Fluctuation theory of the two-dimensional mixed state in type 1 superconductors

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The structure of the surface "mixed" state in type 1 superconductors is elucidated under conditions when, due to the presence of an electric field, the superconductivity in the layer is strongly suppressed and exists in the form of superconducting fluctuations. It is shown that an arbitrarily weak electric field leads to practically complete destruction of the mixed-state layer for currents close to the second critical current. The electromagnetic impedance of the surface in the presence of the mixed-state layer is calculated.

The picture of the destruction of superconductivity by an electric current in a multiply-connected sample possesses an important feature in comparison with the case of a singly-connected superconductor. Namely, in addition to the purely superconducting, normal, and intermediate states, the so-called two-dimensional mixed state should also be realized in the present case.

The idea of the existence of such a "mixed" state was expressed by L. D. Landau as long ago as 1937 (private communication to D. Shoenberg, see[1]) in connection with the question of the destruction of the superconductivity of a hollow cylinder by a current. The point here is that the intermediate state must vanish with an increase of the current because the radius of the intermediate-state region becomes smaller than the radius of the cylinder's inner surface. However, in this connection the purely normal state is also unstable with respect to the onset of superconductivity close to the inner surface, where for an arbitrary value of the current the magnetic field is as small as desired. This superconductivity cannot exist in the form of a layer of the usual superconducting phase since an electric field exists in the normal phase surrounding it, and by virtue of the continuity conditions this electric field must also exist in the superconducting region. Thus, the inside surface of the cylinder must be covered by a "mixed" state layer in which superconductivity and an electric field coexist.[1)] This two-dimensional mixed state was experimentally observed and investigated by I. Landau and Sharvin.[3,4]

In the article by Tekel' and one of the authors,[5] the properties of the two-dimensional mixed state were theoretically investigated under the assumption that a sufficiently weak electric field does not turn out to have any substantial influence on the properties of the layer, which in this connection turn out to be approximately the same as the properties of an ordinary superconducting layer of finite thickness. Such a picture is confirmed by the experiments of I. Landau and Sharvin for weak currents in sufficiently pure metals. It is clear, however, that the existence of an electric field is the most important feature of the mixed state under investigation, and a strong electric field must lead to a significant suppression of the superconductivity.

In the present article the structure of the two-dimensional mixed state will be investigated under conditions when, due to the presence of an electric field, the superconductivity in the layer is almost completely suppressed and exists in the form of small "superconducting" fluctuations. The results obtained in this connection make it possible to clarify under what conditions the electric field plays an essential role. It will be shown, in particular, that the presence of the electric field is extremely important even in an arbitrarily pure metal, when the current is not small. It is precisely the effect of the electric field which determines the maximum value of the current, the value at which the two-dimensional mixed state vanishes and the sample goes over to an essentially purely normal state. Owing to their small conductivity, the picture of the two-dimensional mixed state in type 1 superconducting alloys practically always turns out to be strongly dependent on the electric field.

1. GENERAL RELATIONSHIPS

Let us consider a hollow cylindrical superconductor with outer radius r_2 and inner radius r_1 through which a current J is flowing. In this connection, if the sample exists in a purely normal state, then the magnetic field distribution as a function of the radial distance is determined by the formula

$$H(r)=\frac{2J}{cr}\frac{r^2-r_1^2}{r_2^2-r_1^2}.$$

For our purposes only the region near the inner surface ($r \approx r_1$) will be important; in this region the magnetic field is small and may be written in the form $H(x) = (4J/c)(r_2^2 - r_1^2)^{-1}x$, where $x = r - r_1$ denotes the distance from the inner surface. The corresponding vector potential of the magnetic field can be represented in the form $A_x = A_y = 0$, $A_z = -(2J/c)x^2/(r_2^2 - r_1^2)$, where the z axis is directed along the axis of the sample. In addition to the magnetic field there is also an electric field E, resulting from the finite conductivity of the normal metal and directed along the z axis. With allowance for this field, the total vector potential is given by

$$A_z=-\frac{2J}{c}\frac{x^2}{r_2^2-r_1^2}-cEt. \quad (1)$$

We shall assume that the superconductivity which arises near the inner surface is strongly suppressed by the presence of the electric field and only exists in the form of superconducting fluctuations.[6-10] The conditions under which this will actually occur will be clarified below.

In investigating the superconducting fluctuations we shall use the method which is most convenient for our purposes, based on the introduction of an external force into the temporal equation for the superconducting order parameter. This method was first used by Schmid,[9] and was subsequently extended by Kulik[10] to the temperature region $T < T_c$, in which the smallness of the fluctuations may be due to the presence of the electric field. The latter result was previously derived by a microscopic method in an article by Gor'kov.[11]

With the external force taken into account, the time-dependent equation for the superconducting order parameter ψ has the following form:

$$\frac{\partial\psi}{\partial t}-\nu\left\{\psi-\xi^2\left(\nabla-\frac{2ie}{c}\mathbf{A}\right)^2\psi\right\}=f(\mathbf{r},t). \tag{2}$$

where $\xi = \xi(T)$ is the coherence length, $\nu = 8(T_c - T)/\pi$, and f is the external force satisfying the condition

$$\langle f(\mathbf{r},t)f^*(\mathbf{r}',t')\rangle=4mT\nu\xi^2\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'), \tag{3}$$

where m denotes the electron mass, and the angular brackets denote average values with respect to an ensemble.

Substituting (1) into (2) and changing to the dimensionless quantities

$$\tilde{\mathbf{r}}=\frac{\mathbf{r}}{\xi},\quad \tilde{t}=\nu t,\quad \tilde{f}=\frac{1}{2\nu}\left(\frac{\xi}{mT}\right)^{1/2}f,$$
$$\tilde{\psi}=\frac{1}{2}\left(\frac{\xi}{mT}\right)^{1/2}\psi,\quad \varepsilon=\frac{2eE\xi}{\nu},\quad \beta=\frac{4eJ}{c^2}\frac{\xi^3}{r_2^2-r_1^2}, \tag{4}$$

we obtain

$$\partial\tilde{\psi}/\partial\tilde{t}-\tilde{\psi}+\mathscr{L}\tilde{\psi}=\tilde{f}(\tilde{\mathbf{r}},\tilde{t}),$$
$$\mathscr{L}=-\frac{\partial^2}{\partial\tilde{x}^2}-\frac{\partial^2}{\partial\tilde{y}^2}-\left(\frac{\partial}{\partial\tilde{z}}+i\varepsilon\tilde{t}+i\beta\tilde{x}^2\right)^2, \tag{5}$$

where

$$\langle\tilde{f}(\tilde{\mathbf{r}},\tilde{t})\tilde{f}^*(\tilde{\mathbf{r}}',\tilde{t}')\rangle=\delta(\tilde{\mathbf{r}}-\tilde{\mathbf{r}}')\delta(\tilde{t}-\tilde{t}'). \tag{6}$$

We shall use the dimensionless quantities below, and wherever it does not lead to ambiguities we shall omit the tilde above the corresponding letters.

We shall seek a solution of Eq. (5) in the form

$$\psi=\sum_{k_y,k_z,n}a_{k_yk_z}^{(n)}(t)\psi_{k_yk_z}^{(n)}(\mathbf{r},t), \tag{7}$$

where

$$\psi_{k_yk_z}^{(n)}(t)=L^{-1}\exp\{ik_yy+ik_zz\}\varphi_{k_yk}^{(n)}(x)$$

are the eigenfunctions of the operator $\mathscr{L}$ with eigenvalues $(\lambda + k_y^2)$:

$$\mathscr{L}\psi_{k_yk_z}^{(n)}(t)=(\lambda_n+k_y^2)\psi_{k_yk_z}^{(n)}(t), \tag{8}$$

satisfying the normalization condition

$$\int_0^\infty\varphi_{k_yk}^{(n)}(x)\varphi_{k_yk}^{(m)}(x)dx=\delta_{nm},$$

where $k = k_z + Et$, and L is a normalization length. Equation (8) is equivalent to the following condition, which is imposed on the function φ:

$$d^2\varphi_{k_yk}^{(n)}/dx^2+\{\lambda_n-(k+\beta x^2)^2\}\varphi_{k_yk}^{(n)}=0, \tag{9}$$

from which it is seen that the functions $\varphi_{k_yk}^{(n)}\equiv\varphi_k^{(n)}$, $\lambda_n = \lambda_n(k)$ do not depend on k_y.

Let us consider the most interesting case when the electric field is not too strong. Namely, let us assume that the electrical energy $eE\xi$ is small in comparison with the relaxation frequency ν, that is

$$\varepsilon\ll 1. \tag{10}$$

Substituting the expansion (7) into Eq. (5), after some simple transformations we obtain

$$\partial a_{k_yk_z}^{(n)}/\partial t-a_{k_yk_z}^{(n)}+[\lambda_n(k)+k_y^2]a_{k_yk_z}^{(n)}=f_{k_yk_z}^{(n)}(t), \tag{11}$$

where

$$f_{k_yk_z}^{(n)}(t)=\frac{1}{I}\int d^3\mathbf{r}\exp\{-ik_yy-ik_zz\}\varphi_k^{(n)}(x)f(\mathbf{r},t)$$

are the new components of the external force, satisfying the condition

$$\langle f_{k_yk_z}^{(n)}(t)f_{k_y'k_z'}^{(n')*}(t')\rangle=\delta_{k_yk_y'}\delta_{k_zk_z'}\delta_{nn'}\delta(t-t'). \tag{12}$$

In deriving formula (11) we neglected expressions of the form $a^{(n)}\partial\psi_k^{(n)}/\partial t$ in comparison with $\psi_k^{(n)}\partial a^{(n)}/\partial t$, which is legitimate since the time-dependence of the eigenfunctions $\psi_k^{(n)}$ is due to the dependence k(t) associated with a given k_z. In the case when $\epsilon \ll 1$ this dependence is extremely weak, and it can be neglected in comparison with the explicit time dependence of the coefficients $a^{(n)}$.

The solution of Eq. (11) has the form

$$a_{k_yk_z}^{(n)}(t)=\exp\{-p_{k_yk_z}^{(n)}(t)\}\int_{-\infty}^{t}\exp\{p_{k_yk_z}^{(n)}(t')\}f_{k_yk_z}^{(n)}(t')dt', \tag{13}$$

where

$$p_{k_yk_z}^{(n)}(t)=-\int_0^t[1-\lambda_n(k')-k_y^2]dt',\quad k'=k_z+\varepsilon t'.$$

The density of the fluctuating superconducting current is determined by the well-known Ginzburg-Landau formula:

$$\mathbf{j}=\frac{2e}{m}\operatorname{Im}\left[\psi^*\left(\nabla-\frac{2ie}{c}\mathbf{A}\right)\psi\right], \tag{14}$$

which in our dimensionless units can be rewritten in the form

$$\langle j_z\rangle=\frac{8eT}{\xi^2}\operatorname{Im}\left\langle\psi^*\left(\frac{\partial}{\partial z}+i\varepsilon t+i\beta x^2\right)\psi\right\rangle.$$

Substituting here the expansion (7), the solution (13), and averaging with the aid of formula (12), we find

$$\langle j_z\rangle=\frac{8eT}{\xi^2}\sum_{n=0}^{\infty}\int\frac{dk_y\,dk_z}{(2\pi)^2}(k+\beta x^2)|\varphi_k^{(n)}(x)|^2\mathscr{E}_-^{(n)}(t)\int_{-\infty}^{t}dt'\mathscr{E}_+^{(n)}(t'),$$
$$\mathscr{E}_\pm^{(n)}(t)\equiv\exp\{\pm 2p_{k_yk_z}^{(n)}(t)\}. \tag{15}$$

One can transform the difference $2p^{(n)}(t') - 2p^{(n)}(t)$ in the following way:

$$2p_{k_yk_z}^{(n)}(t')-2p_{k_yk_z}^{(n)}(t)=2\int_{t'}^{t}[1-k_y^2-\lambda_n(k')]dt'$$
$$=2\left\{(t-t')(1-k_y^2)-\frac{1}{\varepsilon}\int_{k-\varepsilon(t-t')}^{k}\lambda_n(k')dk'\right\}.$$

After substituting the last expression into (15), carrying out the simple integration with respect to k_y and introducing the new variable $\tau = \epsilon(t - t')$ we obtain

$$\langle j_z(x)\rangle=\left(\frac{2}{\pi\varepsilon}\right)^{1/2}\frac{eT}{\pi\xi^2}\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}dk(k+\beta x^2)|\varphi_k^{(n)}(x)|^2\cdot$$
$$\times\int_0^\infty\frac{d\tau}{\sqrt{\tau}}\exp\left\{\frac{2}{\varepsilon}\left[\tau-\int_{k-\tau}^{k}\lambda_n(k')dk'\right]\right\}. \tag{16}$$

From this one can obtain the total surface density I_s of the superconducting current by integration with respect to ξdx. The integral

$$\int_0^\infty|\varphi_k^{(n)}(x)|^2(k+\beta x^2)dx$$

which arises in this connection is the average value of the operator $(1/2)\partial\mathscr{L}/\partial k$ with respect to the state $\psi_k^{(n)}$. According to a well-known theorem in quantum mechanics (see[12], the problem in Sec. 11), this average is equal to the derivative of the corresponding eigenvalue $(1/2)\partial\lambda_n/\partial k$. Therefore the surface current density is given by

$$I_s=\left(\frac{2}{\pi\varepsilon}\right)^{1/2}\frac{eT}{2\pi\xi}\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}dk\frac{\partial\lambda_n}{\partial k}\int_0^{\infty}\frac{d\tau}{\sqrt{\tau}}\exp\left\{\frac{2}{\varepsilon}\left[\tau-\int_{k-\tau}^{k}\lambda_n(k')dk'\right]\right\}. \tag{17}$$

The total superconducting current flowing through the sample is obtained by multiplying I_s by $2\pi r_1$.

Below we shall consider two important limiting cases when the fluctuating superconducting current determined by formulas (16) and (17) can be completely calculated.

2. THE DOMAIN OF WEAK CURRENTS

Let the current passing through the sample be such that the dimensionless parameter β is small in comparison with unity. In order of magnitude the value of β is equal to the ratio J/J_{c2}, where J_{c2} denotes the second critical current which was introduced in article[13]. Thus, the topic of discussion is currents which are small in comparison with the second critical current.

For small values of β one can solve Eq. (9) for the functions φ, and the functions $\lambda_n(k)$ can be determined for all important values of k. In fact, if k is positive and satisfies the condition $k \gg \beta^{1/3}$, then Eq. (9) coincides with the Schrödinger equation for an oscillator with mass 1/2, energy $\lambda_n - k^2$, eigenfrequency $(8k\beta)^{1/2}$ and a small anharmonic term of fourth order. Using the well-known formula (see[12], problem 3 in Sec. 38) for the energy levels of the anharmonic oscillator, we find

$$\lambda_n(k)=k^2+(32k\beta)^{1/2}(n+1/4)+3\beta k^{-1}(n^2+1/2n+1/4), \tag{18}$$

where it has been taken into consideration that in our case the functions φ must satisfy the Ginzburg-Landau boundary condition $d\varphi/dx = 0$ at $x = 0$, and therefore only the even parity (even with respect to the transformation $x \to -x$) solutions have any meaning. Accordingly formula (18) is obtained from the formula for the linear oscillator by the substitution $n \to 2n$.

For negative values of k such that $|k| \gg \beta^{1/3}$, the potential energy $(k + \beta x^2)^2$ has a sharp minimum when $x = x_0 \equiv (|k|/\beta)^{1/2}$. Introducing the new variable $x' = x - x_0$, one can easily rewrite Eq. (9) in the form of the Schrödinger equation for an oscillator with mass 1/2, energy λ_n, eigenfrequency $4(|k|\beta)^{1/2}$ and small anharmonic terms of third- and fourth-order. Again using the well-known formulas of quantum mechanics, we obtain

$$\lambda_n(k)=4(|k|\beta)^{1/2}\left(n+\frac{1}{2}\right)-\frac{\beta}{2|k|}(3n^2+3n+1). \tag{19}$$

Since the large factor $2/\epsilon$ occurs in the argument of the exponentials appearing in formulas (16) and (17), a small neighborhood of the maximum of the function

$$F_n(k,\tau)=\tau-\int_{k-\tau}^{k}\lambda_n(k')dk'.$$

gives the major contribution to the integrals over k and τ. The position of the maximum (k_0, τ_0) is determined by the conditions $\lambda_n(k_0) = \lambda_n(k_0 - \tau_0) = 1$, where the derivative $\partial\lambda_n/\partial k$ must be positive for $k = k_0$ and negative for $k = k_0 - \tau_0$. Using formulas (18) and (19) we find

$$k_0=1-(8\beta)^{1/2}\left(n+\frac{1}{4}\right),\quad k_0-\tau_0=-\frac{1}{4\beta(2n+1)^2}-\beta(3n^2+3n+1). \tag{20}$$

At the maximum the value of the function F_n is given by

$$F_n(k_0,\tau_0)=\frac{1}{12\beta(2n+1)^2}+\frac{2}{3}-\frac{4}{3}(8\beta)^{1/2}\left(n+\frac{1}{4}\right) -\left(n^2+\frac{3}{2}n+\frac{5}{12}\right)\beta\ln\beta+O(\beta). \tag{21}$$

Now if we expand the function F_n near its maximum in powers of $k - k_0$ and $\tau - \tau_0$, substitute into (17) and perform the simple integrations, then we obtain the following result upon taking into consideration that, as is clear from (21), the first term with n = 0 gives the major contribution to the summation over n:

$$I_s=\frac{4eT}{\xi}\left(\frac{\varepsilon\beta}{2\pi}\right)^{1/2}\exp\left\{\frac{1}{6\varepsilon\beta}+\frac{4}{3\varepsilon}-\frac{8}{3\varepsilon}\left(\frac{\beta}{2}\right)^{1/2}-\frac{5\beta}{6\varepsilon}\ln\beta\right\}. \tag{22}$$

According to Eq. (16) the coordinate dependence of the density of the fluctuation current is determined by the function $\varphi_k^{(n)}$ with n = 0 and $k \approx 1$, that is, by the wave function of the oscillator ground state, normalized over the interval $(0, \infty)$:

$$\langle j(x)\rangle=2(2\beta/\pi^2)^{1/4}I_s\exp\{-\xi^2x^2/d^2\}, \tag{23}$$

where $d = \xi(2\beta)^{-1/4}$ obviously plays the role of the thickness of the fluctuational "mixed" state layer. In the usual units we have

$$d=\left\{\frac{c^2\xi}{8e}\frac{r_2^2-r_1^2}{J}\right\}^{1/4}. \tag{24}$$

Thus, the thickness of the layer decreases rather slowly with increasing current.

Now let us discuss the question of the range of validity of the obtained results. The major limitation is associated with the fact that we treated the fluctuations as a small perturbation and, in particular, we did not take into account the influence of the fluctuation current on the form of the vector potential which, according to Eq. (1), is determined by only the normal current. In order for this to be correct, the current I_s must be small in comparison with the normal current passing through a region of thickness d near the inner surface:

$$I_s\ll\frac{Jd}{r_2^2-r_1^2}\sim\frac{c^2d}{e\xi^3}\beta\sim\varkappa\beta^{3/4}cH_c, \tag{25}$$

where $H_c \sim c/e\xi^2\kappa$ is the critical magnetic field, and κ is the parameter of the Ginzburg-Landau theory.

In order of magnitude the factor in front of the exponential in formula (22) can be written down in the following form:

$$eT/\xi\sim cH_c(\xi/\xi_0)\varkappa(a_0/\lambda_0)^2,$$

where Δ_0 and λ_0 denote the energy gap and the London penetration depth in a superconductor at zero temperature, $\xi_0 = \hbar v_F/\pi\Delta_0$, and $a_0 \sim \hbar/p_F$ is the interatomic distance.

Thus, the following quantity should be small:

$$(a_0/\lambda_0)^2e^{1/6\varepsilon\beta}\ll 1, \tag{26}$$

where we have omitted some factors that are not essential for an estimate. The parameter ϵ can be expressed in terms of the conductivity σ of the metal's normal phase:

$$\varepsilon=\beta c^2/16\sigma\xi^2(T_c-T),$$

from which it is seen that the argument of the exponential in (26) is equal to $(\beta^2\kappa^2\xi_0/l)^{-1}$ in order of magnitude, where l denotes the free path length of the electrons.

Although the factor in front of the exponential in (26) is extremely small, nevertheless condition (26) cannot be satisfied in pure metals due to the enormous magnitude of the exponential. This means that the electric field is too weak, and therefore in the present case the results of article[5] are actually valid.

However, owing to the smallness of the ratio a_0/λ_0 in type 1 superconducting alloys with $\kappa \sim 1$ and $l \sim \xi_0$, there exists a wide range of values of current through the sample for which condition (26) is satisfied and the properties of the two-dimensional mixed state should be described by the formulas derived above.

3. THE REGION NEAR THE SECOND CRITICAL CURRENT

Now let the current be close to the second critical current. The latter is defined as the maximum current[13] in which superconducting fluctuations which are growing with time can still exist. In our dimensionless units this means that for $J < J_{c2}$ values of n and k exist such that $\lambda_n(k) < 1$. For $J > J_{c2}$ the opposite inequality, $\lambda_n(k) > 1$, holds for arbitrary n and k. If the current is equal to the second critical current, then the minimum value of λ, which is reached for n = 0 and some value $k = k_m$, is exactly equal to unity. Therefore it is clear that the following formula (n = 0) holds for $J \approx J_{c2}$ and $k \approx k_m$:

$$\lambda(k) = 1 + a\frac{J - J_{c2}}{J_{c2}} + \frac{b}{2}(k - k_m)^2, \tag{27}$$

where a and b are certain coefficients, whose order of magnitude is unity.

Substituting (27) into (17) and assuming that the term with n = 0 gives the major contribution, after a simple integration we obtain

$$I_s = \left(\frac{b}{2}\right)^{1/2} \frac{eT}{2\pi\xi} \int_0^\infty d\tau \exp\left\{\frac{2a}{\varepsilon}\frac{J_{c2} - J}{J_{c2}}\tau - \frac{b\tau^3}{12\varepsilon}\right\}. \tag{28}$$

The written expression significantly depends on the relative magnitude of the parameters $(J - J_{c2})/J_{c2}$ and $\epsilon^{2/3}$, and also on the sign of the difference $J - J_{c2}$.

For $J < J_{c2}$ and $(J_{c2} - J)/J_{c2} \gg \epsilon^{2/3}$ the integral in (28) can easily be evaluated by the method of steepest descents, and we obtain

$$I_s = \left(\frac{b}{8a}\right)^{1/4} \frac{eT}{\xi} \left(\frac{\varepsilon}{2\pi}\right)^{1/2} \left(\frac{J_{c2}}{J_{c2} - J}\right)^{1/4} \times \exp\left\{\left(\frac{12}{b}\right)^{1/2} \left(\frac{2a}{3}\right)^{3/2} \frac{2}{\varepsilon} \left(\frac{J_{c2} - J}{J_{c2}}\right)^{3/2}\right\}. \tag{29}$$

If $|J_{c2} - J|/J_{c2} \ll \epsilon^{2/3}$, then one can neglect the first term in the argument of the exponential in (28), and then we easily find

$$I_s = \frac{1}{3}\Gamma\left(\frac{1}{3}\right)\left(\frac{b}{2}\right)^{1/6} \frac{eT}{2\pi\xi}(6\varepsilon)^{1/3}. \tag{30}$$

Finally, for $J > J_{c2}$ and $(J - J_{c2})/J_{c2} \gg \epsilon^{2/3}$ one can neglect, on the other hand, the second term in the exponential in (28). In this connection the superconducting current is given by

$$I_s = \left(\frac{b}{8a^2}\right)^{1/2} \frac{eT}{\xi} \frac{J_{c2}}{J - J_{c2}} \varepsilon.$$

In the present case the electric field is unimportant, and therefore the last formula is identical to the result of article[13].

Since an exact calculation of the coefficients a and b is difficult, we shall use a variational principle for their determination, assuming a trial function of the form

$$\varphi_k(x) = (8\alpha/\pi)^{1/4} e^{-\alpha x^2}.$$

Multiplying Eq. (9) by $\varphi_k(x)$ and then integrating the obtained result with respect to x, we obtain the following result after substituting the explicit form of the functions φ:

$$\lambda = \alpha + k^2 + \frac{k\beta}{2\alpha} + \frac{3\beta^2}{16\alpha^2}. \tag{31}$$

Minimizing this expression with respect to α and k, we obtain

$$k_m = -\left(\frac{\beta}{16}\right)^{1/3}, \quad \alpha(k_m) = \left(\frac{\beta}{2}\right)^{2/3}, \quad \lambda(k_m) = \frac{3}{2}\left(\frac{\beta}{2}\right)^{2/3}.$$

From the condition $\lambda(k_m) = 1$ we obtain the value of the parameter β corresponding to the second critical current, $\beta_{c2} = 2(2/3)^{3/2}$, and in this manner we find

$$J_{c2} = \frac{1}{2}\left(\frac{2}{3}\right)^{3/2} \frac{c^2(r_2^2 - r_1^2)}{e\xi^3}.$$

Expanding (31) in powers of $k - k_m$ and $\beta - \beta_{c2}$ with account of the fact that α is a function of k and is determined from by minimizing expression (31) only with respect to α, we obtain an expansion of the form (27) with a = 2/3 and b = 12/7. It is interesting to note that in the case $J < J_{c2}$, $(J_{c2} - J)/J_{c2} \gg \epsilon^{2/3}$ the coordinate dependence of the superconducting current density is, as is clear from expression (16), determined by the factor

$$(k_m + \beta x^2) e^{-2\alpha(k_m)x^2} = 6^{-1/2}(^8/_3 x^2 - 1) e^{-4x^2/3},$$

which gives zero upon integrating with respect to x. This indicates that the direction of the fluctuation current varies over the depth of the layer, and it also indicates that oppositely directed currents practically cancel each other. The total current I_s is a quantity of a higher order of smallness in $(J_{c2} - J)$ than the current density.

The formulas derived in the present section enable us to elucidate the distinctive features of the picture of the destruction of the mixed-state layer upon increasing the current in pure metals. In fact, we saw above that owing to the smallness of the electric field, the superconducting current in such metals is large for $\beta \lesssim 1$, and in order of magnitude is given by $I_s \sim cH_c$. However, formula (30) indicates that for currents very close to J_{c2} the fluctuation current is given by

$$I_s \sim \frac{eT}{\xi}\varepsilon^{1/3} \sim cH_c\left(\frac{\xi}{\xi_0}\right)\left(\frac{\xi_0}{l}\right)^{1/3} \varkappa^{5/3}\left(\frac{a_0}{\lambda_0}\right)^2,$$

i.e., it is so insignificant that for all practical purposes one can regard the sample as existing in the purely normal phase. Thus, in pure metals and for $J < J_{c2}$, an electric field weakly influences the properties of the mixed-state layer. However, in a sufficiently small neighborhood of J_{c2} any arbitrarily weak electric field leads to practically complete destruction of the layer and to the transition of the metal into the normal state. The width of this neighborhood decreases with increasing purity of the metal. Formula (29) enables us to trace the picture of the destruction in more detail. Let us write the total current through the sample as the sum of the normal current $J_n \approx J$ and the fluctuation current $2\pi r_1 I_s$. According to Eq. (29) the derivative $\partial I_s/\partial J_n$ can be written in the form

$$\frac{\partial I_s}{\partial J_n} = -\left(\frac{12}{b}\right)^{1/2}\left(\frac{2a}{3}\right)^{3/2} \frac{3}{\varepsilon J_{c2}}\left(\frac{J_{c2} - J}{J_{c2}}\right)^{1/2} I_s$$

and therefore the derivative of the total current through the sample with respect to the electric field is given by

$$\frac{\partial J}{\partial E}=\frac{\partial J_n}{\partial E}\left\{1-\left(\frac{12}{b}\right)^{1/2}\left(\frac{2a}{3}\right)^{3/2}\frac{6\pi r_1}{\varepsilon^{1/3}J_{c2}}\left(\frac{J_{c2}-J}{J_{c2}\varepsilon^{2/3}}\right)^{1/2}I_s\right\}. \quad (32)$$

The order of magnitude of the second term inside the curly brackets of the last equation is given by

$$\frac{e\xi^3}{c^2R}I_s\left(\frac{l}{\varkappa^2\xi_0}\right)^{2/3}\left(\frac{J_{c2}-J}{J_{c2}\varepsilon^{2/3}}\right)^{1/2},$$

where R is the characteristic size of the sample. At the boundary of the range of validity of formula (29) from the weak current side, the superconducting current is of the order of κcH_c, as is clear from expression (25). In this connection the second term in Eq. (32) is of the order of

$$\frac{\xi}{R}\left(\frac{l}{\varkappa^2\xi_0}\right)^{2/3}\left(\frac{J_{c2}-J}{J_{c2}\varepsilon^{2/3}}\right)^{1/2},$$

where the last factor is large (albeit logarithmically). Now we see that if the metal is sufficiently pure so that the parameter $(\xi/R)(l/\kappa^2\xi_0)^{2/3}$ is not too small, then a descending section appears in the current-voltage characteristics of the sample near $J = J_{c2}$. Therefore the phenomena of instability and hysteresis should be observed during the transition of the sample into the purely normal state, since these phenomena always arise in the presence of a descending section. Such phenomena were observed in the experiments by I. Landau and Sharvin with regard to the destruction of the mixed-state layer in the presence of strong currents.

4. THE SURFACE IMPEDANCE

The electromagnetic impedance of the inner surface of the sample is an important experimentally observable characteristic of the mixed state under consideration. In order to calculate the impedance we must include the terms describing the time-dependent electromagnetic field in the expression for the operator $\mathscr{L}$ in Eq. (5). As a result the operator $\mathscr{L}$ becomes equal to $\mathscr{L}_0 + \delta\mathscr{L}$, where $\mathscr{L}_0$ is the old value and

$$\delta\mathscr{L}=-(4e\xi/c)\{k_yA_y(\mathbf{r},t)+(k+\beta x^2)A_z(\mathbf{r},t)\}$$

is a small correction proportional to the vector potential $\mathbf{A}(\mathbf{r}, t)$ of the variable field. Since the expression $\delta\mathscr{L}\psi$ can be replaced by $\delta\mathscr{L}\psi_0$ in the linear approximation with respect to the variable field, where ψ_0 denotes the solution without the variable field, one obtains an equation of the form (5) for the corrections $\delta\psi$ to the order parameter with, however, the difference that the random force f must be replaced by $-\delta\mathscr{L}\psi_0$. If we now represent the functions ψ_0 and $\delta\psi$ in the form of the expansion (7) in terms of eigenfunctions with coefficients $a_{k_yk_z}(t)$ and $\delta a_{k_yk_z}(t)$ respectively, and take into consideration that, according to the results obtained above, only the term with n = 0 is important, then we obtain an equation of the form (11) for $\delta a_{k_yk_z}(t)$ where the quantity

$$\delta f_{k_yk_z}(t)=-\frac{1}{L}\int dV\exp\{-ik_yy-ik_zz\}\varphi_k(x)\delta\mathscr{L}\psi_0$$
$$=\frac{4e\xi}{c}a_{k_yk_z}(t)\{k_y\langle A_y\rangle_k+\langle(k+\beta x^2)A_z\rangle_k\},$$

appears on the right-hand side instead of $f_{k_yk_z}$, and also $k = k_z + \epsilon t$, and

$$\langle\ldots\rangle_k=\int_0^\infty|\varphi_k(x)|^2(\ldots)dx.$$

Substituting here the solution (13) for the coefficients $a_{k_yk_z}(t)$ and solving the inhomogeneous equation for $\delta a_{k_yk_z}(t)$ just as above, we find

$$\delta a_{k_yk_z}(t)=\frac{4e\xi}{c}\exp\{-p_{k_yk_z}(t)\}\int_{-\infty}^{t}dt_1\exp\{p_{k_yk_z}(t_1)\}\cdot$$
$$\times[k_y\langle A_y(t_1)\rangle_{k_1}+\langle(k_1+\beta x^2)A_z(t_1)\rangle_{k_1}]a_{k_yk_z}(t_1),$$

where $k_1 = k_z + \epsilon t_1$.

The variable part of the fluctuation current, $\delta\mathbf{j}$, is obtained from the general formula (14) by substituting into it the variable part of the vector potential $\mathbf{A}(\mathbf{r}, t)$ and the quantities $\delta a_{k_yk_z}$, linearizing with respect to the amplitude of the variable field, and using relations (12):

$$\delta j_z=\frac{16e^2T}{c\xi}\left\{-A_z(x,t)\int\frac{dk_y\,dk_z}{(2\pi)^2}|\varphi_k(x)|^2\mathscr{E}_-(t)\int_{-\infty}^{t}dt'\mathscr{E}_+(t')\right. \quad (33)$$
$$\left.+4\int\frac{dk_y\,dk_z}{(2\pi)^2}(k+\beta x^2)|\varphi_k(x)|^2\int_{-\infty}^{t}dt_1\langle(k_1+\beta x^2)A_z(t_1)\rangle_{k_1}\mathscr{E}_-(t)\int_{-\infty}^{t_1}dt'\mathscr{E}_+(t')\right\},$$

$$\delta j_y=\frac{16e^2T}{c\xi}\left\{-A_y(x,t)\int\frac{dk_y\,dk_z}{(2\pi)^2}|\varphi_k(x)|^2\mathscr{E}_-(t)\int_{-\infty}^{t}dt'\mathscr{E}_+(t')\right.$$
$$\left.+4\int\frac{dk_y\,dk_z}{(2\pi)^2}k_y^2|\varphi_k(x)|^2\int_{-\infty}^{t}dt_1\langle A_y(t_1)\rangle_{k_1}\mathscr{E}_-(t)\int_{-\infty}^{t_1}dt'\mathscr{E}_+(t')\right\},$$

where $\mathscr{E}_\pm(t) \equiv \exp\{\pm 2p_{k_yk_z}(t)\}$.

One can evaluate the integrals over k_y, k_z, and t' appearing in Eq. (33) by using the method of steepest descents:

$$\int\frac{dk_y\,dk_z}{(2\pi)^2}|\varphi_k(x)|^2\mathscr{E}_-(t)\int_{-\infty}^{t}dt'\mathscr{E}_+(t')$$
$$=|\varphi_{k_0}(x)|^2\frac{\xi I_s}{4eT}\left[\frac{\partial\lambda}{\partial k}(k_0)\right]^{-1},$$

$$\int\frac{dk_y\,dk_z}{(2\pi)^2}(k+\beta x^2)|\varphi_k(x)|^2\int_{-\infty}^{t}dt_1\langle(k_1+\beta x^2)A_z(t_1)\rangle_{k_1}$$
$$\times\mathscr{E}_-(t)\int_{-\infty}^{t_1}dt'\mathscr{E}_+(t')=\frac{\xi I_s}{4eT}\left[\frac{\partial\lambda}{\partial k}(k_0)\right]^{-1}(k_0+\beta x^2)|\varphi_{k_0}(x)|^2$$
$$\times\int_{t-\tau_0/\epsilon}^{t}dt_1\langle(k_1+\beta x^2)A_z(t_1)\rangle_{k_1},$$

$$\int\frac{dk_y\,dk_z}{(2\pi)^2}k_y^2|\varphi_k(x)|^2\int_{-\infty}^{t}dt_1\langle A_y(t_1)\rangle_{k_1}\mathscr{E}_-(t)\int_{-\infty}^{t_1}dt'\mathscr{E}_+(t')$$
$$=\frac{\varepsilon}{16\tau_0}\frac{\xi I_s}{eT}\left[\frac{\partial\lambda}{\partial k}(k_0)\right]^{-1}|\varphi_{k_0}(x)|^2\int_{t-\tau_0/\epsilon}^{t}dt_1\langle A_y(t_1)\rangle_{k_1}.$$

Here $k_1 = k_0 + \epsilon(t_1 - t)$, and (k_0, τ_0) are the parameters introduced above, determining the position of the saddle point and satisfying the conditions $\lambda(k_0) = \lambda(k_0 - \tau_0) = 1$. The restriction on the range of integration with respect to t_1, which appears on the right-hand side of the last two equations, arose from the condition that the saddle point was located inside the region of integration.

The vector potential of the variable field depends on the time (the usual dimensional time) by means of the factor $e^{-i\omega t}$, which is expressed in terms of our dimensionless time variable $\tilde{t}$ (denoted above by the letter t) by $e^{-i\tilde{\omega}\tilde{t}}$, where $\tilde{\omega} = \omega/\nu$ is the dimensionless frequency of the field. Therefore we have

$$\int_{t-\tau_0/\varepsilon}^{t} dt_1 \langle (k_1+\beta x^2) A_z(t_1)\rangle_{k_1} = e^{-i\tilde{\omega}t} \int_{k_0-\tau_0}^{k_0} \frac{dk_1}{\varepsilon} \exp\left\{ i\frac{\tilde{\omega}}{\varepsilon}(k_0-k_1)\right\} \langle (k_1+\beta x^2) A_z\rangle_{k_1},$$

$$\int_{t-\tau_0/\varepsilon}^{t} dt_1 \langle A_y(t_1)\rangle_{k_1} = e^{-i\tilde{\omega}t} \int_{k_0-\tau_0}^{k_0} \frac{dk_1}{\varepsilon} \exp\left\{ i\frac{\tilde{\omega}}{\varepsilon}(k_0-k_1)\right\} \langle A_y\rangle_{k_1},$$

where the components of the vector potential without any time-dependent factor appear on the right-hand side. Also omitting this factor in the variable part $\delta \mathbf{j}$ of the fluctuation current, we obtain the following result by using the formulas written down above:

$$\delta j_z = \frac{4e}{c} I_s \left[\frac{\partial\lambda}{\partial k}(k_0)\right]^{-1} |\varphi_{k_0}(x)|^2 \Big[-A_z(x) + 4(k_0+\beta x^2) \int_{k_0-\tau_0}^{k_0} \frac{dk_1}{\varepsilon} \exp\left\{ i\frac{\tilde{\omega}}{\varepsilon}(k_0-k_1)\right\} \langle (k_1+\beta x^2) A_z(x)\rangle_{k_1}\Big], \quad (34)$$

$$\delta j_y = \frac{4e}{c} I_s \left[\frac{\partial\lambda}{\partial k}(k_0)\right]^{-1} |\varphi_{k_0}(x)|^2 \Big[-A_y(x) + \frac{\varepsilon}{\tau_0} \int_{k_0-\tau_0}^{k_0} \frac{dk_1}{\varepsilon} \exp\left\{ i\frac{\tilde{\omega}}{\varepsilon}(k_0-k_1)\right\} \langle A_y(x)\rangle_{k_1}\Big].$$

The first terms in formulas (34) describe the interaction of a weak electromagnetic field with the superconducting fluctuations, which are formed without taking this field into account. On the other hand, the second terms describe the influence of the variable field on the process of the growth of the fluctuations itself. The integration with respect to k_1 occurs exactly over that region in which $\lambda(k) < 1$, that is, the fluctuations increase with time.

5. THE SURFACE IMPEDANCE OF ALLOWS

Let us consider the region of weak currents, $\beta \ll 1$. As shown in Sec. 2, in order for our general assumption about the smallness of the fluctuations to be satisfied, here the topic of discussion must involve type 1 superconducting alloys with the free path length of the electrons of order ξ_0 and with the parameter $\kappa \sim 1$. In addition, it follows from the results of Sec. 2 that in this case

$$k_0 \approx 1, \quad \tau_0 \approx 1/4\beta, \quad \partial\lambda/\partial k(k_0) = 2.$$

First let the frequency of the variable field be low enough so that the following inequalities are satisfied:

$$\tilde{\omega} \ll \beta\varepsilon, \quad \delta_n \gg \xi/\beta,$$

where δ_n denotes the skin depth in the normal metal.

In evaluating the integrals over k_1 appearing in formulas (34), one can assume that the spatial dependence of the vector potential is the same as in the normal metal:

$$A(x) = A(0) e^{-\xi x/\delta_n} \approx A(0)(1-\xi x/\delta_n).$$

In actual fact, as we have seen the function $\varphi_1(x)$ is different from zero at distances $d \sim \xi/\beta^{1/4}$ from the surface, a distance which is considerably smaller than δ_n. It is evident from Eq. (34) that here the superconducting current is also different from zero only in a region of thickness d. Meanwhile the values of the vector potential for $x \sim \xi/\beta$ give the major contribution to the integrals over k_1, that is, the major contribution comes from a region where only normal currents exist.

In the zero-order approximation with respect to the small parameters $\tilde{\omega}$ and δ_n^{-1}, the integral appearing in the first formula of (34) is equal to zero since in this case

$$A_z(x) = \text{const}, \quad \langle (k_1+\beta x^2)\rangle = {}^1\!/_2 \partial\lambda/\partial k_1,$$

and the values of the function $\lambda(k)$ are identical (equal to unity) at both limits of integration. Large (in absolute value), negative values of k_1 play a major role in calculating the following approximations. The function $\varphi_{k_1}(x)$ associated with these values of k_1 are localized in small neighborhoods of the points $x = x_0 = (|k_1|/\beta)^{1/2}$; and therefore $A_z(x)$ may be taken outside of the average value sign if we set $x = x_0(k_1)$ in it. The integral over k_1 arising after this has been done can be easily evaluated if it is taken into consideration that the equality $\lambda(k_1) = 2(\beta|k_1|)^{1/2}$ holds for negative values of k_1, and thus we obtain

$$\delta j_z = \frac{2e}{c} I_s A_z(0) |\varphi_1(x)|^2 \left\{ \left(\frac{1}{2\varepsilon}\frac{\xi}{\beta\delta_n} - 1\right) - \frac{i\tilde{\omega}}{6\varepsilon^2\beta}\right\}.$$

We determine the other component of the current in similar fashion:

$$\delta j_y = \frac{2e}{c} I_s A_y(0) |\varphi_1(x)|^2 \left\{ \frac{i\tilde{\omega}}{8\varepsilon\beta} - \frac{\xi}{3\beta\delta_n}\right\}.$$

The surface impedance Z is determined in the usual way as the ratio of the electric field $E = (i\omega/c)A$ at $x = 0$ to the total variable current, integrated with respect to ξdx. Since this total current is equal to the sum of the normal and fluctuation currents, by introducing the impedance Z_n of the normal metal we may write down the following expression:

$$\frac{Z_n}{Z} = 1 + \frac{cZ_n}{i\omega A(0)} \int_0^\infty \delta j(x)\xi\, dx. \quad (35)$$

Substituting the preceding formulas here, we find

$$\frac{Z_n}{Z_z} = 1 - \frac{8\pi e}{c^2} I_s \delta_n \xi \left\{ \left(\frac{1}{2\varepsilon}\frac{\xi}{\beta\delta_n} - 1\right) - \frac{i\tilde{\omega}}{6\varepsilon^2\beta}\right\},$$

$$\frac{Z_n}{Z_y} = 1 - \frac{8\pi e}{c^2} I_s \delta_n \xi \left\{ -\frac{\xi}{3\beta\delta_n} + \frac{i\tilde{\omega}}{8\varepsilon\beta}\right\},$$

where Z_z and Z_y denote the surface impedances of the sample in the cases when the electric field is polarized, respectively, along the axis of the cylinder and in the perpendicular direction. The quantity δ_n represents the complex "skin depth" of the normal metal. The normal skin effect takes place for the conditions under consideration and the equation $\delta_n = \delta/(1-i)$ is valid where $\delta = c/(2\pi\sigma\omega)^{1/2}$ is the usual real skin depth. We emphasize that the second terms in the right-hand sides of the obtained formulas are not, generally speaking, small in comparison with unity.

Now let the frequency of the field be high so that $\delta_n \ll d \sim \xi/\beta^{1/4}$. In this case we have

$$\int_{k_0-\tau_0}^{k_0} \frac{dk_1}{\varepsilon} e^{i\Omega(1-k_1)} \langle (k_1+\beta x^2) A_z(x)\rangle_{k_1} = \int_0^1 \frac{dk_1}{\varepsilon} e^{i\Omega(1-k_1)} k_1 |\varphi_{k_1}(0)|^2 \int_0^\infty A_z(x)\,dx,$$

$$\Omega = \tilde{\omega}/\varepsilon;$$

here it has been taken into account that the functions $\varphi_{k_1}(x)$ with negative values of k_1 are essentially equal to zero on the surface. One can write the last integral on the right-hand side in the form of a product $(\delta_n/\xi)A_z(0)$ and, in addition, it is seen from the results of section 2 that $\varphi_{k_1}(0) = 2^{1/2}(2k_1\beta/\pi^2)^{1/8}$. Therefore, the integral under consideration is given by

$$A_z(0)\frac{2\delta_n}{\varepsilon\xi}\left(\frac{2\beta}{\pi^2}\right)^{1/4} I, \quad I = \int_0^1 dk_1\, k_1^{5/4} e^{i\Omega(1-k_1)}.$$

If the frequency satisfies the condition $\Omega \ll 1$ ($\tilde{\omega} \ll \epsilon$), then I = 4/9, and we have

$$\delta j_z = \frac{2e}{c} I_s |\varphi_1(x)|^2 \left\{ -A_z(x) + \frac{32}{9} \frac{\delta_n}{\varepsilon\xi} \left(\frac{2\beta}{\pi^2}\right)^{1/4} A_z(0) \right\}.$$

In similar fashion we find

$$\delta j_y = \frac{2e}{c} I_s |\varphi_1(x)|^2 \left\{ -A_y(x) + \frac{32}{5} A_y(0) \frac{\beta\delta_n}{\xi} \left(\frac{2\beta}{\pi^2}\right)^{1/4} \right\}.$$

Integrating the obtained expressions with respect to $\xi\,dx$ and using Eq. (35), we obtain

$$\frac{Z_n}{Z_z} = 1 + \frac{64}{9} \frac{e}{i\omega} \delta_n Z_n \frac{I_s}{\varepsilon} \left(\frac{2\beta}{\pi^2}\right)^{1/4},$$

$$\frac{Z_n}{Z_y} = 1 - \frac{4e}{i\omega} \delta_n Z_n \left(\frac{2\beta}{\pi^2}\right)^{1/4} I_s.$$

In the opposite limiting case, $\Omega \gg 1$ ($\tilde{\omega} \gg \epsilon$), it is convenient to represent the integral I in the form of a sum of two integrals, one of which is evaluated by taking the contour along the imaginary axis of the complex k_1 plane from 0 to $-i\infty$, and the second integral is evaluated along a straight line contour parallel to the imaginary axis and running from $1 - i\infty$ to 1. The region near the real axis gives the major contribution to both integrals. As a result the integral I is represented in the form of the sum of a monotonic part, $I_0 = i\epsilon/\tilde{\omega}$, and an oscillating part ΔI, where

$$\Delta I = -e^{-i\pi/8} e^{i\Omega} \Omega^{-9/4} \Gamma(9/4).$$

Then we can easily obtain the following formula for the impedance Z_z:

$$\frac{Z_n}{Z_z} = 1 + \frac{4e}{i\omega} \delta_n Z_n \left(\frac{2\beta}{\pi^2}\right)^{1/4} I_s \left(\frac{4i}{\tilde{\omega}} - 1 + \frac{4}{\varepsilon} \Delta I\right).$$

We obtain the impedance for the other polarization of the field in a quite similar manner:

$$\frac{Z_n}{Z_y} = 1 + \frac{4e}{i\omega} \delta_n Z_n \left(\frac{2\beta}{\pi^2}\right)^{1/4} I_s \left\{ \frac{4i\beta}{\Omega} - 1 - 4i\beta e^{-i\pi/8} e^{i\Omega} \Omega^{-5/4} \Gamma\left(\frac{5}{4}\right) \right\}$$

Thus, at high frequencies of the variable field the surface impedance of the sample is an oscillating function of the parameter $\Omega = \tilde{\omega}/\epsilon$ or (in ordinary units) of the parameter $\pi\omega\sigma(r_2^2 - r_1^2)/2e\xi J$. The oscillations can therefore be observed in the presence of a variation of the current J and a fixed frequency of the field.

6. THE SURFACE IMPEDANCE OF PURE METALS

Using the general formulas of section 4 and the results of section 3, we may calculate the surface impedance of pure metals for values of the current close to the second critical current. In this connection we shall treat the most interesting region $J < J_{c2}$, $(J_{c2} - J)/J_{c2} \gg \epsilon^{2/3}$, where the fluctuating superconducting current is not too small.

Near the second critical current we have

$$k_0 = k_m + \left(\frac{2a}{b}\frac{J_{c2} - J}{J_{c2}}\right)^{1/2}, \quad \tau_0 = \left(\frac{8a}{b}\frac{J_{c2} - J}{J_{c2}}\right)^{1/2}, \quad \frac{\partial\lambda}{\partial k}(k_0) = \left(2ab\frac{J_{c2} - J}{J_{c2}}\right)^{1/2}$$

In addition, the integration in formula (34) takes place over an extremely narrow region near k_m. We can therefore take the average values $\langle \ldots \rangle_{k_1}$ outside of the integral sign, having set $k = k_m$ in them. After doing this the remaining integrals can easily be evaluated. Integrating at once with respect to the coordinate x, we obtain the following formulas for the impedance:

$$\frac{Z_n}{Z_z} = 1 + \frac{4e}{i\omega} Z_n \xi \frac{I_s}{A_z(0)} \left\{ -\left[\frac{\partial\lambda}{\partial k}(k_0)\right]^{-1} \langle A_z(x) \rangle_{km} + \frac{2i}{\tilde{\omega}} (1 - e^{i\Omega\tau_0}) \langle (k_m + \beta x^2) A_z(x) \rangle_{km} \right\},$$

$$\frac{Z_n}{Z_y} = 1 + \frac{4e}{i\omega} Z_n \xi I_s \frac{\langle A_y(x) \rangle_{km}}{A_y(0)} \left[\frac{\partial\lambda}{\partial k}(k_0)\right]^{-1} \left\{ -1 + \frac{i\varepsilon}{\tilde{\omega}\tau_0} (1 - e^{i\Omega\tau_0}) \right\}.$$

From here we obtain the following results for the low-frequency region, when $\tilde{\omega}\tau_0 \ll \epsilon$ and $\delta_n \gg \xi$:

$$\frac{Z_n}{Z_z} = 1 - \frac{4e}{i\omega} Z_n \xi I_s \left(2ab\frac{J_{c2} - J}{J_{c2}}\right)^{-1/2} \cdot \times \left\{ 1 + 8a \frac{\xi}{\varepsilon\delta_n} \frac{J_{c2} - J}{J_{c2}} (k_m \langle x \rangle_{km} + \beta_{c2} \langle x^3 \rangle_{km}) \right\},$$

$$\frac{Z_n}{Z_y} = 1 + \frac{4e}{b\sqrt{\varepsilon}} Z_n \xi I_s.$$

The numerical factors a, b, k_m, and β_{c2} appearing in these formulas were evaluated in section 3 with the aid of a variational method. One can also easily calculate the quantities $\langle x \rangle_{k_m}$ and $\langle x^3 \rangle_{k_m}$ by a similar method. Using the trial function $\varphi_{k_m}(x)$ we find

$$\langle x \rangle_{km} = (3/4\pi)^{1/2}, \quad \langle x^3 \rangle_{km} = (27/64\pi)^{1/2}.$$

At high frequencies, $\delta_n \ll \xi$, one can write down the average values of the vector potential in the form

$$\langle A_{y,z}(x) \rangle_{km} = \frac{\delta_n}{\xi} A_{y,z}(0) \varphi_{km}^2(0),$$

$$\langle (k_m + \beta x^2) A_z(x) \rangle_{km} = \frac{\delta_n}{\xi} A_z(0) k_m \varphi_{km}^2(0),$$

after which substitution into the general formulas written down above gives

$$\frac{Z_n}{Z_z} = 1 + \frac{4e}{i\omega} Z_n \delta_n I_s \varphi_{km}^2(0) \left\{ -\left(2ab\frac{J_{c2} - J}{J_{c2}}\right)^{-1/2} + \frac{2ik_m}{\tilde{\omega}} (1 - e^{i\Omega\tau_0}) \right\},$$

$$\frac{Z_n}{Z_y} = 1 + \frac{4e}{i\omega} Z_n \delta_n I_s \varphi_{km}^2(0) \left(2ab\frac{J_{c2} - J}{J_{c2}}\right)^{-1/2} \left\{ -1 + \frac{i}{\Omega\tau_0} (1 - e^{i\Omega\tau_0}) \right\}.$$

Using the variational method, we obtain the value $(16/3\pi)^{1/2}$ for the quantity $\varphi_{k_m}^2(0)$.

In the high-frequency region the surface impedance of pure metals, just as in the case of the alloys considered above, contains an oscillating part which is a periodic function of the parameter

$$\frac{\pi}{2} \omega \frac{\sigma(r_2^2 - r_1^2)}{e\xi J_{c2}} \left(\frac{8a}{b}\frac{J_{c2} - J}{J_{c2}}\right)^{1/2}$$

with period 2π. Such oscillations can also be observed in connection with a variation of the current and for a fixed frequency of the variable field.

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[1] In order to avoid misunderstandings, we emphasize that in the 1943 article by L. D. Landau [2] the term "mixed state" was also employed in a different sense: in order to denote the states of a type 1 superconductor which arise as a result of multiple branching of the intermediate-state layers in the absence of an electric field.

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