The resistance of thin metallic samples

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The conductivity is calculated of a thin plate with a rough surface in the classical region. Due to specular reflection of glancing electrons the resistance growth with decreasing plate thickness is slower than that for completely diffuse reflection. The dependence of the resistance on the mean characteristics of the surface is analyzed.

The resistance of thin samples has been the subject of a large number of studies, one of the latest being that of Gaĭdukov and Kadlecova[1]. The experimental data are usually reduced with the aid of the Fuchs formulas[2] or their generalizations[3], according to which the resistance can be described by introducing the diffuseness coefficient q of the reflection of the electrons from the surface. If the reflection is purely specular (q = 0), then the conductivity does not depend on the sample thickness b and is equal to the conductivity σ_0 of the unbounded metal. In the case of pure diffuse reflection (q = 1) the conductivity is

$$\sigma=\sigma_0\frac{3b}{4l}\ln\frac{l}{b},$$

if the mean free path l is large in comparison with the thickness b, and $\sigma = \sigma_0$ if $l \ll b$. In the derivation of these results, the electron spectrum was assumed to be quadratic, and the collisions of the electrons with the volume scatterers were described with the aid of the mean free path. We note that the factor b/l characterizes the fraction of electrons that do not collide with the surfaces of the flat sample over the mean free path, and $\ln(l/b)$ takes into account the contribution of the electrons colliding with the surfaces.

In the present note we ascertain the influence of the surface properties on the conductivity of a flat plate. At sufficiently low temperatures, when the phonon emission and absorption processes in collisions between the electrons and the surface are immaterial, the electron reflection is determined by the surface imperfections. Two types of imperfection are possible—impurities and vacancies, and these lead to differences between the properties of the near-surface layer on their volume properties, or various roughnesses. The scale of the roughnesses can be small in comparison with the thickness of the plate, and their influence will be reduced to a certain boundary condition that must be satisfied by the electron distribution function on the "averaged" surface.

A distinction should be made between two limiting cases. In the first case, when the electron wavelength is small in comparison with the average curvature of the surface, one can use the geometrical-optics approximation[4,5]. We consider a different case, when the average height of the roughnesses is small in comparison with the normal component of the electron wavelength, and the roughnesses can be treated by perturbation theory. The latter condition is satisfied if a significant role is played by electrons glancing along the surface. Such electrons are important under conditions of the anomalous skin effect and are observed also in the study of magnetic surface levels[6]. The conductivity of thin plates ($b \ll l$) is similarly determined by electrons with small glancing angles p_x/p (p_x is the normal component of the momentum), such that $b/l \lesssim p_x/p \ll 1$. The plate thickness is assumed to be large enough to be able to disregard size quantization. The influence of the broadening of the quantum levels was investigated by Chaplik and Entin[7].

PRINCIPAL RESULTS

The boundary condition that must be satisfied by the distribution function on the "averaged" surface of the plate was obtained in[8] and is of the form

$$f^{>}(\mathbf{p})=f^{<}(\mathbf{p})+p_x\int p_x'\xi_2(\mathbf{p}-\mathbf{p}')\,[f^{<}(\mathbf{p}')-f^{<}(\mathbf{p})]\frac{d^2p'}{\pi^2}. \qquad (1)$$

The distribution function $f^{>}$ in the left-hand side of (1) describes electrons moving away from the surface, and the function $f^{<}$ describes electrons moving into the surface. These functions depend on one and the same value of the energy, since the energy is conserved when an electron collides with a static surface, and on the two-dimensional vector p lying in the plane of the plate surface. The normal component of the momentum is expressed in terms of the energy and the tangential component:

$$p_x=(2m\varepsilon-p^2)^{1/2},\quad p_x'=(2m\varepsilon-p'^2)^{1/2};$$

for simplicity, the spectrum is assumed to be quadratic. The function $\xi_2(p)$ is the correlator of the roughnesses and is a characteristic of the surface. We shall assume in the estimates that $\xi_2(p)$ differs from zero in the region $p < 1/d$, and $\xi_2(0) \sim a^2d^2$; a is the average height of the roughnesses and d is the average length of the flat sections of the surface.

Condition (1) is identically satisfied for any distribution function that depends only on the energy. This condition ensures the vanishing of the current through the surface at any field orientation.

Inside the plate, $-b/2 < x < b/2$, the dependence of the distribution function on the coordinates is determined by the kinetic equation

$$v_x\frac{df}{dx}+\frac{v_0}{l}f=-ev\mathbf{E}\frac{df_0}{d\varepsilon}, \qquad (2)$$

where f is a nonequilibrium increment, linear in the field E, to the equilibrium value of the distribution function $f_0(\epsilon)$, the derivative of which at low temperatures is proportional to a δ-function: $df_0/d\epsilon = -\delta(\epsilon - \epsilon_0)$; v_0 and ϵ_0 are the Fermi velocity and energy, and l is the mean free path on the Fermi surface. The time-constant homogeneous electric field **E** is chosen parallel to the plate surface.

The problem reduces to a solution of Eq. (2) with boundary conditions (1) at $x = \pm b/2$, and to a subsequent calculation of the current

$$j = \frac{e}{4\pi^3}\int \mathbf{v} f \, d^3p. \quad (3)$$

We assume the sample surfaces to be identical, i.e., described by a single function $\xi_2(\mathbf{p})$. Then the two boundary conditions (1) at $x = \pm b/2$ can be satisfied simultaneously by choosing the solution of Eq. (2) in the form

$$f = \frac{el\mathbf{pE}}{p_0}\left\{1 - q(\mathbf{p})\exp\left[\frac{p_0(b/2 \mp x)}{p_x l}\right]\right\}\delta(\varepsilon - \varepsilon_0) \quad (4)$$

with the function $q(\mathbf{p})$ not known beforehand. The two signs in this formula correspond to the positive and negative projections of the velocity v_x, while p_x is taken to be positive: $p_x = m\,|\,v_x\,|$.

Substituting (4) in (1), we obtain an integral equation for the function $\mathbf{q}(\mathbf{p})$:

$$\left[\exp\left(\frac{bp_0}{lp_x}\right) - 1\right] q(\mathbf{p}) = p_x \int p_x' \xi_2(\mathbf{p} - \mathbf{p}') \times \left\{1 - q(\mathbf{p}) - \frac{\mathbf{p}'\mathbf{p}}{p^2}[1 - q(\mathbf{p}')]\right\}\frac{d^2p'}{\pi^2}. \quad (5)$$

The conductivity of the plate $\sigma = \int j\,dx/Eb$ is expressed in terms of $q(p)$ in the following manner:

$$\sigma = \frac{e^2 l}{4\pi^3 p_0}\int\left[1 - q(p)\frac{lp_x}{bp_0}\left[\exp\left(\frac{bp_0}{lp_x}\right) - 1\right]\right]\frac{p^2}{p_x}d^2p. \quad (6)$$

The quantities under the integral sign depend on the two-dimensional momentum $\mathbf{p}$ and on the Fermi momentum $p_0 = (2m\epsilon_0)^{1/2}$, since one integration in (3) has been carried out with the aid of a δ-function.

If the surface is isotropic in the mean, then the function $q(\mathbf{p})$ depends only on the modulus of the vector $\mathbf{p}$, and it is convenient to regard it as a function of $p_x = (p_0^2 - p^2)^{1/3}$ (this was already used in the averaging over the angle of the vector $\mathbf{p}$ in (6)). We shall show later on that for thin plates ($b \ll l$) the main contribution to the integral (6) is made by the region $p_x \gg p_0 b/l$. Expanding the exponential, we represent (6) in the form

$$\sigma = \frac{e^2 l}{2\pi^2 p_0}\int_0^{p_0}(p_0^2 - p_x^2)[1 - q(p_x)]dp_x + O\left(\frac{b}{l}\right), \quad (7)$$

where the term of order b/l results from integration over the region $0 < p_x/p_0 \lesssim b/l$.

Thus, by obtaining the solution of Eq. (5), we can calculate the conductivity in accordance with formula (7). Further calculations can be carried out in different limiting cases.

SMALL CARRIER GROUPS

We consider the simplest limiting case $p_0 d \ll 1$. The correlator $\xi_2(\mathbf{p} - \mathbf{p}')$ in (5) in the integration region $p' < p_0$ must be regarded under this condition to be a constant on the order of a^2d^2. Inasmuch as an expansion in $p_x a$ is carried out in the derivation (1), and all the electrons with $p_x < p_0$ are important in (5), then the condition $p_0 a \ll 1$ is essential if (1) is to be applicable. This means that we are considering small groups of carriers with a sufficiently small Fermi momentum p_0 (for good surfaces, a and d appear to be of the order of 10–100 atomic distances).

After integration with respect to the angle, the expression containing $\mathbf{p}'\cdot\mathbf{p}$ vanishes. Therefore, accurate to terms of order $(p_0 d)^2$ we have

$$q(p_x) = \frac{1}{\pi^2}p_x\int p_x'\xi_2\,d^2p' \Big/ \left[\exp\left(\frac{bp_0}{lp_x}\right) - 1 + \frac{p_x}{\pi^2}\int p_x'\xi_2\,d^2p'\right] \quad (8)$$

In the region $bp_0/lp_x \ll 1$, expression (8) takes the form

$$q(p_x) = p_x^2/(p_x^2 + p_1^2), \quad (9)$$

where

$$p_1^{-2} = \frac{l}{\pi^2 bp_0}\int p_x'\xi_2(\mathbf{p} - \mathbf{p}')d^2p'. \quad (10)$$

Substituting (9) in (7), we get

$$\sigma = \sigma_0\frac{3p_1^2}{2p_0^2}\left(\frac{p_0^2 + p_1^2}{p_0 p_1}\operatorname{arctg}\frac{p_0}{p_1} - 1\right). \quad (11)$$

From this we get for a relatively thick plate ($p_1 \gg p_0$)

$$\sigma = \sigma_0(1 - p_0^2/5p_1^2),$$

and for a very thin one ($p_1 \ll p_0$)

$$\sigma = \sigma_0 3\pi p_1/4p_0.$$

The quantity p_1 is estimated with the aid of (10):

$$p_1 \sim \left(\frac{b}{l}\right)^{1/2}\Big/ a\,dp_0, \qquad p_0 d \ll 1.$$

We note that in the derivation of (11) it was assumed that $adp_0^2 \ll 1$.

A plot of the relative resistance σ_0/σ against l/b is shown in the figure. The proportionality constant in (10) was chosen such that $p_0^2/p_1^2 = l/2b$. The dashed line shows the dependence of the resistance in the case of completely diffuse reflection, when $f^{>} = 0$ on the surface of the plate for all electrons traveling in the interior of the metal; in this case

$$q(p_x) = \exp(-p_0 b/p_x l).$$

Formula (11) admits of a simple interpretation. At $l \gg b$ the electrons contributing to the conductivity are those traveling at a sufficiently large angle to the surface, $p_1/p_0 \gtrsim p_x/p_0 \gg b/l$. For these electrons, the probability of being diffusely scattered in one collision with the surface, which is proportional to

$$p_x\int p_x'\xi_2\,d^2p'$$

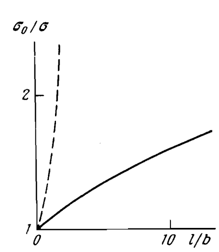
(see (1)), is small. However, the number of collisions $p_x l/p_0 b$ between the surface and an electron traveling at the glancing angle p_x/p_0 and traversing the mean free path l can be large. The conductivity is determined by those electrons for which

$$\frac{lp_x^2}{bp_0}\int p_x'\xi_2\frac{d^2p'}{\pi^2} = \frac{p_x^2}{p_1^2} \lessdot 1.$$

The remaining electrons, for which $p_x \gg p_1$, are reflected from the surfaces of the plate diffusely and make no contribution to the current.

SMOOTH SURFACE

We consider another limiting case, $p_0 d \gg 1$. We make in (5) the substitution $\mathbf{p} - \mathbf{p}' = \mathbf{k}$. Accurate to terms of order $1/pd$, we have $p_x'^2 = p_x^2 + 2\mathbf{p}\cdot\mathbf{k}$, so that



the projection parallel to the momentum p is $k_{\parallel} = (p_x'^2 - p_x^2)/2p$. Since only ξ_2 depends on $k_\perp$ in the integral (5) (the surface is assumed as before to be isotropic in the mean), we can carry out integration with respect to $k_\perp$, introducing the dimensionless function

$$Q(k_{\parallel}d) = \frac{\int \xi_2(\mathbf{k})\,dk_\perp}{\pi a^2 d},$$

that depends on the dimensionless argument $k_{\parallel}d$.

Returning to the variable p_x', we rewrite (5) in the form

$$\left[\exp\left(\frac{p_0 b}{p_x l}\right)-1\right] q(p_x) = \lambda p_x \int_0^{p_0} p_x'^2 Q\left(\frac{p_x'^2 - p_x^2}{2p} d\right) \times\left\{1-q(p_x)-[1-q(p_x')]\left(1-\frac{p_x'^2-p_x^2}{2p^2}\right)\right\}\frac{dp_x'}{p}, \qquad (12)$$

here $\lambda = a^2 d/\pi$, and the function $Q(x)$ is even and differs from zero, mainly in the region $|x| < 1$. We shall assume that the ratio b/l is small and consider the region $p_0 b \ll p_x l$ in which the left-hand side of (12) is equal to $q(p_x)p_0 b/p_x l$. Equation (12) can be solved by iterations, if $q(p_x) \ll 1$ for all $p_x < p_0$. The conductivity is then in the principal approximation equal to the conductivity of a bulky sample, and the small correction is proportional to $-\sigma_0 l a^2/bd^2$. The correction is connected with the electrons for which $p_x \sim p_0$, and for (1) to be valid it is necessary to have $ap_0 \ll 1$, while the iterations are permissible if $a^2 p_0^2 l \ll (p_0 d)^{3/2} b$.

The most interesting case is when the diffuseness is not small, when $q(p_x)$ is close to unity. If this occurs in the region $p_x < (p_0/d)^{1/2}$, then the small p_x play the important role in (12), and upon integration with respect to p_x' the term with $1 - q(p_x')$ makes a small contribution. Omitting this term, we obtain a solution in the form (9) and (10). Estimating p_1 with the aid of (10), we get

$$p_1^2 \approx b(p_0 d)^{1/2}/la^2, \quad p_0 d \gg 1.$$

We can verify that the obtained approximation is valid under the condition $p_1 d \ll 1$, and for applicability of (1) we must have $a^2 p_0 \ll d$. In this case we obtain the following estimate for the conductivity:

$$\sigma \sim \sigma_0 \left(\frac{b}{l}\right)^{1/2} \frac{(p_0 d)^{1/4}}{ap_0}; \quad \frac{b}{l} \ll \left(\frac{b}{l}\right)^{1/2} \frac{(p_0 d)^{1/4}}{ap_0} \ll \frac{1}{p_0 d}. \qquad (13)$$

In the intermediate region, when

$$(p_0 d)^{1/2} \gg \left(\frac{b}{l}\right)^{1/2} \frac{(p_0 d)^{5/4}}{ap_0} \gg 1,$$

the integral equation (12) calls for a special investigation.

DISCUSSION OF RESULTS

An electron moving at a small angle to a rough surface is reflected from it mostly specularly (1). The resistance (11), (13), of this plate therefore increasing with decreasing thickness more slowly than in the case of fully diffuse reflection: the range of thickness in which it remains practically unchanged increases, and at very small thicknesses it increases in proportion to $(l/b)^{1/2}$.

If the glancing-angle region within which the electron is reflected mostly specularly is small in comparison with the angle through which the electron can be scattered by one collision with the surface (i.e., if the distribution function is a more pointed function of the angle than the correlator $\xi_2(\mathbf{p})$), and the electric field is parallel to the surface, then the boundary condition (1) reduces to a dependence of the coefficient of specularity on the angle of incidence in the standard boundary condition

$$f^> = (1-\rho(\theta))f^<, \qquad (14)$$

where

$$\rho(\theta) = \frac{p_x \int p_x' \xi_2(\mathbf{p}-\mathbf{p}')\,d^2p'}{\pi^2}.$$

The condition (14) with an arbitrary function $\rho(\theta)$ was used recently in[9].

We make one more remark concerning the assumed quadratic electron spectrum. The conduction of sufficiently thin plates is determined by the glancing electrons, i.e., by a "belt" on the Fermi surface where $v_x = 0$. The thickness dependence obtained above remains in force also for an anisotropic closed Fermi surface.

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198