Intensity of stimulated Mandel'shtam–Brillouin scattering in a plasma

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SMBS in a plasma is considered under conditions of applicability of the hydrodynamic approximation with allowance for nonlinearity of the excited sound waves. It is shown that the intensity of the scattered radiation may be significant even near the threshold.

In the investigation of the interaction of a powerful electromagnetic radiation with a plasma, one of the principal questions is the intensity of the stimulated Mandel'shtam-Brillouin scattering (SMBS). At the present time, on the basis of general concepts concerning parametric instabilities, there is a sufficiently well developed linear theory describing the initial stage of this phenomenon [1]. It is impossible, however, to construct a complete picture of the SMBS without developing a theory that describes the establishment of that quasistationary state which results from the development of the instability. Considerable progress has been made towards developing such a theory in a number of papers, with account taken of linear [2,3] and nonlinear [4] effects[1)].

In the present paper we consider SMBS under conditions when the quasistationary state is the result of generation of the second harmonic of an acoustic wave that is absorbed in the plasma. The possibility of such a process was indicated in [10], and was considered in [11–13] as applied to current and drift instabilities.

It is shown that under the considered conditions the SMBS intensity can be appreciable even at small excesses over the instability threshold. The time of establishment of the quasistationary state and the effect of collision frequency characterizing the loss of energy by the pump wave have been determined.

1. INITIAL RELATIONS

Our analysis is based on the system of hydrodynamic equations for a plasma in a strong high-frequency field [14], which we supplement with terms that take dissipations into account

$$\partial N/\partial t + \operatorname{div}(N\mathbf{V}) = 0, \tag{1.1}$$

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\mathbf{V} = -\nu_i \mathbf{V} - s^2\nabla \ln N - z\frac{e^2}{2mm_i}\nabla\left\langle\left[\int^t dt'\,\mathbf{E}(t',\mathbf{r})\right]^2\right\rangle, \tag{1.2}$$

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} + c^2 \operatorname{rot}\operatorname{rot}\mathbf{E} + \frac{4\pi e^2}{m}N\mathbf{E} = \frac{4\pi e^2}{m}\nu_e N\int^t dt'\,\mathbf{E}(t',\mathbf{r}). \tag{1.3}$$

where N is the electron density, **V** is the plasma velocity, $s^2 \equiv (zT_e + T_i)m_i^{-1}$, z is the ion charge, **E** is the intensity of the high-frequency electric field, ν_i is the frequency of the collision of the ions with the neutral particles or with ions of a different sort, and ν_e is the frequency of the collisions of the electrons with ions and with neutral particles.

We can eliminate N from (1.1) and (1.2) and write down an equation for **V** [13]. We confine ourselves to allowance for the nonlinear interaction of only the low-frequency waves, and neglect nonlinear dissipative effects. As a result we obtain

$$\left(\frac{\partial^2}{\partial t^2} - s^2\Delta + \nu_i\frac{\partial}{\partial t}\right)\mathbf{V} = -\frac{ze^2}{2mm_i}\nabla\frac{\partial}{\partial t}\left\langle\left[\int^t dt'\,\mathbf{E}(t',\mathbf{r})\right]^2\right\rangle - \frac{\partial}{\partial t}(\mathbf{V}\nabla)\mathbf{V} - \nabla\left(\mathbf{V}\frac{\partial \mathbf{V}}{\partial t}\right). \tag{1.4}$$

In (1.4) we take into account only the largest nonlinear terms, which are quadratic in **V**.

We use the system (1.1), (1.3), and (1.4) to analyze small deviations from the ground state, characterized by a constant electron density N_0 and a pump wave with a specified amplitude

$$\mathbf{E}_0(\mathbf{r},t) = {}^1/_2(\mathbf{E}_0 e^{-i\omega_0 t + i\mathbf{k}_0\mathbf{r}} + \mathbf{E}_0^* e^{i\omega_0 t - i\mathbf{k}_0\mathbf{r}}),$$

We then obtain for the Fourier components of the quantities

$$\delta N = N - N_0, \quad \delta\mathbf{V} = \mathbf{V}, \quad \delta\mathbf{E} = \mathbf{E} - \mathbf{E}_0.$$

the following relations

$$\delta\mathbf{E}(\omega,\mathbf{k}) = \frac{\omega_{Le}^2}{2k^2N_0}\left\{\delta N_+\left(\frac{\mathbf{k}(\mathbf{k}\mathbf{E}_0)}{\omega^2\varepsilon(\omega)} + \frac{[\mathbf{k}[\mathbf{k}\mathbf{E}_0]]}{k^2c^2 - \omega^2\varepsilon(\omega)}\right) + \delta N_-\left(\frac{\mathbf{k}(\mathbf{k}\mathbf{E}_0^*)}{\omega^2\varepsilon(\omega)} + \frac{[\mathbf{k}[\mathbf{k}\mathbf{E}_0^*]]}{k^2c^2 - \omega^2\varepsilon(\omega)}\right)\right\}, \tag{1.5*}$$

$$\varepsilon_s(\omega,k)\,\delta\mathbf{V}(\omega,\mathbf{k}) = -\frac{ze^2}{2mm_i}\frac{\mathbf{k}}{\omega\omega_0^2}(\mathbf{E}_0^*\delta\mathbf{E}_- + \mathbf{E}_0\delta\mathbf{E}_+) + \frac{1}{\omega}\int d\omega'\,d\mathbf{k}'\Big[(\mathbf{k}\,\delta\mathbf{V}(\mathbf{k}-\mathbf{k}',\omega-\omega'))\,\delta\mathbf{V}(\omega',\mathbf{k}') + \frac{\mathbf{k}}{2}(\delta\mathbf{V}(\omega',\mathbf{k}')\,\delta\mathbf{V}(\omega-\omega',\mathbf{k}-\mathbf{k}'))\Big], \tag{1.6}$$

$$\delta N(\omega,\mathbf{k}) \approx \frac{N_0}{\omega}\mathbf{k}\,\delta\mathbf{V}(\omega,\mathbf{k}), \tag{1.7}$$

where

$$\delta N_\pm = \delta N(\omega \pm \omega_0, \mathbf{k} \pm \mathbf{k}_0), \quad \delta\mathbf{E}_\pm = \delta\mathbf{E}(\omega \pm \omega_0, \mathbf{k} \pm \mathbf{k}_0),$$
$$\varepsilon(\omega) = 1 - \frac{\omega_{Le}^2}{\omega^2} + i\frac{\nu_e\omega_{Le}^2}{\omega^3}, \quad \varepsilon_s(\omega,k) = 1 - \frac{k^2s^2}{\omega^2} + i\frac{\nu_i}{\omega}. \tag{1.8}$$

Introducing the potential $\delta\mathbf{V} = \nabla\Phi$, we obtain from (1.5)–(1.7) one equation for $\Phi(\omega, \mathbf{k})$:

$$D(\omega,\mathbf{k})\,\Phi(\omega,\mathbf{k}) = \int d\omega'\,d\mathbf{k}'\,Q(\omega,\mathbf{k};\omega',\mathbf{k}')\,\Phi(\omega-\omega',\mathbf{k}-\mathbf{k}')\,\Phi(\omega'\,\mathbf{k}'), \tag{1.9}$$

where

$$D(\omega,\mathbf{k}) = \varepsilon_s(\omega,k) + \frac{ze^2}{4mm_i}\frac{\omega_{Le}^2k^2}{\omega_0^2\omega^2}\left[\frac{|\mathbf{k}_+\mathbf{E}_0|^2}{k_+^2\omega_+^2\varepsilon_+} + \frac{|[\mathbf{k}_+\mathbf{E}_0]|^2}{k_+^2(\omega_+^2\varepsilon_+ - k_+^2c^2)} + \frac{|\mathbf{k}_-\mathbf{E}_0|^2}{k_-^2\omega_-^2\varepsilon_-} + \frac{|[\mathbf{k}_-\mathbf{E}_0]|^2}{k_-^2(\omega_-^2\varepsilon_- - k_-^2c^2)}\right], \tag{1.10}$$
$$Q(\omega,\mathbf{k};\omega'\mathbf{k}') = \frac{i}{\omega k^2}\mathbf{k}'(\mathbf{k}-\mathbf{k}')\left(\mathbf{k}\mathbf{k}' + \frac{1}{2}k^2\right),$$

with $\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{k}_0$, $\omega_\pm = \omega \pm \omega_0$, $\epsilon_\pm = \epsilon(\omega_\pm)$.

The nonlinear equation (1.9) in the stationary state, when **k** and ω are real, are equivalent to the two equations

$$\Phi(\omega,\mathbf{k})[D(\omega,\mathbf{k}) + D^*(\omega,\mathbf{k})] = \int d\omega'\,d\mathbf{k}'[Q(\omega,\mathbf{k};\omega',\mathbf{k}') + Q^*(-\omega,-\mathbf{k};\omega'-\omega,\mathbf{k}'-\mathbf{k})]\,\Phi(\omega-\omega',\mathbf{k}-\mathbf{k}')\,\Phi(\omega',\mathbf{k}'),$$
$$\Phi(\omega,\mathbf{k})[D(\omega,\mathbf{k}) - D^*(\omega,\mathbf{k})] = \int d\omega'\,d\mathbf{k}'[Q(\omega,\mathbf{k};\omega',\mathbf{k}') - Q^*(-\omega,-\mathbf{k};\omega'-\omega,\mathbf{k}'-\mathbf{k})]\,\Phi(\omega-\omega',\mathbf{k}-\mathbf{k}')\,\Phi(\omega',\mathbf{k}'),$$

from which we can obtain both the dispersion law $\omega(\mathbf{k})$ and the wave amplitudes $\Phi(\omega, \mathbf{k})$.

2. LINEAR THEORY

If the nonlinear terms are disregarded in (1.9), then this equation reduced to the dispersion equation $D = 0$. We shall henceforth be interested in the near-threshold region, when the weak-coupling approximation [14] is valid and the solution of the dispersion equation can be sought in the form

$$\omega = ks(1+\Delta), \quad |\Delta| \ll 1.$$

We introduce the notation

$$\gamma_1 = \frac{\nu_i}{2ks}, \quad \gamma_2 = \frac{\nu_e \omega_{Le}^2}{2\omega_0^2 ks}, \quad \eta = 1 + \frac{c^2(k - 2k_0\cos\chi)}{2s\omega_0},$$

$$a = \frac{\omega_{Li}^2 |v_E|^2}{16\omega_0 ks^3}\left(1 - \frac{k^2}{k_0^2}\cos^2\varphi\right),$$

where $\mathbf{v}_E = e\mathbf{E}_0/m\omega_0$, $\cos\chi = \mathbf{k}\mathbf{k}_0/kk_0$, $\cos\varphi = \mathbf{k}\mathbf{E}_0/kE_0$. The quantity Δ is determined by setting (1.10) equal to zero:

$$(\Delta + i\gamma_1)(\Delta + i\gamma_2 + \eta) + a = 0.$$

Solutions that increase with time ($\operatorname{Im}\Delta > 0$) arise only if

$$\eta^2 < (a - \gamma_1\gamma_2)(\gamma_1 + \gamma_2)^2/\gamma_1\gamma_2 < 1. \tag{2.1}$$

From the inequality (2.1) we obtain the following limitations on the region of unstable wave vectors:

$$k - 2k_0\cos\chi + 2\frac{s\omega_0}{c^2} < \left|\frac{\omega_0}{2k_0c^2\cos\chi}\left(\nu_i + \nu_e\frac{\omega_{Le}^2}{\omega_0^2}\right)\left[\frac{|v_E|^2}{|v_E|^2_{\text{пор}}}\cos\chi(1 - 4\cos^2\chi\cos^2\varphi) - 1\right]^{1/2}\right|, \tag{2.2}$$

where

$$|v_E|^2_{\text{пор}} = 2s\nu_i\nu_e\omega_{Le}^2/\omega_0\omega_{Li}^2 k_0. \tag{2.3}$$

Formula (2.3) determines the minimal threshold field.

It is seen from (2.2) that at specified angles χ and φ and at a specified field intensity there exists in the pump wave a narrow interval of absolute values of wave vectors of unstable waves. Thus, at $\chi = 0$ and $\varphi = \pi/2$ the width Δk of this interval is maximal and equal to

$$\Delta k = \frac{\omega_0}{c^2 k_0}\left(\nu_i + \nu_e\frac{\omega_{Le}^2}{\omega_0^2}\right)\sqrt{\varepsilon}, \tag{2.4}$$

where $\epsilon \equiv (|v_e|^2/|v_E|^2_{\text{thr}}) - 1$ characterizes the excess over the threshold field (2.3). According to (2.4), the interval Δk is sufficiently narrow ($\Delta k \ll k_0$) even at $\epsilon \gg 1$.

If we assume as before that the amplitude of the pump wave is given, then we can obtain from (2.2) the angles at which the instability set in:

$$\frac{|v_E|^2}{|v_E^5|^2_{\text{thr}}}\cos\chi_0(1 - 4\cos^2\chi_0\cos^2\varphi_0) \geqslant 1.$$

Unlike the interval Δk, the region of instability with respect to the angles broadens rapidly with increasing ϵ. At a slight excess above the minimal threshold field (2.3), however, the waves are unstable in a narrow interval of angles $\chi_0 \approx \sqrt{2}\,\epsilon$.

It should be noted that in the case of other mechanisms of sound-wave dissipation, unstable waves can arise in a wide angle interval even at a slight excess over the threshold field.

3. NONLINEAR THEORY

Among the possible nonlinear effects, we are interested in the process of the onset of higher harmonical acoustic waves which draw energy from the unstable waves, stop their growth, and determine the quasistationary states.

We are interested in the near-threshold region ($\epsilon < 1$), when the acoustic waves are unstable in a small solid angle, so that the wave vectors and the frequencies are close respectively to the values $\mathbf{k} = (k_S = 2k_0, 0, 0)$ and $\omega = 2k_0 s \equiv \omega_S$. This makes it possible to use a δ-function approximation for the quantity $\Phi(\omega, k)$ and to confine the analysis to only the first two harmonics:

$$\Phi(\omega, k) = [\Phi_1\delta(\omega - \omega_s)\delta(k_x - k_s) + \Phi_1^*\delta(\omega + \omega_s)\delta(k_x + k_s) + \Phi_2\delta(\omega - 2\omega_s)\delta(k_x - 2k_s) + \Phi_2^*\delta(\omega + 2\omega_s)\delta(k_x + 2k_s)]\delta(k_y)\delta(k_z). \tag{3.1}$$

Substituting (3.1) in (1.9) and equating terms with identical δ-functions, we obtain

$$4\frac{k_0^2}{s^2}|\Phi_1|^2 = -D(\omega_s, k_s)D(2\omega_s, 2k_s).$$

Using (1.8) and (1.10), we get

$$|\Phi_1|^2 = s^2\nu_i^2\varepsilon/8k_0^2\omega_s^2. \tag{3.1}$$

With the aid of (1.7) we obtain from (3.2) expressions for the amplitudes of the perturbations of the electron densities at the fundamental frequency $|\delta N_1|$ and at the second harmonic $|\delta N_2|$:

$$|\delta N_1|^2/N_0^2 = \nu_i^2\varepsilon/2\omega_s^2, \quad |\delta N_2|^2/N_0^2 = 4\nu_i^2\varepsilon^2/\omega_s^2. \tag{3.3}$$

From the condition $|\delta N_2| < |\delta N_1|$, under which we are justified in using the assumption that there are only two harmonics of the acoustic wave, follows the limitation $\epsilon < 1/8$ on the excess-over-threshold parameter.

Formulas (3.3) and (1.5) enable us to find the amplitude of the scattered (Stokes) wave:

$$|\delta E_-|^2/|E_0|^2 = \omega_0^2\nu_i^2\varepsilon/8\nu_e^2\omega_s^2. \tag{3.4}$$

Since we have assumed that the scattered wave is stable, our analysis is valid at $|\delta E_-| < |E_0|$, and according to (3.4) we have

$$\varepsilon < 8\nu_e^2\omega_s^2/\nu_i^2\omega_0^2. \tag{3.5}$$

Let us dwell briefly on the process of establishment of a quasistationary state. Just as in [12], on going to the Fourier representation we take into account the slow dependence of $\Phi(\omega, \mathbf{k})$ on the time. As a result, we obtain from (1.1), (1.3), and (1.4) the following system of equations for the quantities in (3.1):

$$i\frac{2}{\omega_s}\frac{d\Phi_1}{dt} + D(\omega_s, k_s)\Phi_1 = i\frac{8k_0^2}{\omega_s}\Phi_1^*\Phi_2,$$

$$i\frac{1}{\omega_s}\frac{d\Phi_2}{dt} + D(2\omega_s, 2k_s)\Phi_2 = i\frac{2k_0^2}{\omega_s}\Phi_1^2.$$

It follows therefore (cf. [12]) that at $\epsilon < 1$ we have

$$|\Phi_1|^2 = [|\Phi_1(0)|^{-2}e^{-\gamma t} + |\Phi_1(\infty)|^{-2}(1 - e^{-\gamma t})]^{-1}, \tag{3.6}$$

where the quantity $\Phi_1(\infty)$ is defined by (3.2), and $\Phi_1(0)$ is the initial perturbation of the velocity potential. The characteristic time τ of establishment of the stationary state is

$$\tau = 1/\gamma = 1/\nu_i\varepsilon. \tag{3.7}$$

The Appendix gives a general expression for the effective collision frequency (A.2), which determines the energy lost by the pump wave. Using formulas (A.3), (1.8), (3.1) and (3.3), and recognizing that in our case there arises a scattered wave with frequency $\omega_0 - \omega_S$, we obtain from (A.2)

$$\nu_{\text{eff}} = \nu_i^2\omega_0^2\varepsilon/2\nu_e\omega_s^2. \tag{3.8}$$

From a comparison of formulas (3.4) and (3.8) it follows that $\nu_{\text{eff}}/\nu_e = \frac{1}{4}|\delta E_-|^2/|E_0|^2 < 1$.

4. CONCLUSION

Under real conditions the SMBS intensity is determined by those processes that ensure the most effective energy removal from the unstable waves. One of the main parameters characterizing the effectiveness of one process over another is the time of establishment of the quasistationary state.

For the process considered in the present paper, at $\omega_0 = 10^{10}$ sec^{-1}, $\nu_e = 10^6$ sec^{-1}, $\nu_i = 10^5$ sec^{-1}, $\omega_S = 3.6\times 10^7$ sec^{-1}, and $\epsilon = 0.1$ we obtain $\tau = 10^{-4}$ sec in accordance with (3.7). From formulas (3.3), (3.4), and (3.8) we then obtain $|\delta N_1|^2/N_0^2 = 5\times 10^{-7}$, $|\delta E_-|^2/|E_0|^2 = 0.03$, $\nu_{eff}/\nu_e \approx 0.01$. Attention is called to the fact that sufficiently strong scattered fields are produced even near the threshold, at a relatively small perturbation of the electron density. This is due to the fact that the perturbations of the transverse fields add up coherently in the large region of space, equal in order of magnitude to the quantity c/ν_e, so that $|\delta E_-|/|E_0| \sim (\omega_0/\nu_e)(|\delta N|/N_0)$.

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APPENDIX

EFFECTIVE COLLISION FREQUENCY

The energy lost by the electromagnetic wave as a result of scattering and transformation in a turbulent medium can be described with the aid of an effective collision frequency ν_{eff}, which is connected with the extinction coefficient h [15]. In a unit volume and a unit time, an energy $Q_1 = hS$ is released in the form of scattered waves ($S = (c/8\pi)\sqrt{\epsilon(\omega_0)}\,|E_0|^2$ is the energy flux density of the incident wave), and the energy released as a result of electronic collisions is

$$Q_2 = \frac{1}{8\pi}|E_0|^2 \frac{\omega_{Le}^2}{\omega_0^2}\nu_e.$$

To be able to express Q_1 in a form similar to Q_2, we must introduce the collision frequency

$$\nu_{eff} = c\sqrt{\varepsilon(\omega_0)}\, h\omega_0^2/\omega_{Le}^2. \qquad (A.1)$$

If the scattering in an isotropic medium is by slow fluctuations (with a small change of frequency), then the quantity h, neglecting the spatial dispersion, is expressed in terms of the fluctuations of the dielectric constant $\delta\epsilon(\mathbf{r}, t)$ [15], and from (A.1) we have

$$\nu_{eff} = \frac{\omega_0^3}{\omega_{Le}^2}\int d\omega\, d\mathbf{k}(\delta\varepsilon^2)_{\omega,\mathbf{k}}\left\{\frac{[\mathbf{k}+\mathbf{k}_0, \mathbf{e}]^2}{(\mathbf{k}+\mathbf{k}_0)^2}\right.$$
$$\left.\times \mathrm{Im}\left(\varepsilon^*(\omega+\omega_0) - \frac{c^2(\mathbf{k}+\mathbf{k}_0)^2}{(\omega+\omega_0)^2}\right)^{-1} + \frac{(\mathbf{k}+\mathbf{k}_0, \mathbf{e})^2}{(\mathbf{k}+\mathbf{k}_0)^2}\,\mathrm{Im}\frac{1}{\varepsilon^*(\omega+\omega_0)}\right\}, \qquad (A.2)$$

where $\mathbf{e} = \mathbf{E}_0/E_0$ is the polarization vector and $(\delta\epsilon^2)_{\omega k}$ is the spectral density of the square of the fluctuations of the high-frequency dielectric constant, which is connected with the spectral density of the fluctuations of the electron density $(\delta N_e^2)_{\omega k}$ by the relation

$$(\delta\varepsilon^2)_{\omega,\mathbf{k}} = \frac{\omega_{Le}^4}{\omega_0^4}\frac{(\delta N_e^2)_{\omega,\mathbf{k}}}{N_0^2}. \qquad (A.3)$$

In formula (A.2), the first term in the curley brackets describes the loss of pump-wave energy as a result of the scattering (the appearance of transverse fields), while the second term characterizes the transformation (the appearance of longitudinal fields), and it is this term which determines mainly the heating of the plasma.

The electron-density fluctuations can be due to various causes. In particular, they can be produced by the scattered wave itself, owing to parametric instabilities. In this case the slow electron-density perturbations in a plasma are connected with the ion-acoustic waves [16]:

$$\frac{(\delta N_e)^2_{\omega,\mathbf{k}}}{N_0^2} = \frac{1}{2T_e N_0}\frac{W_s(\mathbf{k})}{(2\pi)^3}[\delta(\omega - ks) + \delta(\omega + ks)], \qquad (A.4)$$

where $W_S(k)$ is the spectral energy density of the ion-acoustic waves ($kr_{De} < 1$). From (A.2) we obtain with the aid of (A.3) and (A.4)

$$\nu_{eff} = \frac{\omega_{Le}^2}{\omega_0}\frac{1}{2T_e N_0}\int d\mathbf{k}\frac{W_s(\mathbf{k})}{(2\pi)^3}\left\{\frac{[\mathbf{k}+\mathbf{k}_0, \mathbf{e}]^2}{(\mathbf{k}+\mathbf{k}_0)^2}\right.$$
$$\times\left[\mathrm{Im}\left(\varepsilon^*(\omega_0+ks) - \frac{c^2(\mathbf{k}+\mathbf{k}_0)^2}{(\omega_0+ks)^2}\right)^{-1} + \mathrm{Im}\left(\varepsilon^*(\omega_0-ks) - \frac{c^2(\mathbf{k}+\mathbf{k}_0)^2}{(\omega_0-ks)^2}\right)\right]^{-1}$$
$$\left. + \frac{(\mathbf{k}+\mathbf{k}_0, \mathbf{e})^2}{(\mathbf{k}+\mathbf{k}_0)^2}\left[\mathrm{Im}\frac{1}{\varepsilon^*(\omega_0+ks)} + \mathrm{Im}\frac{1}{\varepsilon^*(\omega_0-ks)}\right]\right\}. \qquad (A.5)$$

If we consider only the transformation of the waves, put $\mathbf{k}_0 = 0$, and assume the condition $\mathrm{Re}\,\epsilon(\omega_0 - ks) = 0$ to be satisfied, then we obtain from (A.5) an expression corresponding to that used in [6] (see also [17-19]).

*$[\mathbf{kE}_0] \equiv \mathbf{k} \times \mathbf{E}_0$.

1)We note that in a number of papers [5-9] they investigated a nonlinear stage of parametric instability of a different type, when Langmuir and sound waves are the growing waves (unlike in SMBS, where transverse waves and sound grow). In [5-8] account was taken of the nonlinear interaction of the wave due to the induced scattering by ions, while in [9] account was taken of the nonlinear frequency shift of the Langmuir waves.

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