On the effect of a dielectric coating on the critical temperature of superconducting films

Yu. A. Uspenskiĭ and G. F. Zharkov

P. N. Lebedev Physics Institute
(Submitted January 10, 1973)
Zh. Eksp. Teor. Fiz. **65**, 1460–1466 (October 1973)

The effect of a dielectric coating on the critical temperature T_c of thin superconducting films is studied within the framework of a model previously proposed by the authors. The dependence of T_c on the parameters of the dielectric is discussed. It is found that the increase of T_c in such system depends strongly on the ratio of the Fermi wavelength to the Debye screening radius in the film. It is shown that although allowance for the dielectric coating does not lead, according to the model, to any significant increase of T_c of the metallic film, there still remain a number of possibilities for further research in this direction.

The problem of obtaining superconducting materials with high transition temperatures has been attracting increasing attention of late (see, e.g., the reviews [1-3] and the literature contained therein). Particular attention is paid in this case to the so called exciton mechanism of superconductivity, when the attraction between the electrons is produced to a considerable degree by exchange of excitations of the electronic type. From this point of view, great interest attaches to the investigation of "sandwich" type systems, namely, the case of a thin superconducting film on which a dielectric coating is deposited. In this case, the increase of the critical temperature should result from the interaction of the electrons contained in the film with the high-frequency excitations in the dielectric.

In an earlier paper [4] (henceforth cited as I) we proposed a simplified model description of such a system, which reflects its principal qualitative features. The interaction between the electrons in a superconducting film of thickness L was described with the aid of the "jellium" model and was characterized by an ionic plasma frequency as a Debye screening radius $r_D = 1/\kappa_D$, which the dielectric was characterized by a permittivity

$$\varepsilon(\omega)=1+(\varepsilon_0-1)\omega_1{}^2/(\omega_1{}^2-\omega^2),$$

with $\omega_i \ll \omega_1$, $\epsilon_0^{1/2}\omega_1 \ll \epsilon_F$, $\epsilon_0 > 1$. We obtained the interaction potential V between the electrons in such a system, and for the Fourier component of the potential

$$V_\omega(\varkappa, z, z') = \int dt \int d\boldsymbol{\rho}\, e^{i\omega t - i\varkappa\rho} V(|\boldsymbol{\rho}-\boldsymbol{\rho}'|, z, z', t)$$

we obtained an expression that could be written in the form[1)]

$$\begin{aligned} V_\omega(\varkappa, z, z') &= 4e^2\alpha(\omega)\int_0^\infty \frac{dq}{q^2-\tilde q^2}\Big\{\cos q(z-z') \\ &- \frac{2}{(\varepsilon^2(\omega)\varkappa^2-\tilde q^2)\sin\tilde q L+2\varepsilon(\omega)\varkappa\tilde q\cos\tilde q L}[\cos qz'\cos\tilde q z(\varepsilon(\omega)\varkappa c(q)-qs(q)) \\ &\times(\varepsilon(\omega)\varkappa s(\tilde q)+\tilde q c(\tilde q))+\sin qz'\sin\tilde q z(\varepsilon(\omega)\varkappa s(q) \\ &+qc(q))(\varepsilon(\omega)\varkappa c(\tilde q)-\tilde q s(\tilde q))]\Big\}; \end{aligned} \tag{1}$$

$$\tilde q^2 \equiv -\varkappa^2-\varkappa_D{}^2\alpha(\omega), \quad \alpha(\omega)\equiv\omega^2/(\omega^2-\omega_i{}^2),$$
$$s(x)\equiv\sin(xL/2), \quad c(x)\equiv\cos(xL/2).$$

The interaction (1) was used in I to estimate the transition temperature for very thin films with $Lk_F < \pi$. In this paper we consider films with more realistic thicknesses $L\kappa_D \gg 1$ and investigate in greater detail the dependence of the parameters of the dielectric.

Just as in the preceding paper, we shall use the weak-coupling approximation to describe the system. Within the framework of this approximation, the conduction electrons in the film can be regarded as situated in a rectangular potential well of depth U_0, and their wave functions then take the form

$$\psi_k(r) = \frac{2}{\sqrt{LS}} e^{i\varkappa\rho}\cos(pz+\delta), \tag{2}$$

where $\mathbf{k} = (\boldsymbol{\kappa}, p)$, $\boldsymbol{\kappa}$ assumes continuous values and $p \geq 0$ assumes discrete values, and $\delta = 0$ or $\delta = \pi/2$ depending on the parity of ψ; the transverse momentum p is determined from the equation

$$\frac{(2mU_0-p^2)^{1/2}}{p} = \begin{cases} \mathrm{tg}[{}^1/{}_2L(2mU_0{}^2-p^2)^{1/2}] & \text{at } \delta=0 \\ -\mathrm{ctg}[{}^1/{}_2L(2mU_0{}^2-p^2)^{1/2}] & \text{at } \delta={}^1/{}_2\pi \end{cases}.$$

The matrix elements of the interaction, taken over these functions,

$$V_\omega(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = \int d\mathbf{r}' \int d\mathbf{r}''\, V_\omega(\mathbf{r}', \mathbf{r}'')\psi_{\mathbf{k}_1}^*(\mathbf{r}')\psi_{\mathbf{k}_2}^*(\mathbf{r}'')\psi_{\mathbf{k}_3}(\mathbf{r}')\psi_{\mathbf{k}_4}(\mathbf{r}'')$$

determine the superconducting properties of the system; here

$$\varkappa_2=-\varkappa_1, \quad \varkappa_4=-\varkappa_3.$$

We note immediately that inasmuch as the energies of the phonon and exciton excitations in the system are much higher than ϵ_F, we can assume, when considering the interaction of the electrons via phonons and excitons, that the momenta of the interacting particles lie on the Fermi surface, i.e., $p_1^2 + \kappa_1^2 = p_2^2 + \kappa_1^2 = p_3^2 + \kappa_3^2 = p_4^2 + \kappa_3^2 = k_F^2$, and the matrix elements for this part of the interaction thus reduce to the simpler ones

$$V_\omega(\mathbf{k}', \mathbf{k}'') = V_\omega(\varkappa'-\varkappa'', p', p'').$$

With this remark taken into account, the transition temperature is determined in the weak-coupling approximation, apart from a numerical factor on the order of unity, by the expression

$$T_c = \bar\omega\exp\{-1/(\lambda-\mu^*)\}. \tag{3}$$

The Coulomb pseudopotential μ^* that enters in the formula depends practically only on $V(\omega \to \infty)$[2)], since there are no electronic plasma oscillations in our model, and the coupling constant averaged over the Fermi surface can be determined from the following chain of equations:

$$\begin{aligned}\lambda &= \sum_{\mathbf{k}',\mathbf{k}''}\frac{-2}{\pi\Sigma}\int_0^\infty\frac{d\omega}{\omega}\mathrm{Im}[V_\omega(\mathbf{k}', \mathbf{k}'')-V_{\omega\to\infty}(\mathbf{k}', \mathbf{k}'')] \\ &= -\frac{1}{\Sigma}\sum_{\mathbf{k}',\mathbf{k}''}(V_{\omega=0}(\mathbf{k}', \mathbf{k}'')-V_{\omega\to\infty}(\mathbf{k}', \mathbf{k}''))=\frac{1}{\Sigma}\sum_{\mathbf{k}',\mathbf{k}''}V_{\omega\to\infty}(\mathbf{k}', \mathbf{k}'');\end{aligned}$$

$$\Sigma=\frac{v_F}{LS}\sum_{\mathbf{k}'}1=\frac{v_F S_F}{(2\pi)^3}, \tag{4}$$

where LS is the normalization volume and S_F is the area of the Fermi surface. We have used here a spectral representation for $V_\omega(\mathbf{k}', \mathbf{k}'')$, took into account the fact that $V_{\omega=0}(\mathbf{k}', \mathbf{k}'') = 0$ in our model, and summed with respect to the momenta $\mathbf{k}' = (\boldsymbol{\kappa}', p')$ and $\mathbf{k}'' = (\boldsymbol{\kappa}'', p'')$ over the entire Fermi surface.

We see thus that the argument of the exponential (3) depends on the interaction only via $V_{\omega\to\infty}(\mathbf{k}', \mathbf{k}'')$, and since $\epsilon(\omega) \to 1$ as $\omega \to \infty$, the argument of the exponential does not change when a dielectric film is coated on the superconducting film.

We consider now the average frequency $\overline{\omega}$. According to[5], it is given by the equation

$$\ln\overline{\omega}=\frac{1}{\lambda}\sum_{\mathbf{k}',\mathbf{k}''}\frac{2}{\pi\Sigma}\int_0^\infty\frac{1}{\omega}\operatorname{Im}[V_\omega(\mathbf{k}',\mathbf{k}'')-V_{\omega\to\infty}(\mathbf{k}',\mathbf{k}'')]\ln\omega\,d\omega. \tag{5}$$

As will be shown below, the quantity

$$\sum_{\mathbf{k}',\mathbf{k}''}\operatorname{Im}[V_\omega(\mathbf{k}',\mathbf{k}'')-V_{\omega\to\infty}(\mathbf{k}',\mathbf{k}'')]$$

for the interaction (1) differs from zero only in two nonoverlapping frequency regions: a) in the region of phonon excitation $0 < \omega < \omega_i$; b) in the region of exciton excitations $\omega_1 < \omega < \omega_{max}^{(e)}$, where

$$(\omega_{max}^{(e)})^2\approx\omega_1^2\frac{(4k_F^2+\varkappa_D^2)^{1/2}+2k_F\varepsilon_0}{(4k_F^2+\varkappa_D^2)^{1/2}+2k_F}.$$

Therefore, recognizing that $\operatorname{Im} V_{\omega\to\infty}(\mathbf{k}', \mathbf{k}'') = 0$, Eq. (5) should be rewritten in the form

$$\ln\overline{\omega}=\frac{\lambda^{(ph)}}{\lambda}\ln\overline{\omega^{(ph)}}+\frac{\lambda^{(e)}}{\lambda}\ln\overline{\omega^{(e)}}, \tag{6}$$

where

$$\lambda^{(ph)}=\frac{2}{\pi\Sigma}\sum_{\mathbf{k}',\mathbf{k}''}\int_0^{\omega_i}\frac{d\omega}{\omega}\operatorname{Im}V_\omega(\mathbf{k}',\mathbf{k}''),$$

$$\lambda^{(e)}=\frac{2}{\pi\Sigma}\sum_{\mathbf{k}',\mathbf{k}''}\int_{\omega_1}^{\omega_{max}^{(e)}}\frac{d\omega}{\omega}\operatorname{Im}V_\omega(\mathbf{k}',\mathbf{k}''),$$

$$\ln\overline{\omega^{(ph)}}=\frac{2}{\pi\Sigma\lambda^{(ph)}}\sum_{\mathbf{k}',\mathbf{k}''}\int_0^{\omega_i}d\omega\frac{\ln\omega}{\omega}\operatorname{Im}V_\omega(\mathbf{k}',\mathbf{k}''),$$

$$\ln\overline{\omega^{(e)}}=\frac{2}{\pi\Sigma\lambda^{(e)}}\sum_{\mathbf{k}',\mathbf{k}''}\int_{\omega_1}^{\omega_{max}^{(e)}}d\omega\frac{\ln\omega}{\omega}\operatorname{Im}V_\omega(\mathbf{k}',\mathbf{k}'').$$

We thus obtain an expression for $\overline{\omega}$:

$$\overline{\omega}=(\overline{\omega^{(ph)}})^{\lambda^{(ph)}/\lambda}(\overline{\omega^{(e)}})^{\lambda^{(e)}/\lambda}. \tag{7}$$

From our earlier paper I, and also from physical considerations, it is obvious that $\overline{\omega^{(ph)}} \approx \omega_i$ and does not depend in any appreciable manner on the dielectric coating. Taking this circumstance into account and combining formulas (3) and (7), we obtain an expression for the change in the critical temperature due to application of a dielectric coating[3]:

$$(T_c)_{\text{coated}}/(T_c)_{\text{uncoated}}=(\overline{\omega^{(e)}}/\overline{\omega^{(ph)}})^{\lambda^{(e)}/\lambda}. \tag{8}$$

The rest of the problem consists of determining the dependence of the quantities that enter in formula (8) on the parameters of the film and of the dielectric. We obtain first $V_\omega(\mathbf{k}', \mathbf{k}'')$, by integrating the interaction (1) with the wave functions (2), and arrive as a result at the expression

$$V_\omega(\mathbf{k}',\mathbf{k}'')=A\alpha(\omega)\sum_{i,j}\Big\{\Big[\frac{s(p_i-p_j)}{p_i-p_j}\cos(\delta_i-\delta_j)+\frac{s(p_i+p_j)}{p_i+p_j}\cos(\delta_i+\delta_j)\Big]\frac{1}{4}(p_i^2+p_j^2-2\tilde{q}^2)+[\cos\delta_i\cos\delta_j(p_is(p_i)c(\tilde{q})-\tilde{q}s(\tilde{q})c(p_i))\mathscr{E}_1(\omega)+\sin\delta_i\sin\delta_j(p_ic(p_i)s(\tilde{q})-\tilde{q}c(\tilde{q})s(p_i))\mathscr{E}_2(\omega)]\Big\}\times(p_i^2-\tilde{q}^2)^{-1}(p_j^2-\tilde{q}^2)^{-1}, \tag{9}$$

$$\mathscr{E}_1(\omega)=\frac{\varepsilon(\omega)\varkappa c(p_j)-p_js(p_j)}{\varepsilon(\omega)\varkappa c(\tilde{q})-\tilde{q}s(\tilde{q})},\qquad \mathscr{E}_2(\omega)=\frac{\varepsilon(\omega)\varkappa s(p_j)+p_jc(p_j)}{\varepsilon(\omega)\varkappa s(\tilde{q})+\tilde{q}c(\tilde{q})},$$

where the following notation is used: $s(x) = \sin(xL/2)$, $c(x) = \cos(xL/2)$,

$$p_i=(p'-p'',\ p'+p'',\ -p'+p'',\ -p'-p''),$$
$$\delta_i=(\delta'-\delta'',\ \delta'+\delta'',\ -\delta'+\delta'',\ -\delta'-\delta'')$$

respectively for $i = 1, 2, 3, 4$, $\kappa = |\boldsymbol{\kappa}' - \boldsymbol{\kappa}''|$, $A = \pi e^2/8L^2S$ and we choose $\tilde{q} = i|\tilde{q}|$ if $0 > \tilde{q}^2 = -\kappa^2 - \kappa_D^2\alpha(\omega)$, corresponding to a causal circling around the poles.

It is easily seen that $V_\omega(\mathbf{k}', \mathbf{k}'')$ has poles of two types, corresponding to two types of excitations in our system:

a) phonon excitations with $0 < \omega < \omega_i$, which are determined by the equations

$$\varepsilon(\omega)\varkappa c(\tilde{q})-\tilde{q}s(\tilde{q})=0,\quad \varepsilon(\omega)\varkappa s(\tilde{q})+\tilde{q}c(\tilde{q})=0;$$

b) exciton excitations with $\omega > \omega_1$, which are determined by the equations

$$\varepsilon(\omega)\varkappa C(\tilde{q})+|\tilde{q}|S(\tilde{q})=0,\quad \varepsilon(\omega)\varkappa S(\tilde{q})+|\tilde{q}|C(\tilde{q})=0,$$
$$S(\tilde{q})\equiv\operatorname{sh}(|\tilde{q}|L/2),\quad C(\tilde{q})\equiv\operatorname{ch}(|\tilde{q}|L/2).$$

If we recognize that $\omega \ll \omega_1$, we can easily obtain the dispersion laws for the two modes of the exciton excitations:

$$\omega^2=\omega_1^2\frac{\tilde{\varkappa}T(\tilde{\varkappa})+\varepsilon_0\varkappa}{\tilde{\varkappa}T(\tilde{\varkappa})+\varkappa}\qquad \omega^2=\omega_1^2\frac{\tilde{\varkappa}+\varepsilon_0\varkappa T(\tilde{\varkappa})}{\tilde{\varkappa}+\varkappa T(\tilde{\varkappa})},$$
$$\tilde{\varkappa}\equiv\sqrt{\varkappa^2+\varkappa_D^2},\quad T(x)\equiv\operatorname{th}(|x|L/2).$$

We note that in the limit $L \gg 2/\kappa_D$, which will be used below, these two modes converge into one: $\omega^2 = \omega_1^2(\tilde{\kappa} + \epsilon_0\kappa)/(\tilde{\kappa} + \kappa)$.

The analysis that follows would be too cumbersome if the depth of the potential well U_0 in the boundary conditions for the wave function (2) were to be arbitrary. We therefore confine ourselves only to the limiting case of an infinitely deep well, i.e., to $U_0 \to \infty$, corresponding to the boundary condition $\psi = 0$ on the film surface. It is necessary then to put $\cos p'L\cos 2\delta' = \cos p''L\cos 2\delta'' = -1$. The expression for $V_\omega(\mathbf{k}', \mathbf{k}'')$ then becomes much simpler and assumes at $\tilde{q}^2 < 0$ the following form (in the case $L\kappa_D > 1$, which is of greatest interest):

$$V_\omega(\mathbf{k}',\mathbf{k}'')=\frac{A\alpha(\omega)}{P_-(\tilde{q})}\Big\{\frac{L}{2}-\frac{2\varepsilon(\omega)\varkappa|\tilde{q}|}{\varepsilon(\omega)\varkappa+|\tilde{q}|}\Big[\frac{1}{P_-(\tilde{q})}-\frac{1}{P_+(\tilde{q})}\Big]\Big\}. \tag{10}$$

In this formula we have used the notation $P_\pm(\tilde{q}) \equiv (p' \pm p'')^2 + |\tilde{q}|^2$ and assumed that the values of p' and p'' lie in the interval $(+k_F, -k_F)$. Putting $\omega \to \infty$ in (10) and using (4), we get

$$\lambda=B\int dk'\int dk''\frac{1}{P_-(\tilde{\varkappa})}\Big\{\frac{L}{2}-\frac{2\varkappa\tilde{\varkappa}}{\varkappa+\varkappa}\Big[\frac{1}{P_-(\tilde{\varkappa})}-\frac{1}{P_+(\tilde{\varkappa})}\Big]\Big\}. \tag{11}$$

We have put here $B = LSA/v_F(2\pi)^3\int d\mathbf{k}$ and replaced summation over the momenta with integration; this should not lead to noticeable errors, since the integrand is of constant sign and varies noticeably over scales larger than κ_D, whereas $\Delta p = \pi/L \ll \kappa_D$.

From (11) we obtain at $\omega > \omega_1$

$$\frac{2}{\pi}\frac{\operatorname{Im} V_\omega(\mathbf{k}',\mathbf{k}'')}{\omega}=-A\left[\frac{1}{P_-(\tilde{\varkappa})}-\frac{1}{P_+(\tilde{\varkappa})}\right]\frac{1}{P_-(\tilde{\varkappa})}$$
$$\times\frac{2\varkappa(\varepsilon_0-1)\tilde{\varkappa}^2}{(\varepsilon_0\varkappa+\tilde{\varkappa})(\varkappa+\tilde{\varkappa})}\delta\left[\omega-\omega_1\left(\frac{\varepsilon_0\varkappa+\tilde{\varkappa}}{\varkappa+\tilde{\varkappa}}\right)^{1/2}\right].$$

From this we obtain, using formulas (6), the expression

$$\lambda^{(e)}=B\int d\mathbf{k}'\int d\mathbf{k}''\left[\frac{1}{P_-(\tilde{\varkappa})}-\frac{1}{P_+(\tilde{\varkappa})}\right]\frac{1}{P_-(\tilde{\varkappa})}\frac{2\varkappa(\varepsilon_0-1)\tilde{\varkappa}}{(\varepsilon_0\varkappa+\tilde{\varkappa})(\varkappa+\tilde{\varkappa})},$$

$$\ln\frac{\overline{\omega^{(e)}}}{\omega_1}=\frac{B}{2\lambda^{(e)}}\int d\mathbf{k}'\int d\mathbf{k}''\ln\left(\frac{\varepsilon_0\varkappa+\tilde{\varkappa}}{\varkappa+\tilde{\varkappa}}\right)\frac{2(\varepsilon_0-1)\tilde{\varkappa}}{(\varepsilon_0\varkappa+\tilde{\varkappa})(\varkappa+\tilde{\varkappa})}\times\left[\frac{1}{P_-(\tilde{\varkappa})}-\frac{1}{P_+(\tilde{\varkappa})}\right]\frac{1}{P_-(\tilde{\varkappa})}. \quad (12)$$

We note certain qualitative features of the obtained expressions. It is seen from (11) that the main contribution to λ is made by the "volume" term

$$\frac{1}{2}BL\int\frac{d\mathbf{k}'\,d\mathbf{k}''}{(\mathbf{k}'-\mathbf{k}'')^2+\varkappa_D^2}=4\pi^2Lk_F^2\ln\frac{\varkappa_D^2}{\varkappa_D^2+4k_F^2}.$$

Therefore, if the boundary conditions for the conduction-electron wave functions differ from the considered case $U_0 \to \infty$, this affects the value of λ little. At the same time, we can expect a relatively strong dependence of $\lambda^{(e)}$ on the boundary conditions as a result of localization of the electron-exciton interaction near the metal-dielectric boundary.

It is also easy to see from (12) that $\lambda^{(e)}$ and $\overline{\ln \omega^{(e)}}$ are monotonically increasing functions of ϵ_0, with

$$\lambda^{(e)}_{\varepsilon_0\to\infty}=B\int d\mathbf{k}'\int d\mathbf{k}''\left[\frac{1}{P_-(\tilde{\varkappa})}-\frac{1}{P_+(\tilde{\varkappa})}\right]\frac{1}{P_-(\tilde{\varkappa})}\frac{2\tilde{\varkappa}^2}{\varkappa+\tilde{\varkappa}},$$

and $\overline{\ln \omega^{(e)}}$ increases in proportion to $\ln \epsilon_0$ at large ϵ_0.

We present the values of the integrals (11) and (12) obtained by computer calculation for the parameters $\epsilon_0 = 30$ and $\kappa_D/k_F = 1$:

$$\lambda=4\pi k_F B\left[\pi Lk_F\ln\frac{\varkappa_D^2+4k_F^2}{\varkappa_D^2}-2(0.985-0.783)\right];$$
$$\lambda^{(e)}=8\pi k_F B[2.175-1.730],$$
$$\ln(\overline{\omega^{(e)}}/\omega_1)={}^1/_2(4.402-3.468)/(2.175-1.730).$$

If we put $k_F = 1\ \text{A}^{-1}$, $L = 10$ Å, and $\omega_1 = 15\,\overline{\omega^{(ph)}}$, then, substituting the corresponding numerical values in (8), we obtain

$$(T_c)_{\text{coated}}/(T_c)_{\text{uncoated}}\,|_{U_0\to\infty}=1.07,$$

i.e., the increase of T_c is small.

We see thus that in this model one can hardly expect a radical increase of T_c as a result of the application of the coating. We note, however, that the integrals in (12) and (13) are sensitive to the parameter κ_D/k_F, and increase quite rapidly when this parameter is decreased. For example, if we take $\kappa_D/k_F = 0.316$[4)] and the previous values of the remaining parameters, then we obtain a somewhat larger increase of T_c:

$$(T_c)_{\text{coated}}/(T_c)_{\text{uncoated}}=1.86.$$

Of course, the foregoing numerical calculations of the change of the critical temperature following application of a dielectric coating are quite arbitrary, since they were made for the approximate model interaction (1) and for electron wave functions that vanish on the film boundary. We hope, however, that if the depth of penetration of the conduction electrons into the dielectric is small, our model accounts adequately for the characteristic properties of the system and leads to correct qualitative results.

We note that if the depth of the potential well is small, then the influence of the coating on T_c is probably larger because of the interaction of the conduction electrons that tunnel from the film into the coating with the exciton excitations in the dielectric. A detailed discussion of this question and a concrete calculation for the case of a semiconductor coating can be found in[6], and also in[7,8], where a number of critical remarks concerning[6] are made. In addition, as already noted in I, insistence with low electron density and with large screening radius (e.g., in superconducting semiconductor films), the role of the boundary effects is relatively more noticeable and can also lead to an appreciable increase of T_c. Finally, mention should be made of the possible appearance in the system of particular surface excitations that can play a decisive role in the superconductivity phenomenon[9]. A more detailed analysis of these effects would be of undoubted interest.

The authors thank V. L. Ginzburg and the participants of the seminar headed by him for useful discussions, and L. V. Parĭskaya for the numerical calculations.

---

1)The plane z = 0 is assumed to pass through the center of the film.

2)We neglect the weak dependence of μ^* on $\bar{\omega}$, which is of the type $\mu^*=\bar{V}_{\omega\to\infty}N(0)\,[1+\bar{V}_{\omega\to\infty}N(0)\ln(\varepsilon_F/\bar{\omega})]^{-1}$, and also the small change of $V_{\omega\to\infty}(\mathbf{k}',\mathbf{k}'')$ due to change in the boundary conditions for the electron wave functions following application of the dielectric coating.

3)We recall that this expression was obtained in the "jellium" model, where $V_{\omega\to 0} = 0$ and λ does not depend on the presence of a dielectric coating. In real metallic films one can more readily expect independence of $\lambda^{(ph)}$ of the presence of a dielectric coating. This would lead to replacement of (8) by $(T_c)_{\text{coated}}/(T_c)_{\text{uncoated}} = (\overline{\omega^{(e)}}/\overline{\omega^{(ph)}})^{\lambda^{(e)}/\lambda}\exp\{\lambda^{(e)}/(\lambda^{(ph)}-\mu^*)(\lambda-\mu)\}$, i.e., to a large increase of T_c in comparison with our model.

4)In the free-electron approximation we have $\varkappa_D^2/k_F^2=4\pi^{-4/3}3^{-1/3}n_0^{-1/3}a_B^{-1}$, where a_B is the Bohr radius and n_0 is the electron concentration. At densities typical of metals, this expression leads to $\kappa_D/k_F \sim 1$, but the effects of the electron-ion interaction can lead to a certain decrease of this quantity.

---

1. V. L. Ginzburg, Usp. Fiz. Nauk 95, 91 (1968) [Contemporary Physics (1969)].
2. V. L. Ginzburg, Usp. Fiz. Nauk 101, 185 (1970) [Sov. Phys.-Uspekhi 13, 335 (1971)].
3. V. L. Ginzburg and D. A. Kirzhnits, Phys. Reports, 4C, 344, 1972.
4. G. F. Zharkov, and Yu. A. Uspenskiĭ, Zh. Eksp. Teor. Fiz. 61, 2123 (1971) [Sov. Phys.-JETP 34, 1132 (1972)].
5. D. A. Kirzhnits, E. G. Maximov and D. I. Khomskii, J. Low Temp. Phys. 10, 79, 1972.
6. D. Allender, F. Bray and J. Bardeen, Phys. Rev. 7B, 1020, 1973.
7. J. C. Phillips, Phys. Rev. Lett. 29, 1551, 1972.
8. J. C. Inkson and P. W. Anderson, Preprint TCM 171, 1973.
9. V. M. Agranovich, A. G. Mal'shukov, and M. A. Mekhtiev, Fiz. Tverd. Tela 14, 839 (1972) [Sov. Phys.-Solid State 14, 716 (1972)].

Translated by J. G. Adashko
150