The dynamics of plasma-column constrictions and the electromagnetic acceleration of ions

V. S. Imshennik, S. M. Osovets, and I. V. Otroshchenko
Institute of Applied Mathematics, USSR Academy of Sciences
(Submitted November 27, 1972)
Zh. Eksp. Teor. Fiz. **64**, 2057-2071 (June 1973)

Another attempt is made in this paper to explain the effect observed long ago of neutron emission by a rapidly contracting Z-pinch. It is postulated that strong electric fields of inductive nature are produced in the constrictions due to the development of $m = 0$ instabilities. The constriction-development calculations carried out in the magnetohydrodynamic approximation show that the potential differences of the electric field are nearly equal to the energy of the accelerated ions observed in experiments. A novel feature of the present formulation of the problem is the allowance for the effect of plasma outflow from the constriction region. This effect is computed with the aid of a two-dimensional system of MHD equations in which all the quantities are radius-profiled in a way suitable for the description of the cumulative phase of the pinch effect. Owing to the outflow of plasma (1—10% of the initial number of particles remains in a constriction at the time of maximum over-voltage), the potential differences attain fairly large values and, also in spite of the presence of a transverse magnetic field, the main condition for direct acceleration of the ions is fulfilled. Electron acceleration, on the other hand, is impeded by this magnetic field. Discussed in the paper are other possible electron-acceleration mechanisms primarily connected with effects which do not fall within the framework of the hydrodynamic treatment and which could explain the appearance of hard x-rays. Such radiation, whose quanta have energies close to the energy of the accelerated ions, is usually observed simultaneously with the neutron radiation.

INTRODUCTION

The effect whereby hard radiation is emitted during the passage of strong currents through a gas discharge in a cylindrical discharge chamber was discovered in 1952. It turned out that for currents $I > 100$ kA, the rapidly contracting plasma column emitted hard x-ray quanta with energies of up to several hundred keV. If, on the other hand, the gas-discharge container was filled with deuterium, then under certain conditions neutron emission was observed as a result of the nuclear reaction

$$d + d = \mathrm{He}^3 + n.$$

These investigations have been described in detail by Artsimovich and his co-authors in[1] and subsequently in Artsimovich's book[2]. It became clear almost immediately after the discovery of this phenomenon that the neutron emission could not be the result of thermonuclear reactions, but owed their origin to a small group of charged particles that got accelerated to energies many times exceeding their temperature[3]. Subsequent experiments (see, for example,[4]) showed that there indeed appear in such a discharge ions that have been accelerated to energies right up to 200 keV. The existence time for the neutron and x-ray emission bursts is very small compared to the existence time of the plasma column and more often pertains to instants close to the second "singularity" on the current curve, i.e., to the time corresponding to maximum compression of the plasma column. In this case several radiation bursts are sometimes observed if, as is often the case, several compressions take place.

It is important to note that the ion- and electron-energy spectra (the latter is measured according to the hardness of the x-rays) roughly coincide. Also close are the values of the maximum ion and electron energies. This gives grounds to suppose that the acceleration of the particles is due to the appearance of local electric fields that are so strong that the potential differences across some distances that are short compared to the length of the column attain hundreds of thousands of electron volts. Experiments designed to locate the place of origin of these radiations (see, for example,[5]) indicate that the accelerated particles do indeed originate from a small region, while measurements pertaining to the spatial distribution of the accelerated particles (of the neutrons and x-ray quanta)[4] give grounds to assert that the electric fields effecting the accelerations are directed along the axis of the column, in such a way that the ions are accelerated towards the cathode while the electrons are accelerated towards the anode.

It was observed shortly after the discovery of the emission effect that the appearance of the radiations is always accompanied by a strongly pronounced nonuniformity in the plasma column along the z axis. The manifestations of the $m = 0$ instability (constrictions) are clearly visible on ultrahigh-speed photographs[6]. Certain propositions about the connection between these phenomena were put forward at the same time. As far as we know, the first attempt to explain the appearance of the accelerated particles in connection with the development of constriction-type instabilities was Trubnikov's work[7]. In other investigations (see, for example,[8]), similar arguments were made and certain computations were also carried out, from which followed possibility of the appearance of local surges, although these surges were, as a rule, inadequate for the explanation of the effect. However, in all these investigations the process of the development of the plasma-column instabilities is itself almost not considered, the treatment being more qualitative than quantitative in nature. The quantitative relations determining the acceleration effect and the conditions for the appearance of this effect remain obscure. In the present paper we shall attempt to more consistently relate the phenomena of column instabilities and particle acceleration by devel-

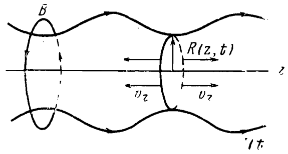

FIG. 1. A qualitative picture of a plasma column with a growing m = 0 instability.

oping a magnetohydrodynamic (MHD) theory of constrictions in a plasma column.

We shall first present a qualitative picture with a view to subsequently formulating the problem in a mathematical form. Let us consider a plasma column which is of the shape shown in the Fig. 1 and which is contracting under the action of the magnetic field of the intrinsic current. At a place of minimum cross section there will be a gas-pressure differential along the axis of the column, in consequence of which particle fluxes will move in both directions from this cross section with some velocity v_z. As a result of this, the total number of particles in the cross section of the constriction will decrease, whereas the magnetic field (and with it the magnetic pressure) will, for a given total current, increase at this place in proportion to the contraction. A two-dimensional MHD theory of such a process was developed for the experiments with the plasma focus[9,10], where the axial nonuniformity of the column was purposely created through the initial conditions. It was established in numerical computations that not more than 10% of the plasma remains in a constriction cross section as compared to the initial quantity.

This effect of plasma efflux from a constriction region was not at all taken into account in[7,8], although it apparently is the decisive factor in the process of the formation of local overvoltages. The question arises as to the duration of the outflow and as to whether such conditions are possible under which so much gas flows out of a constriction that there remains in the cross section only a minimum number N_{min} of electrons necessary for current transport:

$$N_{min} = I / ec. \qquad (1)$$

In this case a region of uncompensated space charge would have been created, with the result that further contraction would have been impeded by the electric field and equilibrium would have set in when the radius R of the plasma column was equal to

$$E_r(R) = B_\varphi(R) = 2I / cR. \qquad (2)$$

Such a regime would have corresponded to the appearance in the circuit of an overvoltage sufficient for the acceleration of the particles to very high energies.

It would appear that such conditions could be realized in a real system. Further MHD computations that were carried out showed, however, that a rupture of the column with total expulsion of the plasma from the constriction region does not occur under all the conditions considered by us. In the computations the plasma does not have time to sufficiently intensely flow out of a constriction, so that the number of particles at maximum compression is still many times larger than it should, according to the formula (1), be in the limiting case. Therefore, the maximum electric fields in a constriction region are by far weaker than the magnetic fields (contrary to the relation (2)) and the space charge here is certainly compensated.

Nevertheless, one important circumstance should be stressed. The above-indicated limiting case of the plasma efflux evidently falls outside the region of applicability of the MHD approximation used in the performed computations. It is also perfectly clear that the obtained result, $N \gg N_{min}$, does not by itself demonstrate the validity of the MHD approximation. The conditions of applicability of the MHD theory can be violated in a constriction cross section even when $N \gg N_{min}$, in which case the equations of magnetohydrodynamics should, beginning from some instant, be replaced by the kinetic equations.

In principle, the outflow of plasma can turn out to be more intense in a more correct kinetic treatment, and doubt can be cast on the above-drawn conclusion that the limiting case is unrealizable. As the following discussion shows, the experimental data definitely indicate an especially important role played by the kinetic phase of the process in the electron acceleration. At the same time ion acceleration can be fully explained in the framework of the MHD theory without invoking additional arguments.

1. THE PROFILING OF THE EQUATIONS OF NONSTATIONARY TWO-DIMENSIONAL MAGNETOHYDRODYNAMICS. THE HYDRODYNAMIC EQUATIONS OF THE PLASMA CONSTRICTION

Although the complete system of equations of the two-dimensional nonstationary problem of magnetohydrodynamics has now been successfully solved[9,10], the approximate profiling of the MHD quantities, which allows us to reduce the complete plasma-column problem to the one-dimensional nonstationary problem, is still useful. The advantages of the one-dimensional formulation of the problem over the two-dimensional formulation are quite obvious (simplicity of realization of the numerical solution, etc.). Comparison of the solutions of the one-dimensional problems with the available solutions of the complete two-dimensional problem will, in principle, enable us to appraise the validity and accuracy of the approximate profiling. As an example of the profiling of the equations of the plasma column, we may cite the well-known "snowplow" model generalized to the two-dimensional case[11]. It is, however, not possible to describe in the framework of such a model the late phase of the contraction and the development of the process in the region near the axis of the plasma column, and, in particular, it is impossible to obtain from it the effect of particle efflux from a constriction region. Another method of profiling is therefore proposed here.

We can assume as a basis of this method the fact that after the focusing and reflection of the shock wave from the axis, the radial distributions of all the MHD quantities become smooth. In the most interesting case of strong skin effect, we can assume that the density ρ, the pressure p, and the axial velocity v_z of the plasma do not at all depend on the radius, while the radial velocity v_r varies linearly with the radius. Inside the extremely thin skin layer, the quantities ρ and p decrease to zero, while the magnetic field B increases from zero to its boundary value (Fig. 2). Naturally, the local satisfaction of the MHD equations is not required for the profiling[1)]. It is sufficient to satisfy the known boundary conditions, while the dependence on the radius can be given somewhat differently from what was

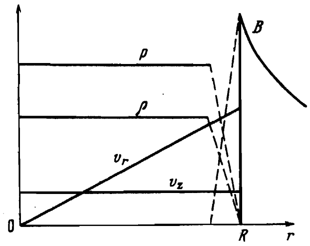

FIG. 2. Scheme for the profiling of the MHD quantities with respect to the radius of the column.

presented above. In the case of weak skin effect, a quadratic dependence of ρ and a linear depenence of B on the radius are, perhaps, more admissible. For the velocities, we can retain the dependences indicated above. In the final analysis, the radius profiling of all the quantities are physically justified if the characteristic radial dimension is considerably less than the characteristic dimension along the axis. Then the detailed dependence of the quantities satisfying the known boundary conditions should not significantly influence the solution of the resulting problem with the independent variables z and t. For example, for the above-indicated cases of strong and weak skin effect, the numerical coefficients in the equations then change only slightly.

Thus, let us assume the following r dependences for all the quantities:

$$v_r = U(z,t)\frac{r}{R(z,t)},\quad v_z = V(z,t),\quad \rho = \rho(z,t),\quad p = p(z,t),\quad B = 0. \tag{3}$$

Inside the skin layer, ρ and p decrease to zero, while B increases from zero to the boundary value B_R. By substituting the functions v_r and v_z from (3) into the system of MHD equations that have been written for the axially symmetric case and integrating all the equations with respect to r from 0 to R(z, t) with allowance for the assumed dependences of ρ, p, and B, we obtain

$$\frac{\partial N}{\partial t} + V\frac{\partial N}{\partial z} + N\frac{\partial V}{\partial z} = 0, \tag{4}$$

$$\frac{\partial R}{\partial t} + V\frac{\partial R}{\partial z} = U, \tag{5}$$

$$\frac{\partial U}{\partial t} + V\frac{\partial U}{\partial z} = \frac{3\pi}{M}\frac{pR}{N} - \frac{3}{8M}\frac{B_R^2 R}{N}, \tag{6}$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial z} = -\frac{\pi}{MN}\frac{\partial}{\partial z}(pR^2) + \frac{1}{4M}\frac{RB_R^2}{N}\frac{\partial R}{\partial z}. \tag{7}$$

The continuity equation (4) contains the linear plasma density $N = \pi\rho R^2/M$, where M is the mass of an ion. Equation (5) is obtained by integrating the equation of the frozen-in magnetic field. The essence of this equation is simple, the substantial derivative of the radius of the column is equal to the radial velocity of the matter at the boundary.

Further development of the model is tied with the choice of the equation of state $p(\rho, T)$ and the law according to which the electric current I(t), which determines the quantity $B_R = 2I/cR$, varies with the time. The pressure inside the plasma can be approximated by two limiting procedures: the adiabatic procedure, in which case $p = p_0(\rho/\rho_0)^\gamma$, and the isothermal procedure, in which case $p = 2k\rho T(t)/M$, so that the plasma temperature does not depend on the z coordinate. In the adiabatic case, the four unknown functions N, R, U, and V are uniquely determined by Eqs. (4)—(7) together with the initial and boundary conditions. In the isothermal case, to determine the plasma temperature, we use in addition the integral energy conservation law in which the absence of leakage or influx of energy into the plasma region due to thermal conduction is implied.

Let us choose dimensionless variables in the following fashion:

$$\zeta = z/R_0,\quad \tau = t/t_0,\quad u = Ut_0/R_0,\quad v = Vt_0/R_0,$$
$$n = N/N_0,\quad \sigma = R/R_0,\quad i = I/I_0. \tag{8}$$

Let us define the unit time t_0 thus:

$$t_0 = \frac{cR_0}{I_0}\left(\frac{N_0 M}{3}\right)^{1/2}. \tag{9}$$

The system of equations (4)—(7) in the dimensionless form and with allowance for the relations (8) and (9) assumes, in the adiabatic case, the form:

$$\frac{\partial n}{\partial \tau} + v\frac{\partial n}{\partial \zeta} + n\frac{\partial v}{\partial \zeta} = 0, \tag{10}$$

$$\frac{\partial \sigma}{\partial \tau} + v\frac{\partial \sigma}{\partial \zeta} = u, \tag{11}$$

$$\frac{\partial u}{\partial \tau} + v\frac{\partial u}{\partial \zeta} = \frac{\varepsilon_0}{2}\frac{n^{\gamma-1}}{\sigma^{2\gamma-1}} - \frac{1}{2}\frac{i^2}{\sigma n}, \tag{12}$$

$$\frac{\partial v}{\partial \tau} + v\frac{\partial v}{\partial \zeta} = -\frac{1}{6}\frac{\varepsilon_0}{n}\frac{\partial}{\partial \zeta}\frac{n^\gamma}{\sigma^{2\gamma-1}} + \frac{1}{3}\frac{i^2}{\sigma n}\frac{\partial \sigma}{\partial \zeta}. \tag{13}$$

Entering into Eqs. (12) and (13) is the dimensionless parameter ϵ_0 which has the meaning of a characteristic ratio of the gas and magnetic pressures:

$$\varepsilon_0 = 6\pi p_0\frac{t_0^2}{MN_0} = 8\pi p_0\frac{1}{B_{R_0}^2}. \tag{14}$$

For the isothermal case with the dimensionless temperature

$$\theta = \frac{t_0^2 k}{R_0^2 M}T$$

the equations of motion have a somewhat different form, and, furthermore, into the system enters the supplementary integral relation (17):

$$\frac{\partial u}{\partial \tau} + v\frac{\partial u}{\partial \zeta} = \frac{6\theta}{\sigma} - \frac{1}{2}\frac{i^2}{\sigma n}, \tag{15}$$

$$\frac{\partial v}{\partial \tau} + v\frac{\partial v}{\partial \zeta} = -\frac{2\theta}{n}\frac{\partial n}{\partial \zeta} + \frac{1}{3}\frac{i^2}{\sigma n}\frac{\partial \sigma}{\partial \zeta}, \tag{16}$$

$$3\frac{d\theta}{d\tau} = \left\{-\frac{1}{3}i^2\int\frac{d\zeta}{\sigma}\left(u - v\frac{\partial\sigma}{\partial\zeta}\right) - \frac{1}{2}\frac{\partial}{\partial\tau}\int d\zeta\, n\left(\frac{1}{2}u^2 + v^2\right)\right\} \Big/ \int n\, d\zeta. \tag{17}$$

The second integral in (17) should be expressed in terms of the space derivatives of the quantities, using the relations (10), (15), and (16), but because of the unwieldiness we shall not give (17) in its final form.

The initial conditions of the problem can be formulated in the simplest way if we neglect the inhomogeneity of the plasma and its motion at $\tau = 0$:

$$n = 1,\ u = 0,\ v = 0,\ \sigma = 1 - \delta_0 \exp(-\alpha_0\zeta^2),\ \theta = \theta_0, \tag{18}$$

where δ_0, α_0, and θ_0 are certain numbers. The small ($\delta_0 \ll 1$) deviation of the radius σ of the column from unity is a necessary small perturbation that develops at $\tau > 0$ into a plasma constriction.

The choice of the initial perturbation precisely in this way needs some justification. Figure 1 shows a typical shape of a contracting pinch at the beginning of the compression process. Such a periodic structure has been observed in ultrahigh-speed photographs: for example, in the photographs shown in[6]. These photographs were taken with a time resolution of 0.5 μsec between the frames, whereas the phenomena determining the maximum-compression process take place in times many times shorter. Therefore, the processes observed in such photographs are processes that have been "averaged" in time. Photographs of considerably higher time resolution taken with an image-converter tube with a light amplifier show a completely different picture (see[16], Fig. 2). It is evident here that although the plasma column has a more or less clearly expressed periodic structure at the beginning of the compression process, this structure is destroyed as the compression goes on. On account of some accidental causes, a "constriction" that contracts more rapidly than the others appears, so that at the moment of maximum compression of this constriction the contraction of the remaining part of the plasma column can be considered as far from being complete. Since in the final analysis we are interested in the processes close to the time of maximum compression of the constriction, we choose as the initial perturbation not a periodic function of z, but a solitary perturbation in a background of symmetrical cylindrical contraction.

As to the choice of the moment to be reckoned as zero time $\tau = 0$, this moment pertains not to the beginning of the discharge-development process, but to the moment of "detachment" of the plasma column from the walls of the discharge chamber. It is shown in[15] that the $m = 0$ instabilities (constrictions) in a fast Z-pinch develop as a result of a Rayleigh-Taylor type of instability. It turns out that such an instability develops just before the detachment and that the higher the current growth rate dI/dt up to the moment of detachment, the shorter the wavelength of the instability. Therefore, the initial values of all the quantities determining the process in question pertain precisely to this moment of time. Thus, R_0 is the radius of the discharge chamber, N_0 is the initial number of particles per unit length, and I_0 is the detachment current. The boundary conditions can be formulated quite simply, taking account of the symmetry of the constriction (18) with respect to the plane $\zeta = 0$:

$$\zeta = 0, \quad v = 0; \quad \zeta = \zeta_0, \quad v = 0 \quad (\tau > 0). \tag{19}$$

A rigid wall at $\zeta = \zeta_0$ is unimportant if $\zeta_0 \gg \alpha^{-1/2}$.

Thus, we are required to numerically solve Eqs. (10)–(13), or (10), (11), and (15)–(17) in the region $0 \le \zeta \le \zeta_0$ $(\tau > 0)$ with the initial conditions (18) and the boundary conditions (19). In a concrete solution it is also necessary to specify the numerical parameters δ_0, α_0, ζ_0, and ϵ_0 (or θ_0), as well as the dimensionless current in the form of a function $i(\tau)$ of the time.

2. DISCUSSION OF THE RESULTS OF THE NUMERICAL SOLUTION OF THE PROBLEM OF THE DYNAMICS OF THE PLASMA CONSTRICTION

Let us assume as typical physical conditions the following set of characteristic quantities:

$$R_0 = 10\,\text{cm}, \quad N_0 = 10^{17}\,\text{cm}^{-1}, \quad I_0 = 1.5 \cdot 10^5\,\text{A}. \tag{20}$$

According to (8) and (18), the initial radius of the column and the linear density figure in (20). The parameter ϵ_0 was set equal to 0.1. This number implies, in accordance with (14) and (20), that the initial plasma temperature $T_0 \approx 30$ eV, i.e., it has quite a reasonable value. Furthermore, it is easily understood that the results of the solution for small values of σ are only slightly sensitive to the quantity ϵ_0. The initial-perturbation parameters from (18) were given the values: $\delta_0 = 0.1$ and $\alpha_0 = 10$. Consequently, the "wavelength" of the initial perturbation in dimensionless variables was of the order of $\alpha^{-1/2} \sim 0.3$, or for the chosen value of $R_0 = 10$ cm, it was ~ 3 cm.

On the one hand, this value corresponds to the experimental data (see, for example,[15,16]) and, on the other, it agrees on the whole with the results of the theoretical analysis of the instability of the boundary of a plasma with a magnetic field. When the viscosity of the plasma is taken into account the wavelength of the instability with the maximum increment obtained in[13] and[14] turns out, under the conditions (20), to be only a little less than the value chosen above. This value also fits well into the framework of the ideas developed in[15] and corresponds to the semiempirical relation cited there and connecting the wavelength with other characteristic parameters. Finally, the parameter ζ_0 in (19) was equal to 3, i.e., it was not small enough to influence the constriction region. The weak dependence of the solution on the choice of the parameters being discussed was verified to some extent in the computations. Thus, we set:

$$\varepsilon_0 = 0.1, \ \alpha_0 = 10, \ \delta_0 = 0.1, \ \gamma = 1.67, \ \zeta_0 = 3. \tag{21}$$

The determination of the form of the current function $i(\tau)$ was a much more critical problem. Here a number of variants was solved with different current functions. Physically, the most interesting variants are the cases of the direct current $i(\tau) = 1$, of the linear dependence of the current on the time, i.e., $i(\tau) = \tau$, and of the self-consistent current when the variation of the inductive reactance of the column is taken into account:

$$i(\tau) = \left\{ \int_0^{\zeta_0} \ln \frac{\sigma_\infty}{\sigma(0,\zeta)} d\zeta + \lambda_0 \right\} \Big/ \left\{ \int_0^{\zeta_0} \ln \frac{\sigma_\infty}{\sigma(\tau,\zeta)} d\zeta + \lambda_0 \right\}. \tag{22}$$

The dependence (22) takes into account the main effect of the variable inductive reactance. The additive parameter λ_0 is introduced in order to take into account the actual length of the column, a length which clearly differs from the length ζ_0 of the computation region. Thereafter, we set $\sigma_\infty = 3$ and for λ_0 we assumed, in particular, the value $\lambda_0 = 6.6$, corresponding to a column "length" (here one can imply the contribution to the constant inductance of the circuit) of about 180 cm (λ_0 $\int_{\zeta_0}^{3\zeta_0} \ln \sigma_\infty d\zeta = 6 \ln 3 \approx 6.6$). Figure 3 shows the plots of $i(\tau)$ that were obtained in two variants of the computation with the self-consistent current (22) for the values $\lambda_0 = 6.6$ (upper curve) and $\lambda_0 = 0$ (lower curve).

It makes sense to take as the basic variant of the computation, bearing in mind the qualitative similarity of the whole series of computations with different $i(\tau)$ and different forms of the equation of state of the plasma, the adiabatic variant with the direct current

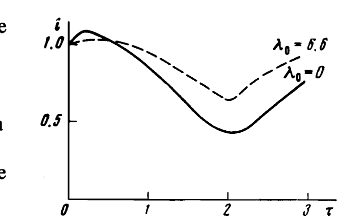

FIG. 3. The dependences of the total current i on the time τ (both quantities are expressed in dimensionless units). The inductive reactance is determined by the formula (22) with the parameter λ_0 corresponding to different lengths of the column.

	Isothermal variant		Adiabatic variant			
					$i(\tau)$ from (22)	
	$i(\tau)=\tau$	$i(\tau)=1$	$i(\tau)=\tau$	$i(\tau)=1$	$\lambda_0=0$	$\lambda_0=6.6$
n_{min}	0.01	0.06	0.01	0.03	0.3	0.04
τ_{min}	1.82	1.48	1.90	1.87	1.95	3.0
σ_{min}	0.0	0.0	0.0	0.0	0.04	0.03

Footnote. In the case $i(\tau) = \tau$, the characteristic value of the current strength I_0 is expressed in terms of the known derivative $\dot{I}_0$ of the current strength by the formula $I_0 = \dot{I}_0 t_0$.

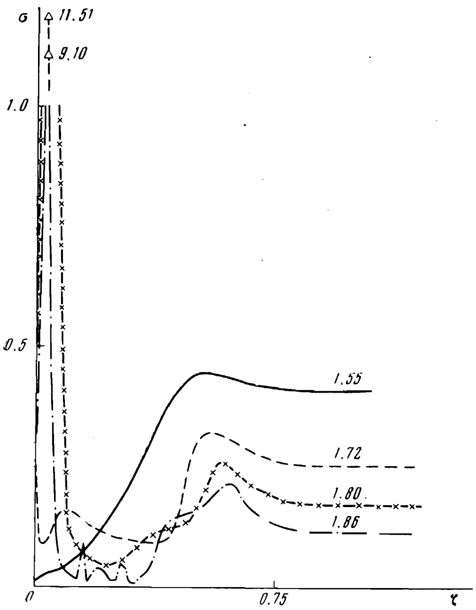

FIG. 4. Profiles of the radius σ of the column as a function of distance along the ζ axis for several instants of time indicated beside the curves (all the quantities are dimensionless).

$i(\tau) = 1$. Figure 4 shows the profiles of $\sigma(\zeta, \tau)$ as a function of ζ for several instants of time in the vicinity of the maximum compression of the column (from $\tau_1 = 1.55$ to $\tau_4 = 1.86$). As was to be expected, a constriction develops near $\zeta = 0$, where the strongest perturbation is assigned at $\tau = 0$. At $\tau_1 = 1.55$ the compression still has quite a regular character, although it is quite considerable ($\sigma(0, \tau_1) \approx 0.01$). The picture subsequently becomes complicated: ejection takes place near $\zeta = 0$ and the maximum compression seems to repeat itself and to move in the direction of increasing ζ, breaking up in the process into discrete foci.

In Fig. 2 in[16] one can clearly see in the last frames, which correspond to maximum compression of the constriction, such plasma "spikes", as well as the elongation and displacement of the constriction relative to the initial phase shown in the preceding frames. Figure 5, which shows the plots of the density $n(\zeta, \tau)$, substantially supplements the considered picture of the process. The plasma-efflux effect accompanies the constriction development. The density decreases appreciably in the first phase of the contraction (at $\tau_1 = 1.55$, $n_{min} \approx 0.2$) and decreases further by almost an order of magnitude when the contraction is repeated (at $\tau_4 = 1.86$, $n_{min} \approx 0.03$). The irregularity of the motion that arises when $\tau \gtrsim 1.80$ is especially manifested in the behavior of the velocities u and v. The axial velocity $v(\zeta)$ changes sign several times, which indicates collisions between the plasma jets. If in the regular phase the quantity v is

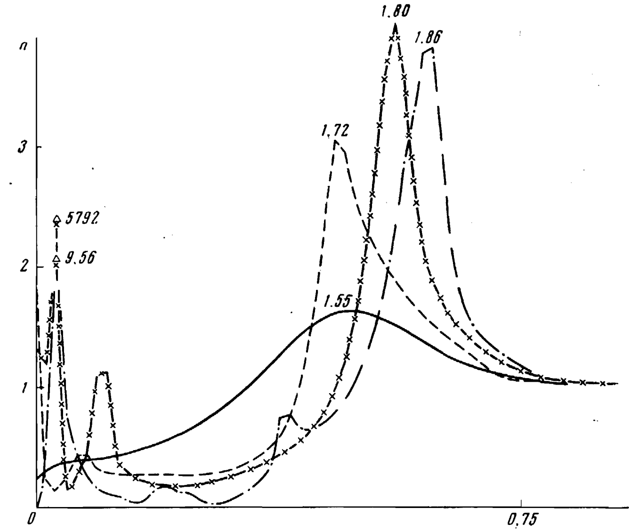

FIG. 5. Profiles of the linear plasma density n as a function of the coordinate ζ for the same instants of time given in Fig. 4.

several times smaller than the quantity u, then they have at $\tau \approx 1.86$ almost the same maximum values: $|u_{max}| \approx |v_{max}| \approx 2$.

A little after $\tau = 1.86$, the radius of the column vanishes for some value of ζ, i.e., there occurs a continuous contraction leading to the rupture of the column. From this moment on the MHD treatment loses meaning[2)]. A striking distinctive feature of the rupture of the column is the preservation, as $\sigma \to 0$, of the finite value of the density n, which, as has already been noted in the Introduction, nevertheless exceeds by far the value $n_{min} \sim 10^{-4}$ corresponding to (1) under the conditions (20). We collect in the Table the n_{min} for the different variants of the computation (the smallest value of n is in fact attained in the rupture). For the variants with the self-consistent current, a rupture of the column does not occur in the computation—in any event not up to $\tau \approx 3$, when the computation was stopped. Also indicated in the Table are the absolute values of σ_{min}, as well as the times τ_{min} for the attainment of the n_{min}, which times, for the first four variants, coincide with the times for the rupture. As can be seen from the Table, the linear density of the plasma in the constriction also decreases by a factor of 10—100 in comparison with the initial density, which is in good agreement with the data of the two-dimensional calculation.

To conclude this section, we note that the accuracy of the calculation was controlled by several methods: a) by varying the number of the computational points along ζ (100 and 200 points were used for $\zeta_0 = 3$ and 200 points for $\zeta_0 = 1.5$); b) by carrying out the computation with a reduced spacing in the time τ; c) by carrying out the computation of the variant with the computational region $-1.5 \le \zeta \le 1.5$ in which the center $\zeta = 0$ of the constriction turned out to be at the middle of the computational interval and not near the boundary. The checking showed that even in the irregular phase of the contraction the accuracy was quite satisfactory.

3. THE INDUCED ELECTRIC FIELD AT THE SURFACE OF THE PLASMA COLUMN. THE PROBLEM OF ION AND ELECTRON ACCELERATION

In the constriction region inside the plasma, there arises an induced electric field whose intensity is, owing to the high conductivity of the plasma[3)], equal to

$$\mathbf{E} = -\frac{1}{c}[\mathbf{v} \times \mathbf{B}]. \tag{23}$$

Since the tangential component of $\mathbf{E}$ is continuous across the surface of the column, we can assert that the estimate (23) pertains, in general, to the tangential field near and on both sides of the plasma surface. It is important to note this because the actual ion and electron acceleration is most likely to occur at the place where the density of matter is least, i.e., in the MHD plasma model, even outside the plasma surface. In the vicinity of the place where the constriction has minimum radius, the tangential field practically coincides with the axial component:

$$E_z = -v_r B_\varphi / c. \tag{24}$$

It follows from (24) that the induced field coincides in direction with the primary field of the discharge only in the contraction phase ($v_r < 0$). Owing to the decrease of both v_r and B_φ, the field decreases rapidly as we move away in either direction from the middle of the constriction. Evidently, we can estimate with a good degree of accuracy the potential difference Φ_z in the direction parallel to the axis of the constriction, using again the method of profiling:

$$\Phi_z = \int_{-\infty}^{+\infty} E_z\,dz = -\frac{2}{c}\int_0^\infty v_r B_\varphi\,dz = -\frac{2}{c}\int_0^\infty \frac{U(z,t)B_R r^2}{R^2(z,t)}\,dz = -\frac{4I}{c^2}\int_0^\infty \frac{Ur^2}{R^3}\,dz. \tag{25}$$

In (25) the velocity and magnetic field are given by linear functions of the radius.

In the case of strong skin effect the quantity B_φ inside the plasma decreases more rapidly, say quadratically. Some overestimate of the quantity Φ_z in (25) is therefore possible, but it is quite excusable, since the actual trajectories of the accelerated particles can be bent parallel to the surface of the column (it can be verified that the lines of force of $\mathbf{E}$ are roughly parallel to this surface).

Further, let us assume in (25) that $r = R_{min}$ at a given moment of time and let us write the integral in dimensionless variables:

$$\Phi_z = -\frac{4I_0R_0}{c^2t_0}\, i(\tau) \int_0^{\zeta_0} \frac{u}{\sigma}\left(\frac{\sigma_{min}}{\sigma}\right)^2 d\zeta = \frac{4I_0R_0}{c^2t_0}\,\Phi. \tag{26}$$

The numerical computation determines the dimensionless function

$$\Phi = \Phi(\tau) = -\,i(\tau)\int_0^{\zeta_0} \frac{u}{\sigma}\left(\frac{\sigma_{min}}{\sigma}\right)^2 d\zeta.$$

Figure 6 shows the plots of $\Phi(\tau)$ for two variants of the computation: the first is an adiabatic variant with $i(\tau) = 1$, while the second is an adiabatic variant with a self-consistent current and with $\lambda_0 = 6.6$. First of all, it should be noted that the two curves are similar, although the details of the motion in the two variants differ quite strongly (see the Table).

The noted similarity of the curves $\Phi(\tau)$ is also characteristic of other variants of the computations. It is characteristic that the plots exhibit two positive maxima, the second of which corresponds to the secondary contraction of the column, the value of this maximum being between 5 and 10. The large negative maximum is connected with the rapid expansion of the plasma in the constriction and does not, apparently, play any role in the acceleration of the particles (see below).

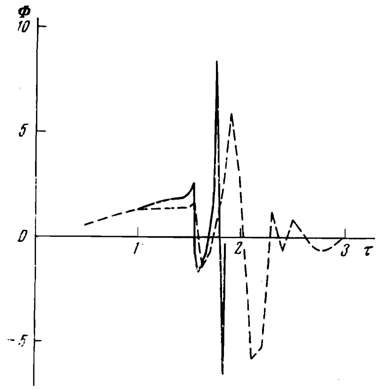

FIG. 6. The dependence of the dimensionless potential difference Φ on the time τ for two different variants of the computation. The continuous curve represents the adiabatic variant with the direct current $i = 1$; the dashed curve, the adiabatic variant with a self-consistent current (the parameter $\lambda_0 = 6.6$).

Let us determine the quantitative value of Φ_z, substituting t_0 from (9) into (26) and expressing all the quantities in convenient units (see (20)):

$$\Phi_z = 0.4 I_0^2 N^{-1/2} \Phi \text{ [kV]}. \tag{27}$$

For the characteristic conditions (20) at the maximum of the function $\Phi = 5-10$ (see Fig. 6), we obtain from (27) that $\Phi_z = 125-250$ kV. It is the excess of Φ over unity by roughly an order of magnitude that accounts for the plasma-constriction effect, since the factor $4I_0R_0/c^2t_0$ follows directly from dimensional considerations and almost corresponds to a cylindrically symmetric compression of the column. The above-obtained value of Φ_z is close to the experimentally-measured energy of the accelerated ions. It is worth noting that under the conditions of a plasma focus, where we can assume $I_0 = 10^6$ A and $N_0 = 2 \times 10^{19}$ cm^{-1}[12], $\Phi_z = 450-900$ kV, i.e., has an appreciably larger value. In the general case the maximum potential difference and, consequently, the energy of the accelerated particles are determined by the simple relation:

$$\Phi_z \approx (2-4) I_0^2 N^{-1/2} \text{ [kV]}, \tag{28}$$

where I_0 can, under real conditions, be identified with the maximum current before the "singularity." In all probability, such a method of determining the current I_0 is more practical than the identification of it with the "detachment current."

It is known that a current break—the so-called "singularity"—can be observed in fast Z-pinches. Starting from some instant, the current decreases, which is connected with the growth of the inductance of the circuit owing to the compression of the plasma column. This is an integrated effect. The amount of maximum compression in the constrictions grated effect. The moment of maximum compression in the constrictions can be slightly shifted since this effect is a local effect. The current grows during the expansion of the column, i.e., during the decrease of the total inductance. Such a process can be repeated and several "singularities"—more often than not two—may be observed. As I_0 in (28), we can take the value of the current before the first, or, correspondingly, the second break.

It is easy to compare the mean thermal energy of the particles in the plasma column with the above-determined accelerated-particle energy corresponding to the potential difference (28). To do this, we shall proceed from the well-known relation for the temperature: $kT = I^2/4c^2N_0$. Let us take the ratio of this quantity to the accelerated particle energy $e\Phi_0\Phi$, where, according to (26), $\Phi_0 = 4I_0R_0/c^2t_0$:

$$\frac{kT}{e\Phi_0\Phi} = \frac{(3M)^{1/2}c}{48e}\frac{1}{N_0^{1/2}}\frac{i^2}{\Phi}. \quad (29)$$

For $N_0 = 10^{17}$ cm^{-1} we find from (29) that

$$kT/e\Phi_0\Phi \approx 1.3\cdot 10^{-2} i^2/\Phi.$$

Further, assuming $i \approx 1$ and $\Phi \approx 6.5$, we find that $kT/e\Phi_0\Phi \approx 2\times 10^{-3}$, i.e., that the mean thermal energy of the accelerated particles.

Thus, we have computed the potential difference produced in the constriction. The question arises: Under what conditions can ions moving across a magnetic field be accelerated in an electric field? The corresponding computations have been performed and will be reported separately. Here, however, we restrict ourselves to some qualitative estimates.

In order that the magnetic field will not bend the trajectories of the ions, the inequality $r_L \gg R$ must be satisfied, if allowance is made for the fact that the acceleration length $\sim R$. From this follows the well-known limitation on the linear density of a deuterium plasma:

$$N < \frac{3}{16}\frac{Mc^2}{e^2} \approx 2.5\cdot 10^{15} \text{ cm}^{-1}$$

(to show this, we again have to use the condition $B^2/8\pi = 2k\rho T/M$). The velocity of the injectable ions is assumed to be equal to the mean thermal velocity $v = (3kT/M)^{1/2}$ in the vicinity of the constriction. It can then be seen that the last inequality can be satisfied only because of the outflow of plasma from the constriction (see Sec. 2). Concrete computations of the trajectories of ions in the electromagnetic-field configuration under consideration confirm the criteria and indicate at the same time the threshold nature of the effect of trajectory bending, in connection with which the inequality sign has been relaxed for the quantity N. For the negative maximum, the mean value of N again increases several times, since the constriction is filled with matter during its expansion. Furthermore, the corresponding acceleration length also increases several times in comparison with the second positive maximum. These two factors facilitate the deceleration of the accelerated particles and the bending of their trajectories, so that the acceleration of the particles in the opposite direction is hindered as compared to the acceleration in the direction of the primary field of the discharge.

Let us estimate the existence time of the potential-difference pulse. Since for the conditions (20) $t_0 \approx 2.2\times 10^{-7}$ sec, $\Delta t \approx 0.03\times 2.2\times 10^{-7} \approx 7\times 10^{-9}$ sec (see the continuous curve in Fig. 6). A deuteron with velocity $\sim 10^7$ cm/sec has quite enough time to traverse an acceleration length of ~ 0.1 cm.

The results obtained here answer only some of the questions posed. The computed value of the potential difference in a constriction agrees quite well with the measured energy of the accelerated ions if we assume that the ions are accelerated at the periphery of the constriction, where the potential difference is highest. Estimates show that because of the effect of particle outflow from the vicinity of the constriction, the Larmor ion radius becomes comparable with, and can even exceed the radius of the plasma column at the constriction.

Owing to this circumstance, ions even with energy that is small compared to the mean thermal energy of the plasma particles can move unimpeded in the direction of the z axis across the magnetic field. In the course of such a motion the ions acquire from the electric field an energy equal to the potential difference in the constriction. This allows us to explain the mechanism of neutron emission from a Z-pinch, the characteristic angular distributions of the accelerated ions, and the neutrons obtained as a result of the d-d reaction. There remains, nevertheless, phenomena that do not fall within the framework of the obtained results. Neutron emission in a Z-pinch is usually accompanied by a burst of hard x-rays. The appearance of this radiation can be due only to the presence of accelerated electrons in the discharge. It is exactly here that the difficulties of the above-considered theory begin.

It can easily be shown that electrons in the electric field (24) cannot, in principle, move along the field and cannot consequently gain energy. In order for the Larmor radius of the electrons having the mean thermal energy to be larger than the radius of the constriction at the time of the injection of the electrons, the linear plasma density in the constriction should satisfy the very rigid condition: $N < mc^2/e^2 \sim 10^{12}$ cm^{-1}, which is totally incompatible with the MHD calculations. In all the experiments that have been performed, when neutron and hard x-ray emissions were observed, the initial number of particles in a cross section of the column was 10^{17}–10^{18} cm^{-1}. In accordance with the results of the computations (see Sec. 2 and the Table), in the cross section of the constriction remains 1–10% of the initial number of particles, i.e., 10^{15}–10^{17} cm^{-1}, which by far exceeds the quantity N that guarantees the direct acceleration of the electrons.

Thus, to explain the acceleration of the electrons, it is necessary to trace in the subsequent computations further depletion of matter in the constriction. This cannot be done in the framework of magnetohydrodynamics; this is evident from the fact that one of the conditions of its applicability is the smallness of the Larmor radius of the ions in comparison with the radius of the plasma column. This leads to the condition $e^2N/Mc^2 > 1$. But in actual fact the computations were carried out up to the stage when this quantity become smaller than unity. This is indicated by the "rupture" of the column obtained in the computations. The subsequent computations should have been performed in the framework of the solution of the kinetic equations. It is to be hoped that owing to the efflux effect, the possibility of a direct acceleration of the electrons will emerge in such a treatment.

The possibility of a direct acceleration of the electrons emerges upon the violation of the conditions for quasineutrality of the plasma. The outflow cannot continue until the constriction is exhausted of all particles because there cannot be a break in the current in the discharge, owing to the large inductance of the circuit. Since the electrons are supposed to be the current carriers in the discharge, the minimum number of them in the cross section of the constriction is determined by the formula (1): $N_{min} \approx I/ec$, i.e., for I = 150 kA (hard x-rays have not been observed for weaker currents), $N_{min} \approx 3 \times 10^{13}$ cm^{-1}. This number is roughly an order of magnitude higher than that required from the condition $e^2N/mc^2 < 1$. The acceleration of the electrons can nevertheless be realized for $N \gtrsim N_{min}$ in the electric fields of the uncompensated space charge (see the relation (2)). An electric field E_z then exists at all points of the cross section of the constriction, including the point on the axis of the column where no magnetic field exists.

It would be premature to present any valid quantitative relations that characterize such a state of strong violation of the quasineutrality of the plasma. It should be pointed out that under the conditions of strong violation of quasineutrality, because of the shortness of the time of existence of such a state, the quasistationary treatment turns out to be also inapplicable, i.e., the displacement currents must in this case be taken into account. As a result of this, only part of the current will flow through the constriction, and this will lead to a change in the structure of the electric and magnetic fields in the vicinity of the constriction. As to the acceleration of the ions in such a regime, the times here are apparently so short that the ions simply do not have time to accelerate in appreciable numbers. However, tracks of ions that had been accelerated to energies considerably higher than the estimates of the present paper were observed by Brezhnev in his experiments[4], although the resolution in these experiments was not sufficient for a sharp fixation of the tracks. Such ions were too few to make an appreciable contribution to the neutron radiation.

---

1)It is worth noting that the indicated dependences of the quantities on the radius satisfy the complete system of MHD equations in the presence of high viscosity and thermal conductivity. Such dependences are actually obtained in the two-dimensional plasma-focus calculation with allowance for dissipative processes (see Fig. 7 in [12].

2)It would appear that the rupture of the column could be avoided by returning to the complete two-dimensional problem in which, in addition, dissipative effects are included. But this, unfortunately, is not so. The rupture of the column was obtained in the two-dimensional problem both with [12] and without [9] allowance for dissipation. In both problems the rupture occurred in the secondary-contraction phase. The violation of the conditions of applicability of the MHD approximation was established in [12].

3)It is assumed that the magnetic Reynolds number is large, i.e., $R_m = Rv/\nu_m \gg 1$ (ν_m is the coefficient of diffusion of the magnetic field). In this case the thickness of the skin layer $d^2 \approx \nu_m t = \nu_m R/v \ll R^2$ and the induced field by far exceeds the field due to the finite conductivity

$$\frac{vBc}{4\pi j\nu_m} \approx \frac{vd}{\nu_m} \approx \left(\frac{Rv}{\nu_m}\right)^{1/2} = R_m^{1/2} \gg 1.$$

It has been shown in two-dimensional constriction calculations under plasma-focus conditions (even with a reduced value of the conductivity as compared to the classical value) that the strong skin effect obtains [12].

1 L. A. Artsimovich, A. M. Andrianov, E. I. Dobrokhotov, S. Yu. Luk'yanov, I. M. Podgornyĭ, V. N. Sinitsyn, and N. V. Fillippov, Atomnaya énergiya 1, No. 3, 76 (1956) [Sov. J. At. Energy 1, No. 3 (1956)].

2 L. A. Artsimovich, Upravlyaemye termoyadernye reaktsii (Controlled Thermonuclear Reactions), Fizmatgiz, 1961 (Eng. Transl., Gordon and Breach, New York, 1964), Chap. 5.

3 S. M. Osovets, in: Fizika plazmy i problemy upravlyaemykh termoyadernykh reaktsiĭ (Plasma Physics and the Problems of Controlled Thermonuclear Reactions), Vol. 2 Izd. AN SSSR, 1958, p. 165.

4 B. G. Brezhnev, Izv. Akad. Nauk SSSR, OTN, énergetika i avtomatika, No. 2, 54 (1960).

5 N. A. Borzunov and D. V. Orlinskiĭ in: Fizika Plazmy i problemy upravlyaemykh termoyadernykh reaktsiĭ (Plasma Physics and the Problems of Thermonuclear Reactions), Vol. 2, Izd. AN SSSR, 1958, p. 150.

6 N. A. Borzunov, D. V. Orlinskiĭ, and S. M. Osovets, Atomnaya énergiya 4, 149 (1958) [Sov. J. At. Energy **4**, No. 4 (1958)].

7 B. A. Trubnikov, in: Fizika plazmy i problemy upravlyaemykh termoyadernykh reaktsiĭ (Plasma Physics and the Problems of Controlled Thermonuclear Reactions), Vol. 4, Izd. AN SSSR, 1958, p. 87.

8 O. Anderson, W. Baker, S. Colgate, J. Ise, Jr., and R. Pyle, translation in: Problemy sovremennoĭ fiziki (Problems of Modern Physics), Vol. 1, 1958, p. 117.

9 V. F. D'yachenko and V. S. Imshennik, Zh. Eksp. Teor. Fiz. 56, 1766 (1969) [Sov. Phys.-JETP **29**, 947 (1969)].

10 D. E. Potter, Phys. Fluids 14, 1911 (1971).

11 G. Basque, A. Jolas, and J. P. Watteau, Phys. Fluids 11, 1384 (1968).

12 N. V. Filippov, V. I. Agafonov, I. F. Belyaeva, et al., Plasma Physics and Controlled Nuclear Fusion Research, Vol. 1, IAEA, Vienna, 1971, p. 573.

13 S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Oxford Univ. Press, Oxford, 1961.

14 V. S. Imshennik, Dokl. Akad. Nauk SSSR **204**, 1335 (1972) [Sov. Phys.-Doklady 17, 576 (1972)].

15 S. M. Osovets, E. I. Pavlov, and V. I. Sinitsyn, Zh. Eksp. Teor. Fiz. 64, 1228 (1973) [Sov. Phys.-JETP **37**, 625 (1973)].

16 V. A. Aleksandrov, N. G. Koval'skiĭ, S. Yu. Luk'yanov, V. A. Rantsev-Kartinov, and M. M. Stepanenko, Zh. Eksp. Teor. Fiz. 64, 1222 (1973) [Sov. Phys.-JETP **37**, 622 (1973)].

Translated by A. K. Agyei
231