\frac{2e^2}{(2\pi\hbar)^3}\frac{\sqrt{\pi}}{4}\int\frac{mdp_x}{\gamma}\sum_{\mu=1,2}\frac{v_\alpha(\eta_\mu)v_\beta(\eta_\mu)}{[|v_z'(\eta_\mu)|D_0\Omega]^{1/2}},$$

$$N_{i\beta}=\frac{2e}{(2\pi\hbar)^3}\frac{\sqrt{\pi}}{4}\int\frac{mdp_x}{\gamma}\sum_{\mu=1,2}\frac{\Lambda_{iz}(\eta_\mu)v_\beta(\eta_\mu)\,\mathrm{sign}\,v_z'(\eta_\mu)}{[|v_z'(\eta_\mu)|D_0\Omega]^{1/2}}. \qquad (3.21)$$

For the quadratic, isotropic dispersion law $\epsilon = p^2/2m$, the expressions for the tensors (3.16) and (3.21) are considerably simpler:

$$A_{\alpha\beta}=\frac{3}{4}\frac{Ne^2}{mv\gamma}\delta_{\alpha\beta},\quad C_{yy}=\frac{3}{2}C_{xx}=\frac{Ne^2}{mv\gamma}\frac{9\pi}{5\Gamma^2(0.25)},$$

$$C_{xy}=C_{yx}=0,\quad M_{i\beta}=-\delta_{iz}\delta_{\beta y}\sqrt{\frac{2}{\pi}}\frac{Ne}{\gamma},$$

$$N_{i\beta}=\delta_{iz}\delta_{\beta y}\frac{Ne}{\gamma}\frac{\Gamma^2(0{,}25)}{12\pi\sqrt{\pi}},$$

where N is the electron concentration.

It is not difficult to see that in this case the electromagnetic field excites only longitudinal acoustic vibrations. The transverse acoustic wave does not exist in the approximation under consideration: $U_t \sim (qD_0)^{-3/2}$.

4. Let us use for the solution of the Maxwell equations for the Fourier component $\mathscr{E}_\alpha(k)$ of the field the method developed by Hartmann and Luttinger [9]. In order not to have to solve a system of integral equations, let us in the case of diffuse reflection diagonalize the tensor $A_{\alpha\beta}$. This is easily accomplished by rotating the axes in the xy plane through the angle θ:

$$\mathrm{tg}\,2\theta=2A_{xy}/(A_{xx}-A_{yy}).$$

In the new system of coordinates the equation for the field can be written in the form

$$k^2\mathscr{E}_i(k)+\frac{\beta_i}{k}\left\{\mathscr{E}_i(k)-\int_0^\infty\frac{dk_1}{\pi k_1}\psi\left(\frac{k}{k_1}\right)\mathscr{E}_i(k_1)\right\}=-2E_i'(0), \qquad (4.1)$$

$$i=1,\ 2,\qquad \rho=0.$$

Here we have introduced the notation:

$$\beta_i=-\frac{4\pi i\omega}{c^2}\sum_{\alpha,\beta=x,y}a_{i\alpha}a_{i\beta}A_{\alpha\beta},\quad a_{11}=a_{22}=\cos\theta,\quad a_{12}=-a_{21}=\sin\theta.$$

An integral equation with a kernel of the type (4.1) is solved with the aid of the Mellin transformation. Its solution has the form:

$$\mathscr{E}_i(k)=-\frac{2E_i'(0)}{[k_0^{(i)}]^2}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dz\,M_i(z)\left[\frac{k}{k_0^{(i)}}\right]^z, \qquad (4.2)$$

$k_0^{(i)} \equiv |\beta_i|^{1/3}$. The function $M_i(z)$ is regular in the vertical strip of the complex plane

$$-2.75<\mathrm{Re}\,z<1. \qquad (4.3)$$

except at the singular point $z=-2$. The singularity of $M_i(z)$ at $z=-2$ is a simple pole with a residue equal to unity.

The function $M_i(z)$ is the solution of the difference equation:

$$M_i(z-3)+\alpha_i(1-\cos^{-1}\pi z)M_i(z)=0$$

($\alpha_i \equiv \exp[i \arg \beta_i]$), and is described by the formula

$$M_i(z)=\frac{\pi}{3}\exp\left[-\frac{z+2}{3}\ln\alpha_i\right]\exp\left\{\int_{-2}^{z}G'(t)\,dt\right\}\Big/\sin\frac{\pi}{3}(z+2),$$

$$G'(z)=-\pi\frac{2z+3\cos\pi z}{3\sin 2\pi z}+V(z)\equiv G_0'(z)+V(z). \qquad (4.4)$$

The arbitrary periodic function $V(z) = V(z+3n)$ (n is an integer) is chosen such that the function $M_i(z)$ has the requisite analytic properties in the interval (4.3). In other words, it should annul those singularities of the function $G_0'(z)$ that arise in the strip (4.3) under consideration.

Let us substitute the Fourier component $\mathscr{E}_i(k)$, (4.2), of the field into the formula (3.18) for the sound amplitude. The integration over k_i presents no difficulty. The second and third terms, which contain rapidly oscillating functions under the integral sign, are small compared to the first term. We obtain after the integration

$$U_{i\,\mathrm{def}}(q)=\frac{1}{\lambda_{izzi}}\sum_{\beta,s}\frac{M_{i\beta}a_{\beta s}}{q}\frac{E_s'(0)}{[k_0^{(s)}]^2}\int_{c-i\infty}^{c+i\infty}dz\frac{M_s(z)}{\cos\pi z}\left[\frac{q}{k_0^{(s)}}\right]^z\frac{\cos(qD_0+\Delta)}{\sqrt{qD_0}}$$

The asymptotic behavior of the function $|M_s(z)|$ for $z \to \infty$ ensures the uniform convergence of the contour integral. This makes possible the computation of the asymptotic form of the amplitude $U_{i\,\mathrm{def}}(q)$ for small q in the form of a series—a sum of the residues of the integrand. In the magnetic-field region (1.1)–(1.2), the acoustic wave vector $q \ll k_0^{(s)}$ (this corresponds to the condition for the anomalous skin effect, $\delta \ll \lambda$), and therefore we can restrict ourselves to the first terms of the series:

$$U_{i\,\mathrm{def}}=\frac{1}{\lambda_{izzi}}\sum_{\substack{\beta=x,y\\ s=1,2}}\frac{a_{\beta s}M_{i\beta}}{2\beta_s}E_s'(0)\frac{\cos[qD_0+\Delta]}{(qD_0)^{1/2}}\left\{1+O\left(\frac{q}{k_0^{(s)}}\right)\right\},\quad \rho=0. \qquad (4.5)$$

The asymptotic form of the field $\mathscr{E}_i(q)$ is computed in similar fashion, and takes the form

$$\mathscr{E}_i(q)=-\frac{E_i'(0)}{\beta_i}q\left\{1+O\left(\frac{q}{k_0^{(i)}}\right)\right\},\quad \rho=0.$$

5. In the case of specular reflection, it is convenient to write the equation for the field in the following form:

$$k^2\mathscr{E}_i(k)-\frac{4\pi i\omega}{c^2}A_{is}\frac{\mathscr{E}_s(k)}{k}-i\zeta_i(kD_0)^{1/2}\int_0^\infty\frac{dk_1}{k_1^{1/2}}$$

$$\times\left\{\frac{1}{|k-k_1|^{1/2}}-\frac{1}{(k+k_1)^{1/2}}\right\}\frac{\mathscr{E}_\beta(k_1)}{k}=-2E_i'(0),\quad \rho=1,\qquad (5.1)$$

$$\zeta_i=\sum_{\alpha,\beta}\frac{4\pi\omega}{c^2}b_{i\alpha}b_{i\beta}C_{\alpha\beta},\quad b_{11}=b_{22}=\cos\Phi,\quad b_{12}=-b_{21}=\sin\Phi.$$

Here Φ is the angle of rotation that diagonalizes the tensor $C_{\alpha\beta}$.

Let us compare in Eq. (5.1) the terms that are due to the bulk and surface conductivities. When the condition (1.2) is fulfilled, the second term is $(kD_0)^{1/2}$ times smaller than the third term, and it can be neglected. This means that the dominant contribution to the "skin" current is made by the surface electrons whose trajectories lie entirely within the skin layer (slipping electrons). If we substitute into Eq. (5.1) the solution obtained under such an assumption, then it turns out that it is valid for all k satisfying the inequalities $D_0^{-1}\ll k\ll\varkappa_i$, where $\varkappa_i \equiv(\zeta_i\sqrt{D_0})^{2/5}k_\delta^{-1}$. We are interested in precisely this region

The solution to the integral equation (5.1) has been obtained by Kaner and Makarov [10]:

$$\mathscr{E}_i(k)=-\frac{2E_i'(0)}{\varkappa_i^2}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dt\,M(t)\left[\frac{k}{\varkappa_i}\right]^t,\quad -2.25<c<0.5,$$

$$M(t)=\exp\left[\frac{t+2}{5}(i\pi-\ln 2\pi+4\ln 0{,}4)\right]\cos\frac{\pi t}{2}\Gamma(t+1)\qquad (5.2)$$

$$\times\Gamma\left[-\frac{2}{5}\left(t-\frac{1}{2}\right)\right]\Gamma\left[-\frac{2}{5}\left(t-\frac{3}{2}\right)\right],\quad \rho=1.$$

The integrand has poles of order one at $t_l=-2l$ $(l=1, 2, \ldots)$ along the left semiaxis and at $t_m=(1+5m)/2$, $t_n=(3+5n)/2$ $(m, n = 0, 1, 2, \ldots)$ along the right semiaxis. Its behavior as $|t|\to\infty$ allows us to find the asymptotic forms of the field for small and large k. For $k=q\ll\varkappa_i$, we obtain

$$\mathscr{E}_i(q)=-\frac{iE_i'(0)}{5\varkappa_i^2}\left(\frac{q}{\varkappa_i}\right)^{1/2}\left\{1+O\left(\frac{q}{\varkappa_i}\right)\right\},\quad \rho=1.\qquad (5.3)$$

Let us substitute the obtained results (5.2) and (5.3) into the formula (3.20) for the sound amplitude $U_{i\,\mathrm{def}}$:

$$U_{i\,\mathrm{def}}=\lambda_{izzi}^{-1}\left\{-\sum_{\beta,s}b_{\beta s}M_{i\beta}\frac{iE_s'(0)}{5\varkappa_s^2q}\left(\frac{q}{\varkappa_s}\right)^{1/2}\frac{\cos(qD_0+\Delta)}{(qD_0)^{1/2}}\right.$$

$$\left.-\sum_{\beta,s}b_{\beta s}N_{i\beta}(qD_0)^{1/2}\frac{2E_s'(0)}{\varkappa_s^2q}I\left(\frac{q}{\varkappa_s}\right)\right\}.\qquad (5.4)$$

The integral

$$I(x_s)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dt\,x_s^t\exp\left\{\frac{t+2}{5}[i\pi-\ln 2\pi+4\ln 0.4]\right\}$$

$$\times\sin\frac{\pi}{2}\left(z-\frac{1}{2}\right)\Gamma\left(z+\frac{1}{2}\right)\Gamma\left[-\frac{2}{5}\left(z-\frac{1}{2}\right)\right]\Gamma\left[-\frac{2}{5}\left(z-\frac{3}{2}\right)\right]$$

can be evaluated by the same method used in the evaluation of the integral (5.2). Its asymptotic form for small $x_s=q/\varkappa_s\ll1$ is given by the expression

$$I(x_s)=\frac{5\sqrt{2\pi}\,\Gamma(-2/5)}{2\Gamma(2/5)}\exp[0.7(i\pi-\ln 2\pi+4\ln 0.4)]x_s^{3/2}.$$

In the magnetic-field region (1.1)–(1.2) being investigated, the parameter $|q^2D_0/\varkappa_s|\ll1$, and the second term in the expression (5.4) is small compared to the first. Consequently,

$$U_{i\,\mathrm{def}}=-i\lambda_{izzi}^{-1}\sum_{\beta,s}b_{\beta s}M_{i\beta}\frac{E_s'(0)}{5\zeta_i}\frac{\cos(qD_0+\Delta)}{qD_0},\quad \rho=1.\qquad (5.5)$$

6. The formulas (4.5) and (5.5) describe the amplitude of the transverse acoustic wave. In the case of longitudinal sound we must compare the amplitudes due to the induction and deformation forces. It is not difficult to see that for diffuse electron scattering from the boundary, the amplitude $|U_{\mathrm{ind}}|$ is $(qD_0)^{1/2}$ times smaller than $|U_{z\,\mathrm{def}}|$, while for specular scattering they are of the same order of magnitude:

$$U_z=U_{z\,\mathrm{def}},\quad \rho=0,$$

$$U_z=-i\lambda_{zzzz}^{-1}\left\{\frac{\Omega mc^2}{4\pi\omega qe}2E_y'(0)+\sum_{\beta,s}\frac{b_{\beta s}M_{i\beta}}{5\zeta_i qD_0}E_s'(0)\cos(qD_0+\Delta)\right\},\quad \rho=1.$$

Thus, we obtain that an electromagnetic wave in a metal excites longitudinal and transverse acoustic vibrations whose amplitude is an oscillating function of H^{-1}. The period of the oscillations is found from the condition $q\Delta D_0(H)=2\pi$, i.e.,

$$\Delta(H^{-1})=e\lambda/2cp_{ext},\quad 2p_{ext}\equiv[p_{y\,max}-p_{y\,min}]p_x=p_{x\,ext},$$

and is equal to the period of the oscillations of the geometrical resonance in sound absorption, but it is out of phase by $\pi/2$. The nature of the oscillations is elucidated by the distinctive features of the periodic motion of the electrons in the magnetic field and their interaction with the acoustic wave. The period and the phase of the oscillations do not depend on the nature of electron reflection from the metal surface. The surface electrons have an appreciable influence on the distribution of the electromagnetic field in the skin layer and, consequently, on the amplitude of the oscillations.

In conclusion, I express my profound gratitude to É. A. Kaner for a discussion of the work.

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*$[\dot{u}H]\equiv\dot{u}\times H$.

[1)]In [6], the monotonic term—the contribution of the integration region not containing stationary-phase points—is retained. On account of the inequalities (1.1), the consideration of this term leads to small corrections.

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