Measurement of the relaxation time on acceleration of vessels with helium II and superfluidity in pulsars

J. S. Tsakadze and S. J. Tsakadze
Physics Institute, Georgian Academy of Sciences
(Submitted September 26, 1972)
Zh. Eksp. Teor. Fiz. **64**, 1816-1823 (May 1973)

The behavior of superfluid helium on variation of the rotation velocity of vessels of various geometry is investigated experimentally. In this way an attempt is made to simulate relaxation processes accompanying sudden acceleration of the solid shell of a pulsar.

DESCRIPTION OF SETUP

We used the procedure previously employed in a number of studies[1,2] to investigate the peculiarities of the spinning of liquid helium. In these studies, the vessel with the liquid, freely supported by a magnetic suspension, was imparted a rotational impulse, and then rotated freely (gradually slowing down as a result of friction). The time interval from the start of the rotation to the assumption of uniform attenuation of the vessel determines the characteristic time, and the form of the $\omega_0(t)$ curve in this interval determines the relaxation process occurring in the "vessel + liquid" system.

We measured the relaxation times using a similar procedure for spherical and cylindrical vessels with liquid helium. In different experiments, the characteristic dimensions and boundary conditions were different. The experiments were performed both with twisting of initially immobile vessels, and with abrupt increase of the angular velocity of the vessel. Unlike the cited papers[1,2], in this study the support-free magnetic suspension was replaced by a simpler device used earlier by one of the authors[3], which had given good account of itself.

A schematic diagram of the instrument is shown in Fig. 1. A hollow glass sphere 1 (or a thin-wall bucket of Plexiglas), together with a brass disk 2 having a relatively large moment of inertia, were secured firmly on a straight shaft 3 made of stainless tubing of 3 mm diam. The sphere (or bucket) served as vessel for the helium and could be filled through a narrow opening of ~0.5 mm diameter in its upper part. The small opening ensured a negligible decrease of the amount of helium II carried away from the vessel, in the form of a film, during the course of the experiment. The vessel was refilled with liquid prior to each measurement.

A steel ball 7 of 2 mm diameter was soldered to the second end of the tube 3. A similar ball served as the end piece of the pole of electromagnet 8 mounted in the upper part of the Dewar cover. When the electromagnet coil was connected to a dc line fed from a storage battery, the balls were attracted to each other and the suspended vessel could rotate freely. The small contact area of the highly-polished surfaces of the balls minimized the friction between the moving and stationary parts of the instrument: the vessel, accelerated to an angular velocity $\sim 4\ \text{sec}^{-1}$ rotated for 70–80 minutes before stopping fully. Contributing to the small friction at the points of contact with the balls was also the fact that minimum current was made to flow through the electromagnet coil. The current was regulated in such a way that the lifting force of the electromagnet exceeded only slightly the rotating part of the instrument.

The shaft 3 carried also a firmly secured rotor 6 of a small induction motor, the stator of which consisted of coils 5, which produced a rotating magnetic field (there were six such coils). The initially stationary instrument was accelerated to $\omega_0 \sim 7\ \text{sec}^{-1}$ within ~10 sec. By connecting the stator coils to the line for a short time it was possible to increase instantaneously the rotary speed of the vessel. There were of course no vertical oscillations of the rotating vessel. The amplitude of the radial oscillations, on the other hand, did not exceed ~1 mm after the instrument was fully adjusted.

A mirror 9, on which a focused light beam was incident through window 10, was glued to the upper part of the tube 3. The light beam reflected from the mirror was incident on an FSK-1 photoresistor connected in a special electronic circuit for the measurement (with the aid of a ChZ-4 frequency meter) and automatic recording (with the aid of a digital-printing TsPM-1 instrument) the time between two successive pulses produced when the light beam illuminated the photoresistance. The period of revolution was measured accurate to 10^{-3} sec. The experimental data were reduced with a computer.

In the experiment, the vessel 1 was lowered with the aid of catch 4 into the helium bath. After the vessel 1 was filled with liquid, it was raised into the helium vapor and suspended in the field of electromagnet 8. The moment of inertia of the liquid was 15–20% of the total moment of inertia of the rotating part of the instrument. The entire setup was mounted on a floating concrete foundation, thereby protecting it reliably against mechanical disturbances.

The experiments were performed in the following manner: the small electric motor was turned on, the vessel rotation was accelerated (or changed) to the required value, the motor was turned off, and the in-

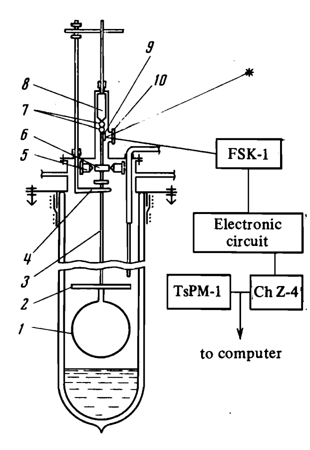

FIG. 1. Schematic diagram of the instrument. The electronic circuit shapes and amplifies the pulses and controls the triggering of the ChZ-4 frequency meter.

strument was left alone. As the initially-immobile liquid was gradually set in motion by the walls of the rotating vessel, the rotary speed of the system decreased, first abruptly and then more and more slowly, and ultimately the speed began to decrease exponentially, owing to the friction against the balls 7 and against the gaseous medium. As is well known, free rotation with small damping follows the equation

$$\omega_0 \sim e^{-\gamma t/J},$$

where γ is the damping coefficient and J is the moment of inertia of the vessel. Figure 2 shows a typical plot of the rotary velocity ω_0 against the time t for a sphere with a smooth internal surface, but in the case of motion from standstill (from $\omega_0 = 0$ to $\omega_0 = 4.2\ \mathrm{sec}^{-1}$) and in the case when the speed is increased from 3 to 4.2 sec^{-1}. The upper plots (straight lines) pertain to the rotation of an empty vessel, and the lower to a vessel filled with liquid helium. The function $\omega_0(t)$ is similar in form also for a cylindrical vessel.

EXPERIMENTS WITH SPHERICAL VESSEL

To perform experiments in a spherical geometry, we chose from among glass vessels prepared in the glass-blowing shop a sphere with minimum differences of the diameters measured in different directions in the equatorial plane. The experiments were performed with a sphere of radius 3.4 ± 0.05 cm. We investigated initially the temperature dependence of the relaxation time in a sphere with a smooth surface, filled with liquid helium. The corresponding results are shown in Fig. 3. The upper curve shows the acceleration of an initially immobile instrument to $\omega_0 = 4\ \mathrm{sec}^{-1}$, and the lower curve the acceleration of a uniformly damped instrument from $\omega_0 = 3\ \mathrm{sec}^{-1}$ to $\omega_0 = 4\ \mathrm{sec}^{-1}$. We were unable to note a difference in the initial character of the spinning between helium I and helium II, as was observed by Reppy et al.[2]. At the λ point, however, both in the case of twisting of an immobile instrument and in the case of its acceleration, jumplike changes in the relaxation time were observed.

In the next experiment, the sphere was filled with crushed Plexiglas with particle dimensions ~0.5 mm, in order to bind the normal component of helium II. The $\omega_0(t)$ plot at T = 1.74°K is shown in Fig. 4. From the linearity of this plot we see that in this case the relaxation time is less than the time needed to wind up the vessel (~10 sec). A similar result was obtained also at T = 1.47°K and when the instrument was accelerated from $\omega_0 = 3\ \mathrm{sec}^{-1}$ to $\omega_0 = 4\ \mathrm{sec}^{-1}$.

In addition, experiments were performed under conditions of good pinning of the vortices on the moving solid surface. To this end, Plexiglas particles with dimensions 0.05 cm were glued to the inner surface of the sphere. The temperature dependence of the relaxation time is shown for this case in Fig. 5. Under conditions of a rough surface, the relaxation times decreased appreciably in comparison with a sphere with smooth surface (to ~20 sec), and were independent of the temperature within the limits of the measurement accuracy. Incidentally, the practical equality of the relaxation times in helium I and helium II may mean that the shortening of the relaxation process is due in this case not so much to the pinning of the vortices (although this cannot fail to have an effect) as to the turbulization of the normal component. To clarify this question fully, we are planning measurements at very

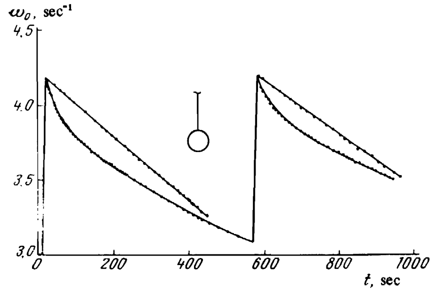

FIG. 2. Dependence of the angular velocity on the time for a smooth sphere. Straight lines—empty vessel, curves—vessel with helium II (T = 1.57°K).

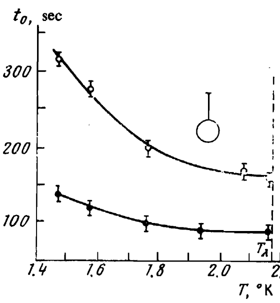

FIG. 3

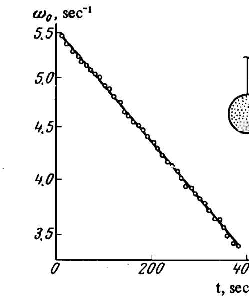

FIG. 4

FIG. 3. Dependence of the relaxation time on the temperature: ○—acceleration of vessel from standstill to $\omega_0 = 4\ \mathrm{sec}^{-1}$; ●—acceleration of a uniformly attenuating instrument from $\omega_0 = 3\ \mathrm{sec}^{-1}$ to $\omega_0 = 4\ \mathrm{sec}^{-1}$.

FIG. 4. Dependence of the angular velocity of rotation on the time for a porous sphere (the ordinate scale is logarithmic); T = 1.74°K.

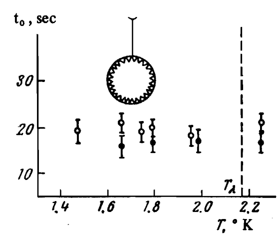

FIG. 5

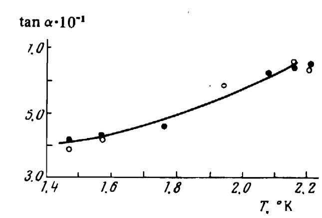

FIG. 6

FIG. 5. Dependence of relaxation time on the temperature for a sphere with a rough internal surface: ○—acceleration of instrument from standstill to $\omega_0 = 4\ \mathrm{sec}^{-1}$; ●—acceleration of instrument from 3 to 4 sec^{-1}.

FIG. 6. Temperature dependence of the slope of the linear sections of plots such as in Fig. 2 for a sphere with smooth surface (tan α is proportional to the rotation damping coefficient γ).

low temperatures, when the amount of the normal component is vanishingly small.

As already noted, after the liquid and the vessel begin to rotate at equal speed, the rotation damping coefficient should remain unchanged. Its value depends not only on the external dragging force but also on the intensity of the interaction of the vessel with the liquid. Indeed, the liquid overtakes the decelerating vessel and its interaction with the shell has an accelerating character, i.e., it decreases the damping. But this statement is valid only in the mean. A more detailed analysis leads to the conclusion that as the vessel becomes decelerated, the superfluid component overtakes it more and more strongly, until the metastable excess of the vortices present in it disintegrates (see the experimental data by Andronikashvili et al.[4] and by Packard and Sanders[5]). This should lead to oscillations of the damping, which are superimposed on the mean constant damping, or to an alternating-sign variation of the mean value. This in turn gives rise to oscillations superimposed on the average dependence of ω_0 on t. The experimental curves of Fig. 2 show a slight hint of such oscillations. We are planning to study this question in greater detail, too.

Figure 6 shows the temperature dependence of the slope of the linear sections of the $\omega_0(t)$ curves similar to those shown in Fig. 2, for a sphere with a smooth surface. These slopes are proportional to the rotation damping coefficient γ. It is seen from Fig. 6 that with decreasing temperature the attenuation of the liquid + vessel system decreases. This means that the larger the amount of the superfluid component in the liquid helium in the rotating state, the stronger it is coupled to the vessel. This coupling of the vessel with the liquid is via the quantized Onsager-Feynman vortices, which are drawn to the surface (owing to the drop in pressure at the core of the vortex) with a force that coincides with the vortex tension, which in turn is proportional to the amount ρ_s of the superfluid component of the liquid helium.

It should be noted that the interaction of the superfluid liquid with a solid (in particular with an oscillating disk executing axially-torsional oscillations in rotating liquid helium), effected via the tension of the vortices, is very effective—see the data by Andronikashvili and one of the authors[6], where such an interaction gives rise to a stronger damping of the oscillations of the disk than its interaction with the normal component of the rotating liquid helium. It is also shown in[6] that by varying the cleanliness of the disk surface it is possible to vary the coupling of the vortices with this surface. When the slippage coefficient is decreased by making the wall rough, the coupling of the superfluid component with the vessel should increase, and the damping coefficient should decrease. Indeed, whereas for a sphere with a smooth surface we have the ratio $\gamma(2.2°K)/\gamma(1.47°K) = 1.6$, for a sphere with a rough surface this ratio is equal to 2. Control experiments with an empty vessel have shown that the rotation damping coefficient is insensitive to pressure changes from 50 to 1 mm Hg.

EXPERIMENTS WITH CYLINDRICAL VESSEL

The bucket for these experiments was machined from organic glass in one setting of the lathe. The beaker radius was 35 mm, its height 40 mm, and the wall thickness was 0.5 mm. The beaker cover was also made of Plexiglas and glued to the beaker. The liquid helium was poured into the vessel through a hole of 0.5 mm diameter in the cover.

Experiments similar to those described above were performed also in the case of the cylindrical geometry. The results agree fully with the relations illustrated in Figs. 2, 3, and 5 in the case of smooth and rough surfaces, the latter produced by gluing particles of Plexiglas measuring 0.05 cm to the bottom and to the cover of the beaker. Just as in the case of the sphere, the rotation damping coefficient of the beaker with the liquid decreases with temperature. Whereas for a bucket with a smooth bottom and cover we have the ratio $\gamma(2.2°K)/\gamma(1.47°K) = 1.7$, for a bucket with rough end surfaces the ratio equals 2.

We have bound the normal component of the liquid also in the cylindrical vessel. To this end, the bucket was made in the form of a stack of ribbed disks 0.02 cm thick. The distance between disks was 0.05 cm. The obtained $\omega_0(t)$ dependence is similar to that shown in Fig. 4. This means, in full agreement with the results of Esel'son et al.[1], that the corresponding relaxation times are shorter than the time required to wind up the vessel. Whereas for a bucket at $T = 1.47°K$ the relaxation time is $t = 130$ sec, for the same vessel with a coaxially inserted additional cylindrical surface, spaced equal distances away from the walls and from the rotation axis, the relaxation time decreases to $t = 65$ sec.

DISCUSSION OF RESULTS AND COMPARISON WITH THE BEHAVIOR OF PULSARS

We can conclude from our experiments that the jump of the relaxation time at the λ point (Fig. 3) indicates an appreciable influence of the superfluid component of the liquid on the duration of the redistribution of the angular momentum between the liquid and the solid surface of the shell.

It is natural to assume that the change of the pressure regime of the superfluid component in a certain volume calls for a time proportional to the linear dimensions of this volume. Indeed, introduction of an additional surface into the bucket, which reduces the linear dimension of the vessel to half its value, decreases the relaxation time also by a factor of two. Experiments with a porous sphere show that at small characteristic dimensions of the liquid the relaxation time is immeasurably small. Thus, $t_0 \propto R$, where R is the linear dimension of the volume occupied by the liquid. Further, in a rotating sphere with helium II, the coupling of the normal component with the vessel and the interaction of its different layers are due to viscosity (therefore, it is impossible to couple the normal component with a vessel, in order to observe the relaxation only in the superfluid component, without introducing additional solid surfaces, and the latter "bind" also the superfluid component because of the transcriticality of the rotation and the presence of vortices). On the other hand, the coupling between the normal and the superfluid components is via the mutual friction, through scattering of the normal component by the vortex axes. One can therefore assume that $t_0 \propto (\omega_0 c_n)^{-1}$, where c_n is the concentration of the normal component, and the angular velocity ω_0 characterizes the number of vortices. Consequently, the relaxation time should satisfy the condition

$$t_0 = KR / \omega_0 c_n. \tag{1}$$

This semiempirical formula agrees indeed with our data, and within the range of variation of all the quantities contained in it, which were realized in the experiments described in this article, we obtained $K \approx 10 \pm 3\ \text{cm}^{-1}$. A more detailed investigation may yield a more accurate dependence of t_0 on the parameters that determine it. However, as will be shown below, even in this rough approximation the relation (1) can be used to compare phenomena occurring in rotating vessels containing liquid helium II, on the one hand, with the kinetics of the rotation of pulsars after their acceleration.

At the present time, the pulsar model proposed by Ruderman[7] is being widely discussed. According to this model, a neutron liquid with density $\rho \sim 10^{14}\ \text{g/cm}^3$ is contained inside a solid spherical envelope of the star, of radius $R \sim 10$ km. As first shown by Migdal[8], this liquid is in the superfluid state. The temperature of the λ transition for a neutron liquid is $T \sim 10^{10}$–10^{12}°K, whereas the pulsar temperature is $\sim 10^{8}$°K and is quite uniform over the entire star. Thus, the temperature of the neutron liquid in pulsars is lower than T_λ by at least two orders of magnitude. At the corresponding temperature in liquid helium, the phonon density of the normal component is vanishingly small ($\rho_n^{ph}/\rho = 10^{-10}$) and ρ_n is determined in practice by the impurity content. The neutron superfluid liquid of a pulsar contains an admixture of a superfluid proton quantum liquid and a normal electron quantum liquid. The concentration of each is $\sim 1\%$[9].

The coupling between the pulsar shell and the superfluid neutron liquid is quite weak and is effected (as in helium II) via quantized vortices, the possibility of formation of which in a rotating superfluid neutron liquid was first noted by Ginzburg and Kirzhnits[10]. For a neutron star, the critical velocity of vortex formation is $\omega_{c1} \sim 10^{-14}$ sec^{-1}, while the velocity at which the superfluidity vanishes is $\omega_{c2} \sim 10^{20}$ sec^{-1}. Thus, pulsars (whose angular velocity ranges from 1.7 to 190 sec^{-1}) should be filled with quantized vortices parallel to their rotation.

According to the two-component neutron-star model proposed by Baym et al.[11] the mutual friction between the normal component of the star (hard shell and admixture of "normal" electrons) and the superfluid neutron liquid is due to scattering of electrons by the cores of the vortices and is characterized by a relaxation time

$$\tau = \tau_n \omega_{c2} / \omega_0, \qquad (2)$$

where ω_0 is the pulsar rotary velocity and τ_n is the relaxation time, under the assumption that the neutron liquid is normal. We note that in this case $\tau_n \sim 10^{-11}$ sec., i.e., if the pulsar were to consist of a normal neutron liquid, the process of its acceleration after a starquake would be practically instantaneous.

For a superfluid pulsar, τ increases strongly in comparison with τ_n. At $\omega_{c2}/\omega_0 \sim 10^{18}$ we obtain $\tau \sim 10^7$ sec. Indeed, after the known "first" accelerations of the pulsars PSR 0833-45[12] (in the constellation Vela-X) and NP 0532[13] (in the Crab nebula) there were observed relaxation processes characterized by respective macroscopic times

$$\tau_{\text{Vela-X}} = 3.7 \cdot 10^7 \text{ sec}, \quad \tau_{\text{Crab}} = 6 \cdot 10^5 \text{ sec}$$

(see, e.g., references 23 and 24 in[14]).

If we assume that in pulsars the interaction between the n- and s-components is effected in the same manner as in rotating helium II, then formula (1) should be valid also for a pulsar, and the coefficient K should have approximately the same value as in helium. Then using the value $K = 10$ and the values assumed for pulsars, $R \sim 10^6$ cm, $c_n = 0.01$, $\omega_{0\,\text{Crab}} = 190$ sec^{-1}, and $\omega_{0\,\text{Vela-X}} = 70$ sec^{-1}, we obtain $\tau'_{\text{Crab}} \approx 5 \times 10^6$ sec and $\tau'_{\text{Vela-X}} = 10^7$ sec, which agrees in order of magnitude with the observed relaxation times of the two pulsars.

The result shown in Fig. 5 demonstrates that roughnesses of the surface increase the coupling between the shell and the liquid, as a result of which the relaxation time falls off sharply. This in turn makes the agreement with (1) worse. Further, when the pulsar rotation attenuates, in spite of the weak coupling between its "crust" and the vortex, the angular momentum returned to the shell by the superfluid component should be quite appreciable. Indeed, as shown in[6], the coefficient of slippage of the vortices relative to the surface is smaller the smaller their relative velocity.

It can be stated in conclusion that the results favor the possibility of simulating the behavior of pulsars in low-temperature laboratories. The preliminary data presented here confirm the notion that the pulsar is a neutron star that is almost completely filled with a superfluid liquid.

The authors consider it their pleasant duty to thank É. L. Andronikashvili for stimulated interest in the work. The authors thank Yu. G. Mamaladze for a systematic discussion of the results and M. K. Zaĭonchik for help in the construction and adjustment of the electronic part of the instrument. The authors thank R. E. Packard and I. E. Serbin for calling their attention to the possibility of velocity oscillations of a decelerating vessel filled with a superfluid liquid.

[1] B. N. Esel'son, K. D. Sinel'nikov, B. G. Lazarev, and A. D. Shvets, Zh. Eksp. Teor. Fiz. 31, 912 (1956) [Sov. Phys.-JETP 4, 774 (1957)].
[2] J. D. Reppy, D. Depatie, and C. J. Lane, Phys. Rev. Lett. 5, 12 (1960).
[3] Dzh. S. Tsakadze, Zh. Eksp. Teor. Fiz. 42, 985 (1962) [Sov. Phys.-JETP 15, 681 (1962)].
[4] É. L. Andronikashvili, G. V. Gudzhabidze, and Dzh. S. Tsakadze, Zh. Eksp. Teor. Fiz. 50, 51 (1966) [Sov. Phys.-JETP 23, 34 (1966)].
[5] R. E. Packard and T. M. Sanders, Phys. Rev. Lett., 22, 823 (1969).
[6] É. L. Andronikashvili and Dzh. S. Tsakadze, Zh. Eksp. Teor. Fiz. 37, 562 (1959) [Sov. Phys.-JETP 10, 397 (1960)].
[7] M. Ruderman, Nature, 218, 1128 (1968).
[8] A. B. Migdal, Zh. Eksp. Teor. Fiz. 37, 249 (1959) [Sov. Phys.-JETP 10, 176 (1960)].
[9] J. Nemeth and D. W. L. Sprung, Phys. Rev., 176, 1496 (1968).
[10] V. L. Ginzburg and D. A. Kirzhnits, Zh. Eksp. Teor. Fiz. 47, 2006 (1964) [Sov. Phys.-JETP 20, 1346 (1965)].
[11] G. Baym, G. J. Pethick, D. Pines, and M. Ruderman, Nature, 224, 872 (1969).
[12] V. Radhakrishan and R. N. Manchester, Nature, 222, 228 (1969).
[13] P. Boynton, E. Groth, P. Patridge, and D. Wilkinson, I. A. U. Circular No. 2179 (oktober 30), 1969.
[14] D. Pines, Proc. 12-th Intern. Conf. Low Temp. Phys., Kyoto, Japan, 1970.

Translated by J. G. Adashko
195