Effect of magnetic breakdown on the Hall effect in beryllium

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The Hall efffect in beryllium is investigated under magnetic breakdown conditions. The breakdown field strength is estimated to be $H_0 \approx 110$ kOe on the basis of the magnetic field dependence of the Hall constant. Giant oscillations of the symmetric (with respect to the field) component ρ_{yx} are observed. The change in the phase shift between the oscillations of ρ_{yx} and ρ_{xx} which is observed on deviation of the hexagonal axis of the sample from the direction of the magnetic field, can be explained on the basis of a coherent model for the breakdown.

INTRODUCTION

Investigation of the galvanomagnetic properties in strong magnetic fields is now a very effective method of studying the behavior of electrons in metals. Such investigations, as is well known, make it possible to determine the topological features of the Fermi surface of a metal and to obtain the values of some of its parameters [1,2].

In a number of metals, the character of the carrier motion can undergo serious changes when the magnetic field is increased. Such changes may be connected with magnetic breakdown [3], which has been the subject of a rather large number of theoretical and experimental studies. Modern theoretical studies of magnetic breakdown are based on two concepts, stochastic and coherent. In the stochastic approach, the transitions of an electron from one classical orbit to another are regarded as independent of one another [4]. In the coherent approach, the phase relations are taken into account for the wave functions of the electrons that are multiply scattered by the magnetic-breakdown regions [5,6].

Whereas the stochastic model is a satisfactory approximation when it comes to describing the monotonic part of the resistivity tensor [7], some features of the behavior of the oscillating part make it necessary to resort to the recently developed coherent model [8]. In this paper we investigate the influence of magnetic breakdown on the off-diagonal element (ρ_{yx}) of the resistivity tensor of beryllium; to this end, we use a method developed by us to record a signal proportional to ρ_{yx} as a function of ρ_{xx} (the diagonal element).

2. MEASUREMENT METHOD

All the measurements were performed on three samples of beryllium with dimensions $\approx 0.3 \times 0.8 \times 5$ mm. To ascertain the influence of the purity of the sample on the measurement results, we used single-crystal beryllium with various resistivity ratios $\rho(300°K)/\rho(4.2°K)$, from 150 to 400.[1)] It turned out that the purity of the samples employed had practically no influence on the measurement results (only a small increase was observed in the amplitude of the oscillating components of the signals picked off samples having the larger resistivity ratio).

The mounting of the samples is shown schematically in Fig. 1. The samples were cut and mounted in such a way that the measuring current J was parallel to the [$\bar{1}$010] axis, and the [0001] axis was perpendicular to the plane of the sample. To obtain a uniform distribution of the current over the sample and to avoid the effects observed in [9], each current lead consisted of four wires welded with the aid of a laser. The potential terminals were welded either with a laser or with a spark; the distance between them was $\lesssim 1$ mm. The accuracy (equipotentiality) of the Hall terminals was monitored by the value of the voltage U_{yx} in the absence of a magnetic field. To determine the contribution of the resistivity signal to U_{yx} with increasing magnetic field, we used the $\rho_{xx}(H)$ plot measured with the same sample. The contribution of the resistivity signal to U_{yx} in fields $\gtrsim 20$ kOe, determined by this method, did not exceed 0.5%. After welding of the current and potential terminals, the sample was mounted in a holder placed between permendur concentrators (the device used to rotate the sample in the magnetic field was described in detail earlier [10]). The investigations were carried out at a temperature 4.2°K in the field of a superconducting magnet.

Figure 2 shows a typical plot of $U_{yx}(H)$ when the sample is so rotated that the angle α between the [0001] axis of the sample and the direction of the magnetic field is 2° [2)] (in this case, as in all other measurements, the current was perpendicular to the magnetic field). The oscillations of U_{yx} begin in the same field region as the oscillations of the resistivity. When the U_{yx} and U_{xx} signals were recorded simultaneously, a phased shift relative to the magnetic field was observed between them.

For detailed measurements of the monotonic and oscillating parts of U_{xx} and U_{yx} we used the following method: we determined the angle α of inclination of the hexagonal axis of the sample to the direction of the magnetic field, the signals U_{xx} and U_{yx} were partially balanced with potentiometers, and the unbalance signals were fed through photoamplifiers to the X and Y coordinates of an automatic X-Y recorder. The magnetic field was varied in a small range ($\approx 2-3$ kOe), and an "untwisting" ellipse was obtained on the automatic recorder (the "untwisting" is due to the field dependence of the monotonic signal components). A typical plot of this type is shown in Fig. 3. Assuming that the variations of the monotonic components of the signals at such a small variation of the magnetic field are linear in the field, it was possible to eliminate the untwisting (corresponding in Fig. 3 to the dashed "non-untwisting" ellipse in Fig. 3). From the shapes and positions of the ellipse obtained by this method we determined the following quantities:

1) The values of the monotonic components of the signals U_{yx} and U_{xx} (from the position of the center of the ellipse).

2) The amplitudes A_{yx} and A_{xx} of the oscillating components of the voltages (from the lengths of the size of the circumscribed rectangle).

3) The phase difference θ between the U_{yx} and U_{xx} oscillations ($|\sin\theta| = a/b$ (Fig. 3); in addition, we determined the direction in which the ellipse is traced and the position of its major axis).

In the present study we investigated the angular de-

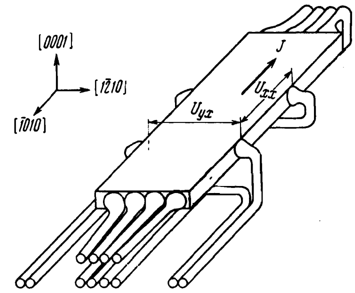

FIG. 1. Mounting of samples for the measurement of the Hall emf and the resistivity.

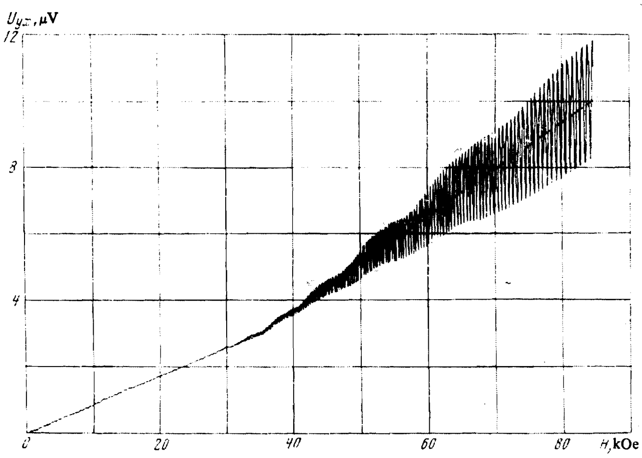

FIG. 2. Typical plots of U_{yx} against magnetic field. The sample is so turned that the angle α between the hexagonal axis of the sample and the magnetic field is 2°. The measurement current is J = 0.2 A.

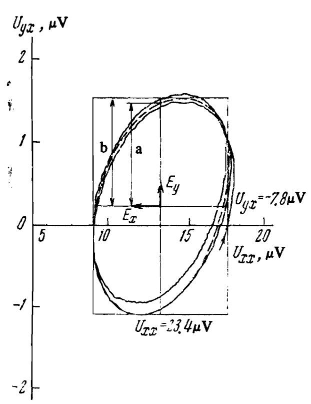

FIG. 3. Typical plots of U_{yx} against U_{xx}, obtained with an x-y recorder in the case when the magnetic field is varied from 84 to 87 kOe. E_y = 8 μV and E_x = −10 μV are the values of the balance voltages of the potentiometers to which the signals U_{yx} and U_{xx} are applied. The amplitudes of the oscillating components are A_{yx} = 2.1 V and A_{xx} = 8.45 μV. For this case we have $|\sin\theta| = a/b \approx 0.95$ and the phase difference is θ = 288°.

pendences of the monotonic and oscillating parts of the signals U_{yx} and U_{xx}. Using the described method, we could carry out the measurements of the oscillating parts U_{yx} and U_{xx} and the phase difference between them only at small (≈6°) deviations of the hexagonal axis of the sample from the magnetic field direction. The reason is that when the angle α was increased the monotonic part of U_{xx} increased rapidly and the oscillation amplitudes A_{yx} and A_{xx} decreased; the untwisting of the ellipse then became so strong that it was impossible to reconstruct the shape of the non-untwisting ellipse with sufficient accuracy.

If we plot U_{yx} against U_{xx} as the magnetic field is varied in a wide range, then we obtain a complicated figure made up of ellipses. An experimental plot of this type is illustrated in Fig. 4.

To separate the symmetrical and antisymmetrical components of U_{yx}, we used reversal of the magnetic field and calculated the antisymmetrical component of U_{yx}:

$$E_1 = {}^1/_2[U_{yx}(+\mathrm{H}) - U_{yx}(-\mathrm{H})]$$

and the symmetrical component of U_{yx}.

$$E_2 = {}^1/_2[U_{yx}(+\mathrm{H}) + U_{yx}(-\mathrm{H})].$$

3. EXPERIMENTAL RESULTS AND DISCUSSION

Antisymmetrical Components of the Signal U_{yx} (E_1)

As is well known [1], expansion of the nondiagonal element of the conductivity tensor in the magnetic field takes in general the form

$$\sigma_{yx} = -\frac{ec\Delta n}{H} + \frac{a_2}{H^2} + \frac{a_3}{H^3} + \ldots, \tag{1}$$

where $\Delta n = n_1 - n_2$ (n_1 is the number of electrons and n_2 is the number of holes). Since beryllium is a compensated metal ($n_1 = n_2$), the first term of the series vanishes, and the Hall constant $R_H = E_1D/JH$ (D is the sample thickness and J is the measuring current) is given by

$$R_H = \frac{\rho_{yx}}{H} \approx \frac{\rho_{xx}^2}{H}\left(\frac{a_2}{H^2} + \frac{a_3}{H^3} + \ldots\right). \tag{2}$$

The dependence of the Hall constant on the magnetic field is shown in Fig. 5. In fields up to 30 kOe, R_H does not depend on the field and $R_H(0) = (4.25 \pm 0.1) \times 10^{-12}$ Ω-cm/Oe. In fields exceeding 30 kOe, R_H decreases with increasing field. Since it is precisely in these fields that magnetic breakdown takes place and leads to the formation of open trajectories in the basal plane, it is natural to attribute this field dependence of R_H to the unbalance of the electron and hole volumes of the Fermi surface, which results from the breakdown (in

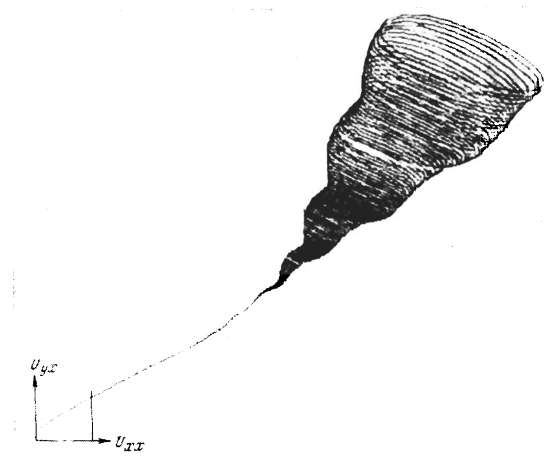

FIG. 4. Dependence of U_{yx} on U_{xx} obtained by increasing the magnetic field from 0 to 88 kOe. The sample was so turned that the angle α between the hexagonal axis of the sample and the magnetic field was 2°.

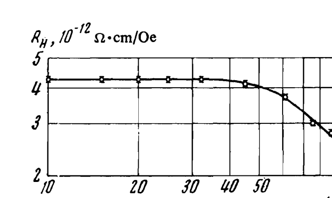

FIG. 5. Dependence of the Hall constant on the magnetic field.

the case of breakdown $\Delta n \neq 0$ and increases with increasing breakdown probability W). As $W \to 1$ the unbalance is

$$(\Delta n)_{W\to 1} = S d_{max} / 4\pi^3$$

(S is the cross-sectional area of the Brillouin zone, d_{max} is the maximum thickness of the layer of open trajectories). For beryllium, d_{max} is determined by the dimension of the neck of the hole part of the Fermi surface in the second zone; in accordance with [11] we have $d_{max} = 3.6\times 10^6\ cm^{-1}$ and $(\Delta n)_{W\to 1} = 0.96\times 10^{21}\ cm^{-3}$.

If the influence of the magnetic breakdown on R_H reduces to an unbalance, then it is easy to obtain the following dependence of Δn on the field:

$$\frac{\Delta n}{(\Delta n)_{W\to 1}} = \frac{H^2[R_H(0) - R_H(H)]}{ec\rho_{xx}^2(H)(\Delta n)_{W\to 1}}. \qquad (3)$$

This dependence, obtained on the basis of the experimental data, is shown by the solid curve of Fig. 6. The dashed lines in the figure are theoretical plots for different values of the breakdown field H_0 ($W = \exp(-H_0/H)$) on the basis of calculations performed in [7]. It follows from Fig. 6 that the field dependence of R_H can be attributed to the unbalance due to magnetic breakdown, and that the breakdown field is $H_0 = 110 \pm 10$ kOe.

In earlier studies [12,13] the breakdown field H_0 was determined by other methods and the following values were obtained: $H_0 \approx 130$ kOe from measurements of the de Haas-van Alphen effect [12] and $H_0 = 110$ kOe from the temperature and field dependences of the resistivity oscillations [13].

The dimensions of the Fermi surface of beryllium have now been determined with sufficient accuracy [11]. Calculating by means of the Chambers formulas [14]

$$H_0 = \frac{\pi\hbar c}{2\sqrt{2}\,e}(k_g^3 k_r)^{1/2}, \qquad (4)$$

where k_g is the minimal distance between the orbits in k-space and k_r is the curvature radius of the orbits, we can obtain $H_0^{theor} = 113$ kOe, which is in good agreement with the estimate of H_0 from measurements of the Hall constant [3)].

We observed no oscillations of the antisymmetrical component of the signal U_{yx} (E_1). The absence of oscillations of E_1 can be regarded as a consequence of the relatively weak influence of magnetic breakdown on the off-diagonal elements of the conductivity tensor [6].

Symmetrical Component of the Signal U_{yx} (E_2)

For compensated metals ($n_1 = n_2$) the signal U_{yx} can contain a symmetrical component (in the field). If the current **J** and the magnetic field **H** are directed arbitrarily relative to the principal axes of the conductivity

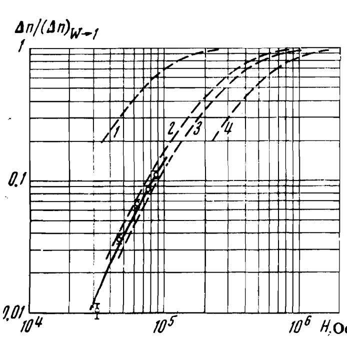

FIG. 6. Variation of the unbalance $\Delta n/(\Delta n)_{W\to 1}$ with the field. Dashed—theoretical plots for different values of the breakdown field H_0: 1—H_0 = 30 kOe, 2—H_0 = 100 kOe, 3—H_0 = 120 kOe, 4—H_0 = 200 kOe.

tensor, then U_{yx} contains a term quadratic in the magnetic field; this term characterizes the anisotropy of the resistivity (the difference between the diagonal elements ρ_{xx} and ρ_{yy} of the resistivity tensor). Thus, for example, if the current is inclined at an angle α to one of the principal axes and the magnetic field is parallel to the other, then

$$E_2 \approx \frac{J}{D}\sin\alpha\cos\alpha[\rho_{xx}(0) - \rho_{yy}(0)]\left(1 + \frac{H^2}{H_1^2}\right), \qquad (5)$$

where $H_1 = m^*c/e\tau$ (τ is the relaxation time) and $\rho_{xx}(0)$ and $\rho_{yy}(0)$ are the diagonal elements of the resistivity tensor in the absence of a field [1].

Figure 7 shows plots of the monotonic part of U_{yx} against the angle α between the hexagonal axis of the sample and the magnetic field, obtained after reversing the field, and Fig. 8 shows the dependence of the monotonic part E_2 (the symmetrical component of U_{yx}) on the field when the angle $\alpha = 1.5°$. From Fig. 7 and 8 it follows that in our case the symmetrical component of U_{yx} is described by expression (5).

The relaxation time τ which enters in the definition of H_1 can be determined from measurements of R_H and $\rho_{xx}(H)$ in weak magnetic fields. For the investigated samples $\tau \approx 10^{-11}$ sec and $H_1 \approx 3-5$ kOe. Thus, the term $(H/H_1)^2$ in (5) gives an enhancement, by hundreds of times, of the relatively small ($(\rho_{xx} - \rho_{yy})/\rho_{xx} \approx 10^{-3}$) difference of the diagonal elements of the resistance tensor.

It was already noted above that we observed oscillations of only the symmetrical component of U_{yx} (E_2). Since the monotonic part E_2 is described by expression (5), it is natural to assume that the oscillating part of the symmetrical component U_{yx} is determined by the difference of the diagonal elements of the resistivity tensor. Incidentally, this is confirmed by the experimentally observed dependence of the amplitude of the oscillations of the signals U_{yx} on the angle α (Fig. 9).

Let

$$\rho_{xx}^{osc} = k\sin(F/H + x), \quad \rho_{yy}^{osc} = k\sin(F/H + y)$$

(F is the oscillation frequency); then elementary operations yield

$$\theta = {}^1/_2\pi\,\mathrm{sign}\,\alpha + {}^1/_2(x - y) + N\pi, \qquad (6)$$

where θ is the phase difference between the oscillations of U_{yx} and U_{xx}, and α is the angle between the hexagonal axis of the sample and the magnetic field. Thus, when the angle α goes through zero, a phase jump equal to π should be observed.

The dependence of the phase difference between the oscillations of U_{yx} and U_{xx} on the angle α is shown in Fig. 10. On going through zero, one observes a phase jump close to π ($\approx 0.85\pi$)[4)]. With further change of the angle α, a monotonic increase of the phase difference θ is observed. This increase cannot be understood on the basis of the stochastic model of magnetic breakdown[4], since variation of the angle α between the hexagonal axis of the sample and the magnetic field causes the cross sections of all the electron cigars in the third Brillouin zone of beryllium to vary in absolutely the same manner. It is precisely these sections that cause the giant magnetoresistance oscillations, and the Lifshitz-Onsager quantization rules produce one and the same initial phase $\pi/4$ for them (ρ_{xx}^{osc} and ρ_{yy}^{osc}).

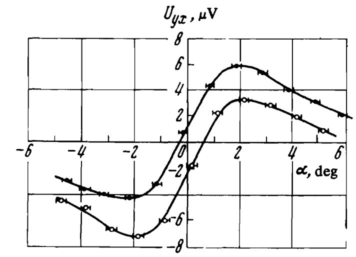

FIG. 7

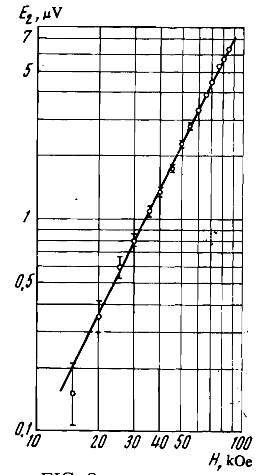

FIG. 8

FIG. 7. Dependence of the monotonic part of U_{yx} on the angle α between the hexagonal axis of the sample and the magnetic field for two magnetic-field directions: ○—field +H, ●—field −H. Measuring current J = 0.4 A. The field ⟨H⟩ = 60.5 kOe.

FIG. 8. Dependence of the symmetrical component of U_{yx} on the magnetic field. The sample was so turned that the angle between the hexagonal axis of the sample and the magnetic field was α = 1.5°. The measuring current was J = 0.2 A.

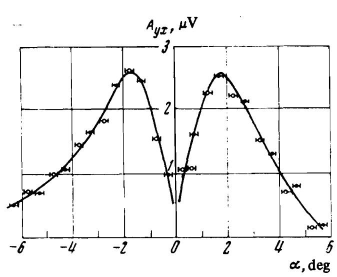

FIG. 9

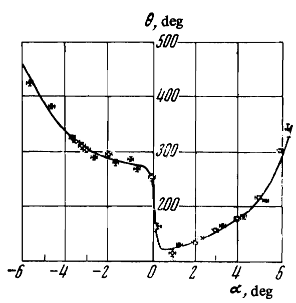

FIG. 10

FIG. 9. Dependence of the oscillation amplitude A_{yx} on the angle α between the hexagonal axis of the sample and the magnetic field for two directions of the magnetic field: ○—field +H, ●—field −H. Measurement current J = 0.4 A. Field ⟨H⟩ = 75.4 kOe.

FIG. 10. Dependence of the phase difference between the oscillations of U_{yx} and U_{xx} on the angle α between the hexagonal axis of the sample and the magnetic field for two directions of the magnetic field: ○—field +H, ●—field −H. Mean value of the magnetic field ⟨H⟩ = 85.6 kOe.

A different situation arises in the coherent model, when the motion of the electrons over the cigars is synchronized with the motion in the corona in such a way that the wave function of the electron, corresponding to the open direction, is of the Bloch type, and the phase jump of the wave function on going from the corona to the cigar is determined by the breakdown probability [5]. In this case the initial phase of the resistivity oscillations contains two contributions, the first determined by the periodicity of the reciprocal lattice, and the second determined by the breakdown probability.

According to Slutskin [5], at W ≈ 1 the first contribution is given by

$$y_1 = P_{x0} bc / e\hbar H, \quad (7)$$

where P_{x0} is the generalized momentum of the electron and b is the reciprocal-lattice period. When the angle α changes, the initial phase of the oscillations ρ_{yy}^{osc} increases, since $b(\alpha) = b(0°)/\cos\alpha$, and the initial phase of the oscillations ρ_{xx}^{osc} remains unchanged. Thus,

$$\Delta\theta = \frac{P_{x0} c b(0°)}{e\hbar H} \frac{1}{\cos\alpha},$$

which is in qualitative agreement with Fig. 10, and a numerical estimate of the coefficient at $1/\cos\alpha$ agrees with the experimentally observed value.

As noted above, the initial phase of the oscillations also depends on the breakdown probability. If W ≪ 1, then

$$y_2 = {}^1/_4\pi + {}^1/_2\gamma - {}^1/_2\gamma \ln(\gamma/2) + \arg\Gamma(i\gamma/2), \quad (8)$$

where $\gamma = c\hbar d_{max}/eH$. This second contribution should lead to a change in the experimentally observed oscillation frequency with changing field. If γ > 1 (for beryllium this condition is satisfied at H < 100 kOe), then $y_2 \approx 0.17/\gamma$, and an estimate for the change of the frequency in a field ≈30 kOe yields $\Delta F_1 \approx 10^{-5}$ F.

Unfortunately, observation of such small frequency changes is complicated in the case of beryllium by low-frequency beats due to the formation of diamagnetic domains [16] (the corresponding value is $\Delta F_2 \approx 10^{-3}$F).

The authors consider it their pleasant duty to thank A. A. Slutskin and M. I. Kagonov for a discussion of the results.

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1)The purest samples were graciously furnished us by V. E. Ivanov and B. G. Lazarev, for which the authors express their gratitude.

2)The amplitude of the U_{yx} oscillations was maximal at α = 2°.

3)Sellmyer et al. [15] determined H_0 from the field dependence of the monotonic part of ρ_{xx}. Unfortunately, only the first term of the expansion of σ_{yx} in the field, connected with the unbalance, was taken into account in [15]. As indicated above, however, in weak fields (in the absence of breakdown) this term is much smaller than the sum of the next terms of the series (1). Therefore, the value of the breakdown field $H_0 \approx 30$ kOe given in [15] cannot be regarded as correct.

4)A somewhat smaller value of the phase jump may be due to the difference between the mechanisms that lead to a lowering of the symmetry, for example, to a nonuniform distribution of the dislocations or to formation of a diamagnetic domain structure in the sample [16] (an estimate based on the work of Privorotskiĭ and Azbel' [17] gives a domain-structure period ≈0.1 mm, which is commensurate with the sample dimensions).

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