Parametric instabilities in a plasma containing two types of ions

A. B. Kitsenko and K. N. Stepanov
Physico-technical Institute
Ukrainian Academy of Sciences
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The stability of a homogeneous plasma which consists of electrons and of two types of ions and which is located in an alternating electric field with a frequency of the order of the ion cyclotron frequency is investigated. The relative velocity of the ions in the field in the direction perpendicular to the magnetic field is assumed to be smaller than or of the order of the ion thermal velocity. Parametric excitation of longitudinal oscillations moving almost perpendicular to the magnetic field is considered. Oscillation excitation is due to relative motion of the ions. The oscillation increments are determined for the frequencies of ion-ion hybrid resonance, ion cyclotron oscillations and ion-ion sound.

1. INTRODUCTION

If a plasma contains two sorts of ions, new low-frequency oscillation modes appear[1-4]. The excitation of these modes of electromagnetic waves in a dense plasma with large dimensions can be quite effective[5], and these modes can be used successfully for high-frequency heating of the plasma. We refer to experiments[6,7] in which effective high-frequency heating was produced in a plasma containing a mixture of ions of two sorts, using electromagnetic waves with frequencies on the order of the cyclotron frequencies of the ions. In these experiments, the absorption of the waves and the rate of plasma heating were large in a wide range of variation of the constant magnetic field, and had a maximum corresponding to the frequency of the ion-ion hybrid resonance.

The relative motion of the particles in the field of a low-frequency electromagnetic wave can be the cause of numerous short-wave instabilities of the two-stream and parametric type[8,9]. If the growth increment of these oscillations is much larger than the pump frequency ω_0, then the adiabatic approximation is valid, and in this case the instability has a pure two-stream character. An important feature[10] of such instabilities is that the turbulence level can be very high, since the role of the nonlinear terms in the equations of motion of the electrons is small for such oscillations. This leads to a very rapid damping of the pump wave and to turbulent heating of the plasma. If the relative velocity of the ions of the two sorts exceeds a certain critical value on the order of the thermal velocity of the ions, then the growth increment of the oscillations is of the order of the lower hybrid frequency or the ion Langmuir frequency. On the other hand, if the relative velocity of the ions is smaller than the thermal velocity, then the growth increments of the oscillations become smaller than the cyclotron frequency of the ions. In this case the adiabatic approximation cannot be used, and the resultant instabilities are parametric. However, as before, the level of the oscillations in the turbulent state, when ion-cyclotron parametric instabilities develop, is much higher than the level of the oscillations of the low-frequency instabilities in a plasma consisting of electrons and of ions of a single sort.

In the present paper we study parametric instabilities of a plasma containing ions of two sorts in a homogeneous alternating electric field, when the frequency ω_0 of the pump wave is of the order of the ion cyclotron frequency. It is assumed that the relative velocity of the ions is smaller than the thermal velocities, and that the wavelength of the developing oscillations is much smaller than the characteristic distances over which the electric field of the pump wave and the plasma density vary. The plasma pressure is assumed to be small in comparison with the magnetic pressure. In this case, the considered oscillations can be regarded as potential.

2. DISPERSION EQUATION

The development of potential oscillations in a plasma situated in a constant homogeneous magnetic field $\mathbf{B}_0$ and an alternating electric field $\mathbf{E} = \mathbf{E}_0 \sin \omega_0 t$ is described by the equation[12]

$$\varphi(\omega)+\sum_{\alpha}\sum_{s,m=-\infty}^{\infty} J_{s+m}(a_{E\alpha})J_m(a_{E\alpha})e^{-is(\delta_\alpha+\pi)}\delta\varepsilon_\alpha(\omega+m\omega_0)\varphi(\omega-s\omega_0)=Q;$$

where $\varphi(\omega)$ is the Laplace transform of the oscillation potential:

$$\varphi(\omega)=\int_0^\infty \varphi(t)e^{i\omega t}\,dt, \qquad (2.2)$$

$Q(\omega)$ is a quantity proportional to the initial perturbation of the particle distribution function, J_m is a Bessel function

$$a_{E\alpha}=\frac{|e_\alpha|}{m_\alpha}\left[\left(\frac{k_\parallel E_{0\parallel}}{\omega_0^2}+\frac{\mathbf{k}_\perp\mathbf{E}_{0\perp}}{\omega_0^2-\omega_{B\alpha}^2}\right)^2+\frac{\omega_{B\alpha}^2(\mathbf{B}_0[\mathbf{k}\mathbf{E}_0])^2}{B_0^2\omega_0^2(\omega_0^2-\omega_{B\alpha}^2)^2}\right]^{1/2},$$

$$\operatorname{ctg}\delta_\alpha=\frac{\omega_0}{\omega_{B\alpha}}\left(\frac{k_\parallel E_{0\parallel}}{\omega_0^2}+\frac{\mathbf{k}_\perp\mathbf{E}_{0\perp}}{\omega_0^2-\omega_{B\alpha}^2}\right)\left[\frac{\mathbf{B}_0[\mathbf{k}\mathbf{E}_0]}{B_0(\omega_{B\alpha}^2-\omega_0^2)}\right]^{-1}. \qquad (2.3)^*$$

The contribution of the particles of sort α to the longitudinal dielectric constant is determined by the usual expression

$$\delta\varepsilon_\alpha(\omega)=\frac{\omega_{p\alpha}^2}{k^2v_{T\alpha}^2}\left[1+i\sqrt{\pi}\,z_{0\alpha}\sum_{s=-\infty}^{\infty}A_s(x_\alpha)w(z_{s\alpha})\right], \qquad (2.4)$$

where

$$A_n(x)=e^{-x}I_n(x), \quad x_\alpha=k_\perp^2\rho_\alpha^2=k_\perp^2v_{T\alpha}^2/\omega_{B\alpha}^2,$$

$$w(z)=e^{-z^2}\left(\frac{k_\parallel}{|k_\parallel|}+\frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}\,dt\right), \quad z_{s\alpha}=\frac{\omega-s\omega_{B\alpha}}{\sqrt{2}\,k_\parallel v_{T\alpha}},$$

$$\omega_{B\alpha}=e_\alpha B_0/m_\alpha c, \quad v_{T\alpha}=(T_\alpha/m_\alpha)^{1/2},$$

$\mathbf{k}$ is the wave vector of the oscillations, $k_\parallel = k\cos\theta$, $k_\perp = k\sin\theta$, and θ is the angle between $\mathbf{k}$ and $\mathbf{B}_0$. The subscript α = e denotes electrons, and the subscript α = i = 1, 2 denotes ions of the first and second sorts with charges e_1 and e_2, respectively, and with masses m_1 and m_2.

We shall consider oscillations for which $\theta \approx \pi/2$ and the frequency of the oscillations is much lower than the electron cyclotron frequency. We confine ourselves below to oscillations in two cases.

A. Hydrodynamic approximation for the electrons. If $|z_{ne}| \gg 1$ and $k\rho_e \ll 1$, then

$$\delta\varepsilon_e = \frac{\omega_{pe}^2}{\omega_{Be}^2} - \frac{\omega_{pe}^2}{\omega^2}\cos^2\theta. \qquad (2.5)$$

We assume that the term $(\omega_{pe}^2/\omega^2)\cos^2\theta$ in (2.5) is small in comparison with $\delta\epsilon_i$, owing to the smallness of $\cos^2\theta$. This case corresponds to "cold" electrons, the role of which reduces to polarization of the plasma under the influence of the electric field of the oscillations propagating perpendicular to the external magnetic field.

B. "Hot" electrons (small phase velocities or short waves). If $|z_{0e}| \ll 1$ or $k\rho_e \gg 1$, then

$$\delta\varepsilon_e = \omega_{pe}^2 / k^2 v_{Te}^2. \qquad (2.6)$$

In this case the role of the electrons reduces to Debye screening of the space charge of the oscillations.

In either case, $\delta\epsilon_e$ does not depend on the frequency, so that in the term with $\alpha = e$ of Eq. (2.1) we can sum over s and m, and this yields the term $\delta\epsilon_e\varphi(\omega)$:

$$\sum_{s,m} J_{s+m}(a_{Ee}) J_m(a_{Ee}) e^{-is(\delta_e+\pi)} \cdot \delta\varepsilon_e\varphi(\omega - s\omega_0) = \delta\varepsilon_e\varphi(\omega). \qquad (2.7)$$

Taking (2.7) into account, we can easily transform (2.1) into

$$[1 + \delta\varepsilon_e + \delta\varepsilon_1(\omega)]\Phi(\omega) + \sum_n a_n(\omega)\Phi(\omega - n\omega_0) = Q(\omega), \qquad (2.8)$$

where

$$\Phi(\omega) = \exp\left\{i\frac{\omega}{\omega_0}(\delta+\pi)\right\}\sum_{s=-\infty}^{\infty} J_s(a_{E1}) e^{-is(\delta_1+\pi)}\varphi(\omega - s\omega_0), \qquad (2.9)$$

$$a_n(\omega) = \sum_{m=-\infty}^{\infty} J_m(a_E) J_{m+n}(a_E)\,\delta\varepsilon_2(\omega + m\omega_0), \qquad (2.10)$$

and the quantities a_E and δ are given by

$$a_E = \left\{\left[\sum_{i=1,2}(-1)^i\frac{e_i}{m_i}\left(\frac{k_\parallel E_{0\parallel}}{\omega_0^2} + \frac{\mathbf{k}_\perp \mathbf{E}_{0\perp}}{\omega_0^2 - \omega_{Bi}^2}\right)\right]^2 + \left[\sum_{i=1,2}(-1)^i\frac{e_i\omega_{Bi}}{m_i\omega_0(\omega_0^2 - \omega_{Bi}^2)}\frac{\mathbf{B}_0[\mathbf{kE}_0]}{B_0}\right]^2\right\}^{1/2} \qquad (2.11)$$

$$\operatorname{ctg}\delta = \sum_{i=1,2}(-1)^i\left(\frac{k_\parallel E_{0\parallel}}{\omega_0^2} + \frac{\mathbf{k}_\perp \mathbf{E}_{0\perp}}{\omega_0^2 - \omega_{Bi}^2}\right) \times \left[\sum_{i=1,2}(-1)^i\frac{e_i\omega_{Bi}}{m_i\omega_0(\omega_{Bi}^2 - \omega_0^2)}\frac{\mathbf{B}_0[\mathbf{kE}_0]}{B_0}\right]^{-1}. \qquad (2.12)$$

We note that (2.8) contains only the relative velocity u of the ions of the two sorts ($a_E = ku/\omega_0$). The motion of the electrons relative to the ions does not enter in the fundamental equation (2.8) in the considered approximation. Replacing ω in (2.8) by $\omega - \nu\omega_0$ ($\nu = 0, \pm 1, \pm 2, \ldots$), we obtain an infinite system of difference equations for the quantities $\Phi(\omega - n\omega_0)$. Equating the determinant of this system to zero, we obtain a dispersion equation that determines the complex frequency of the considered oscillations:

$$\operatorname{Det}\|a_{\nu,n}\| = 0, \qquad (2.13)$$

where

$$a_{\nu,n} = \delta_{\nu,n} + a_{n-\nu}(\omega - \nu\omega_0).$$

Equation (2.13) will be investigated subsequently in a number of special cases.

3. "COLD" IONS (LONG-WAVE OSCILLATIONS)

We consider first parametric excitation of oscillations for which the ions can be regarded as "cold", i.e., $k\rho_i \ll 1$,

$$z_{ni} = (\omega - n\omega_{Bi}) / \sqrt{2}k_\parallel v_{Ti} \gg 1 \quad (n = 0, \pm 1).$$

In this case

$$\delta\varepsilon_i(\omega) = -\omega_{pi}^2 / (\omega^2 - \omega_{Bi}^2). \qquad (3.1)$$

We assume that in the general case the ion masses and densities are comparable in order of magnitude. We confine oureselves to oscillations for which $a_E \ll 1$. We then obtain from (2.13) the following approximate dispersion equation:

$$\varepsilon(\omega)\varepsilon(\omega - \omega_0) - {}^1\!/_4 a_E^2[\delta\varepsilon_1(\omega - \omega_0) - \delta\varepsilon_1(\omega)]^2 = 0, \qquad (3.2)$$

where

$$\varepsilon(\omega) = 1 + \delta\varepsilon_e + \delta\varepsilon_1(\omega) + \delta\varepsilon_2(\omega).$$

In the zeroth approximation ($a_E = 0$), the solution of the dispersion equation $\epsilon(\omega) = 0$ determines the natural frequencies of the longitudinal plasma oscillations with "cold" ions $\omega = \omega_\alpha$, where

$$\omega_\alpha(k) = \pm\omega_\pm(k),$$

$$\omega_\pm^2 = {}^1\!/_2(\omega_{B1}^2 + \omega_{B2}^2 + \omega_1^2 + \omega_2^2) \pm {}^1\!/_2[(\omega_{B1}^2 + \omega_{B2}^2 + \omega_1^2 + \omega_2^2)^2 - 4(\omega_{B1}^2\omega_{B2}^2 + \omega_{B1}^2\omega_2^2 + \omega_{B2}^2\omega_1^2)]^{1/2}. \qquad (3.3)$$

For the frequencies ω_1 and ω_2 we have in case A

$$\omega_i^2 = \omega_{LHi}^2 \equiv \omega_{pi}^2(1 + \omega_{pe}^2/\omega_{Be}^2)^{-1}, \qquad (3.4)$$

and in case B

$$\omega_i^2 = \omega_{si}^2 \equiv k^2 v_{si}^2 / (1 + k^2 r_{De}^2), \qquad (3.5)$$

where

$$v_{si} = (T_e/m_i)^{1/2}, \quad r_{De} = (T_e/4\pi e^2 n_0)^{1/2} = v_{Te}/\omega_{pe}. \qquad (3.6)$$

In case A, the frequency ω_+ is the frequency of the second (lower) hybrid resonance, and ω_- is the frequency of the third (non-ionic) hybrid resonance [1,2]. Usually $\omega_{pi} \gg \omega_{Bi}$, and then the expressions for the frequencies of the lower hybrid resonance and of the ion-ion hybrid resonance simplify:

$$\omega_+^2 = \omega_{LH}^2 = \frac{\omega_{p1}^2 + \omega_{p2}^2}{1 + (\omega_{pe}/\omega_{Be})^2}, \qquad (3.7)$$

$$\omega_-^2 = \omega_{JJ}^2 = \frac{\omega_{B2}^2\omega_{p1}^2 + \omega_{B1}^2\omega_{p2}^2}{\omega_{p1}^2 + \omega_{p2}^2}. \qquad (3.8)$$

In the case B, the frequency ω_+ corresponds to fast ion sound in a magnetized plasma containing ions of a single sort. We note that $\omega_+ > \max(\omega_{B1}, \omega_{B2})$. In the region of sufficiently large wave vectors in a dense plasma, when $\omega_{s1}^2 + \omega_{s2}^2 \gg \omega_{Bi}^2$, the expression for the frequency ω_+ coincides with the frequency of the ion-sound oscillations in a nonmagnetized plasma containing ions of two sorts:

$$\omega_+^2 = \omega_s^2 = \omega_{s1}^2 + \omega_{s2}^2. \qquad (3.9)$$

In the region where $\omega_{s1}^2 + \omega_{s2}^2 \ll \omega_{Bi}^2$, we have $\omega_+ = \max(\omega_{B1}, \omega_{B2})$. The frequency ω_- corresponds to a new branch, which appears when the ions of the second sort are added. In the region of large wave numbers, where $\omega_{s1}^2 = \omega_{s2}^2 \gg \omega_{Bi}^2$, we have $\omega_- = \omega_{JJ}$. In the region of long waves, where $\omega_{s1}^2 + \omega_{s2}^2 \ll \omega_{Bi}^2$, we have $\omega_- = \min(\omega_{B1}, \omega_{B2})$.

The term $\sim a_E^2$ in (3.2) becomes significant under parametric-resonance conditions, when

$$\omega_\alpha - N\omega_0 \approx \omega_\beta \quad (N = \pm 1) \tag{3.10}$$

where ω_α and ω_β are the natural frequencies defined by formula (3.5). Putting

$$\omega = \omega_\alpha + \varepsilon, \quad \delta = {}^1/_2(\omega_\alpha - N\omega_0 - \omega_\beta),$$

we obtain from (3.2)

$$\varepsilon = -\delta \pm (\delta^2 - \gamma_m^2)^{1/2}, \tag{3.11}$$

where

$$\gamma_m^2 = -\frac{1}{4} a_E^2 \frac{[\delta\varepsilon_1(\omega_\alpha) - \delta\varepsilon_1(\omega_\beta)]^2}{\varepsilon'(\omega_\alpha)\varepsilon'(\omega_\beta)}, \tag{3.12}$$

$$\varepsilon'(\omega) = \sum_{i=1,2} \frac{2\omega\omega_{pi}^2}{(\omega^2 - \omega_{Bi}^2)^2}. \tag{3.13}$$

We see therefore that instability can set in if $\gamma_m^2 > 0$. To this end, it is necessary that ω_α and ω_β have opposite signs and belong to different branches, i.e., in the case of instability we have

$$|\omega_1(k)| + |\omega_2(k)| \approx \omega_0. \tag{3.14}$$

The maximum value of the growth increment $\gamma = \gamma_m$ is reached at $\delta = 0$. If $n_1 \sim n_2$ and $m_1 \sim m_2$, then we obtain at $\omega_1 \sim \omega_2 \sim \omega_{Bi} \sim \omega_0$ the order-of-magnitude estimate

$$\gamma_m \sim a_E\omega_{Bi} \sim ku. \tag{3.15}$$

If $\omega_1 \gg \omega_2$ ($\omega_1 \gg \omega_{LH} \gg \omega_2 \approx \omega_{JJ}$ or $\omega_1 \approx \omega_s \gg \omega_2 \approx \omega_{JJ}$), then

$$\gamma_m \sim a_E\omega_{pi}, \tag{3.16}$$

i.e.,

$$\gamma_m \sim ku(1 + \omega_{pe}^2/\omega_{Be}^2)^{1/2} \quad (\omega_1 \approx \omega_{LH} \approx \omega_0),$$

$$\gamma_m \sim ku(1 + 1/k^2 r_{De}^2)^{1/2} \quad (\omega_1 \approx \omega_s \approx \omega_0).$$

However, if the pump frequency is large in comparison with ω_-, Eq. (3.2) no longer holds. In this case the dispersion equation can be represented in the form

$$1 + \frac{a_E^2}{4} \frac{\delta\varepsilon_1(\omega)\delta\varepsilon_2(\omega)}{\varepsilon(\omega)} \left[\frac{1}{\varepsilon(\omega - \omega_0)} + \frac{1}{\varepsilon(\omega + \omega_0)}\right] = 0. \tag{3.17}$$

Recognizing that $\omega_1 \approx \omega_0$, we seek the solution of this equation under the assumption that $\omega \ll \omega_0$. We then obtain

$$\omega^2 = \frac{1}{2}[(\omega_0 - \omega_1)^2 + \omega_2^2] \pm \frac{1}{2}\left\{[(\omega_1 - \omega_0)^2 - \omega_2^2]^2 - 2a_E^2 \frac{\omega_{p1}^2\omega_{p2}^2(\omega_0 - \omega_1)}{\varepsilon'(\omega_1)(\omega_{p1}^2 + \omega_{p2}^2)}\right\}^{1/2} \tag{3.18}$$

or

$$\omega^2/\omega_2^2 = {}^1/_2(1 + x^2) \pm {}^1/_2[(1 - x^2)^2 - 4\eta x]^{1/2}, \tag{3.19}$$

where

$$x = \frac{\omega_0 - \omega_1}{\omega_2}, \quad \eta = a_E^2 \frac{\omega_{p1}^2\omega_{p2}^2}{2\omega_2^3\varepsilon'(\omega_1)(\omega_{p1}^2 + \omega_{p2}^2)}. \tag{3.20}$$

The instability sets in if

$$x_1 < x < x_2 \quad (x_1 < 1 < x_2), \tag{3.21}$$

where $x = x_{1,2}$ are the roots of the equation

$$(1 - x^2)^2 - 4\eta x = 0, \tag{3.22}$$

and also in the region

$$-\eta < x < 0. \tag{3.23}$$

In the region (3.21), the maximum increment is equal to

$$\gamma_m = {}^1/_2\omega_2|1 + x^2 - 2\sqrt{x(x + \eta)}|^{1/2}, \tag{3.24}$$

where x is determined from the equation

$$4x(x + \eta)(x^2 - 1) = \eta^2. \tag{3.25}$$

In the limiting cases we obtain from (3.25) and (3.24)

$$\gamma_m \approx {}^1/_2\omega_2\eta^{1/2}, \quad \eta \ll 1, \tag{3.26}$$

$$\gamma_m \approx 2^{-4/3}\sqrt{3}\omega_2\eta^{1/3}, \quad \eta \gg 1. \tag{3.27}$$

In the region (3.23), the maximum value of the increment is

$$\gamma_m = \omega_2\left[\frac{2|x|(x + \eta)}{x^2 + 1 + \sqrt{(x^2 - 1)^2 - 4\eta x}}\right]^{1/2}, \tag{3.28}$$

where x is determined from the equation

$$2x^3 + 2x + \eta = 0. \tag{3.29}$$

In the limiting cases we obtain from (3.28) and (3.29)

$$\gamma_m \approx {}^1/_2\eta\omega_2, \quad \eta \ll 1, \tag{3.30}$$

$$\gamma_m \approx 2^{-2/3}\eta^{1/3}\omega_2, \quad \eta \gg 1. \tag{3.31}$$

A comparison of expressions (3.15) with (3.27) and (3.31) shows that at $\omega_1 \approx \omega_0 \gg \omega_- \sim \omega_{JJ}$, the growth increment can increase strongly. It should be noted, however, that under these conditions parametric excitation of other low-frequency oscillation modes is possible, with growth increments on the order of

$$\gamma' \sim (a_E')^{2/3}\omega_{LH} \sim \omega_{LH}, \tag{3.32}$$

if $a_E' \sim ku_e/\omega_0 \sim 1$. The principal role in the excitation of these oscillations is played by the motion of the electrons relative to the ions. If $\omega_0 \gg \omega_{Bi}$, then the amplitude u_e of the oscillations of the electron velocity in the alternating electric field of the pump wave is much larger than the amplitude of the relative velocity of the ions

$$u_e \sim cE_0/B_0 \sim u\omega_0/\omega_{Bi} \gg u. \tag{3.33}$$

If, for example, $\omega_0 \sim \omega_{LH} \sim (\omega_{Be}\omega_{Bi})^{1/2}$, then

$$u_e \sim (m_i/m_e)^{1/2}u. \tag{3.34}$$

Comparing (3.32) with (3.27) and (3.31), we obtain

$$\gamma'/\gamma_m \sim (u_e/u)^{2/3} \sim (\omega_0/\omega_{Bi})^{2/3} \gg 1. \tag{3.35}$$

It is obvious that under these conditions the principal role is played by the electron-ion parametric instabilities investigated in [11-14]. Incidentally, for a final answer to this question it is necessary to investigate the nonlinear stage of development of the oscillations.

Unlike the case $\omega_0 \gg \omega_{Bi}$, if the pumping is by waves of frequency $\omega_0 \sim \omega_{Bi}$, the relative velocity of the electrons is of the order of the relative velocity of the ions. In this case the growth increments of the parametric instabilities, an important role in the onset of which is played by the oscillations of the electrons relative to the ions, turn out to be of the same order of magnitude as the growth increments (3.15) of the ion-ion parametric instabilities. The role of the latter, however, is more important during the nonlinear stage, since, as noted earlier, the turbulence level for them is much higher than for electron-ion parametric instabilities.

4. EXCITATION OF CYCLOTRON WAVES

For cyclotron waves we have $|z_{si}| \gg 1$, and $\delta\epsilon_i$ takes the form

$$\delta\varepsilon_i(\omega) = \Omega_i^2\left[1 - \sum_{s=-\infty}^{\infty} A_s(k^2\rho_i^2)\frac{\omega}{\omega - s\omega_{Bi}}\right],$$

$$\Omega_i^2 = \frac{\omega_{pi}^2}{k^2 v_{Ti}^2}. \tag{4.1}$$

The zeroth-approximation ($a_E = 0$) dispersion equation, $\epsilon(\omega) = 0$ determines the frequency $\omega = \omega_\alpha(k)$ of the plasma cyclotron oscillations [8], which are modified somewhat because of the presence of ions of two sorts.

At $k\rho_i \ll 1$, the frequency ω is close to

$$\omega = s\omega_{Bi}\left[1 + \frac{\Omega_i^2 A_s(k^2\rho_i^2)}{1 + \delta\varepsilon_e + \Omega_1^2 + \Omega_2^2}\right]. \tag{4.2}$$

If the condition $s_1\omega_{B1} = s_2\omega_{B2}$ is satisfied for certain values $s = s_1$ and s_2, then we obtain in place of (4.2)

$$\omega = s_1\omega_{B1}\left[1 + \frac{\Omega_1^2 A_{s1}(k^2\rho_1^2) + \Omega_2^2 A_{s2}(k^2\rho_2^2)}{1 + \delta\varepsilon_e + \Omega_1^2 + \Omega_2^2}\right]. \tag{4.3}$$

At $k\rho_i << 1$, the frequency ω is close either to $\omega_\pm$ or to $s\omega_{Bi}$:

$$\omega = s\omega_{Bi}\left[1 + \Omega_i^2 A_s(k^2\rho_i^2)\left(1 + \delta\varepsilon_e - \frac{\omega_{p1}^2}{s^2\omega_{Bi}^2 - \omega_{B1}^2} - \frac{\omega_{p2}^2}{s^2\omega_{Bi}^2 - \omega_{B2}^2}\right)^{-1}\right] \tag{4.4}$$

If the condition $s_1\omega_{B1} = s_2\omega_{B2}$ is satisfied, then we have in place of (4.4)

$$\omega = s_1\omega_{B1}\left[1 + (\Omega_1^2 A_{s1}(k^2\rho_1^2) + \Omega_2^2 A_{s2}(k^2\rho_2^2)) \times \left(1 + \delta\varepsilon_e - \sum_{i=1,2}\frac{\omega_{pi}^2}{s_1^2\omega_{B1}^2 - \omega_{Bi}^2}\right)^{-1}\right]. \tag{4.5}$$

Formulas (4.2)—(4.5) do not hold when the frequency $s\omega_{Bi}$ is close to one of the frequencies $\omega_\pm$. In this case $\omega = \omega_\pm + \delta\omega$, where

$$\delta\omega = {}^1/_2(s\omega_{Bi} - \omega_\pm) \pm {}^1/_2[(s\omega_{Bi} - \omega_\pm)^2 + \Omega/\varepsilon'(\omega_\pm)]^{1/2};$$
$$\Omega = \Omega_i^2 s\omega_{Bi} A_s(k^2\rho_i^2), \quad s_1\omega_{B1} \neq s_2\omega_{B2}, \tag{4.6}$$
$$\Omega = s_1\omega_{B1}\Omega_1^2 A_{s1}(k^2\rho_1^2) + s_2\omega_{B2}\Omega_2^2 A_{s2}(k^2\rho_2^2), \quad s_1\omega_{B1} = s_2\omega_{B2}.$$

At a pump frequency $\omega_0 \sim \omega_{Bi}$ and at $a_E \ll 1$, we can use the approximate dispersion equation (3.2), from which we find that at resonance $\omega_\alpha - \omega_0 \approx \omega_\beta$ the value of $\epsilon = \omega - \omega_\alpha$ is determined by formulas (3.11) and (3.12), where $\delta\epsilon_i$ is given by (4.1) and

$$\varepsilon'(\omega) = \sum_{i=1,2}\sum_{s=1}^{\infty}\Omega_i^2\frac{4s^2\omega_{Bi}^2 A_s(k^2\rho_i^2)\,\omega}{(\omega^2 - s^2\omega_{Bi}^2)^2}. \tag{4.7}$$

The instability sets in when ω_α and ω_β have opposite signs, i.e., $|\omega_\alpha| + |\omega_\beta| = \omega_0$. The order of magnitude of the maximum growth increment is

$$\gamma \sim a_E\frac{\omega}{k\rho_i} \sim \frac{u}{v_{Ti}}\omega \quad (k\rho_i > 1, \ \omega \approx s\omega_{Bi}), \tag{4.8}$$

$$\gamma \sim a_E(k\rho_i)^{2s}\omega \quad (k\rho_i < 1, \quad \omega \approx s\omega_{Bi}, \quad s \geqslant 2). \tag{4.9}$$

If $k\rho_i \ll 1$ and $\omega \approx \omega_\pm$, then γ_m is given by (3.16). From a comparison of (3.16) and (4.9) with (4.8) it follows that the growth increment of the cyclotron oscillations at $k\rho_i \gg 1$ greatly exceeds the growth increment of the long-wave oscillations ($k\rho_i \ll 1$).

In the case of a high pump frequency ($\omega_0 \gg \omega_{Bi}$), the expressions obtained in this section for the growth increment are no longer valid, since the dispersion equation (3.2) no longer holds. In this case we can use the dispersion equation (3.17). Recognizing that $\omega_0 \gg \omega_{Bi}$ and assuming $|\omega| \ll \omega_0$, we obtain, using (4.1),

$$\varepsilon(\omega \pm \omega_0) = 1 + \delta\varepsilon_e - \sum_{i=1,2}\frac{\omega_{pi}^2}{(\omega \pm \omega_0)^2}; \tag{4.10}$$

$$\delta\varepsilon_i(\omega) = \Omega_i^2[1 - A_0(k^2\rho_i^2)]. \tag{4.11}$$

Substituting (4.10) and (4.11) in (3.17), we get

$$\omega^2 = \delta^2 + 2\delta\gamma_m, \tag{4.12}$$

where $\delta = \omega_+ - \omega_0$ and

$$\gamma_m = \frac{a_E^2}{2\varepsilon'(\omega_\pm)}\frac{\Omega_1^2\Omega_2^2[1 - A_0(k^2\rho_1^2)][1 - A_0(k^2\rho_2^2)]}{1 + \delta\varepsilon_e + \sum_{i=1,2}\Omega_i^2[1 - A_0(k^2\rho_i^2)]}. \tag{4.13}$$

The instability sets in at $\delta < 0$. The growth increment reaches the maximum value $\gamma = \gamma_m$ at $\delta = -\gamma_m$. In order of magnitude, we have for the increment (4.13) the estimate

$$\gamma \sim \omega_+ u^2/v_{Ti}^2. \tag{4.14}$$

We note that in the considered case ($\omega_0 \gg \omega_{Bi}$) the velocity of the electrons in the field of the pump wave is much larger than the relative velocity of the ions and, as indicated in the preceding section, an important role is assumed by electron-ion parametric instabilities with a growth increment much larger than (4.14).

5. EXCITATION OF OSCILLATIONS BY RESONANT IONS

We shall assume that either the concentration of the ions of the second sort is small ($n_2 \ll n_1$) or that their temperature is much higher than the temperature of the ions of the first sort ($T_2 \gg T_1$). Then $|\delta\epsilon_1| \gg |\delta\epsilon_2|$ (here $\delta\epsilon_i$ is defined by expressions (4.1) and the ions of the second sort exert little influence on the dispersion of the cyclotron waves. The dispersion equation, which takes into account the presence of the particles of the second sort, is

$$1 + \delta\varepsilon_e + \delta\varepsilon_1(\omega) + a_0(\omega) = 0, \tag{5.1}$$

where

$$a_0(\omega) = \Omega_2^2 + \Omega_2^2 i\sqrt{\pi}\sum_{s,m=-\infty}^{\infty} J_m^2(a_E)(z_{02} + m\zeta)\,w(z_{s2} + m\zeta)A_s(k^2\rho_2^2) \tag{5.2}$$

and $\zeta = \omega_0/\sqrt{2}k_\parallel v_{T_2}$. In the zeroth approximation we obtain from the equation $1 + \delta\epsilon_e + \delta\epsilon_1 = 0$ the frequencies $\omega = \omega_\alpha(k)$ of the cyclotron oscillations in the absence of ions of the second sort. If the resonance conditions

$$\omega_\alpha \approx s_0\omega_{B2} - m_0\omega_0, \tag{5.3}$$

are satisfied for certain values $m = m_0$ and $s = s_0$ in the sum (5.2), then retaining only the resonant term in (5.1) and assuming that $\gamma \ll k_\parallel v_{Ti}$, we obtain for the growth increment the expression

$$\gamma = -\Omega_2^2\frac{\sqrt{\pi}}{\delta\varepsilon_1'(\omega_\alpha)}\frac{s_0\omega_{B2}}{\sqrt{2}k_\parallel v_{T2}}J_{m_0}^2(a_E)A_{s_0}(k^2\rho_2^2)\exp\{-(z_{s_02} + m\zeta)^2\}, \tag{5.4}$$

where $\delta\epsilon_1'(\omega)$ is given by the term in the right-hand side of (4.7) with i = 1.

It follows from (5.1) that the instability sets in at $s_0 < 0$ (and $m_0 < 0$). If $T_1 \sim T_2 \sim T_e$, $\omega_{B1} \sim \omega_{B2} \sim \omega_0$, $|m_0| \sim |s_0| \sim 1$, and $\omega_{pe} \sim \omega_{Be}$, then it is easy to obtain the following estimates for the maximum values of the growth increment (5.1):

$$\gamma \sim \left(\frac{n_2}{n_1}\right)^{1/2}\left(\frac{m_e}{m_i}\right)^{(1-l)/2}\left(\frac{u}{v_{Ti}}\right)^{l}\omega_{Bi} \quad \text{when} \quad \frac{u}{v_{Ti}} \leqslant \left(\frac{m_e}{m_i}\right)^{1/2}, \tag{5.5}$$

$$\gamma \sim \left(\frac{n_2}{n_1}\right)^{1/2}\frac{u}{v_{Ti}}\omega_{Bi} \quad \text{when} \quad \left(\frac{m_e}{m_i}\right)^{1/2} \leqslant \frac{u}{v_{Ti}} \leqslant \min\left\{\left(\frac{m_e}{m_i}\frac{n_1}{n_2}\right)^{1/2}, 1\right\} \tag{5.6}$$

$$\gamma \sim \left(\frac{n_2}{n_1}\right)^{1/2}\left(\frac{v_{Ti}}{u}\right)^{1/2}\omega_{Bi} \quad \text{when} \quad \frac{u}{v_{Ti}} \geqslant \max\left\{1, \frac{m_i}{m_e}\frac{n_2}{n_1}\right\}, \tag{5.7}$$

$$\gamma \sim \left(\frac{m_e}{m_i}\right)^{1/2}\omega_{Bi} \quad \text{when} \quad \left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{n_1}{n_2}\right)^{1/2} \leqslant \frac{u}{v_{Ti}} \leqslant \frac{n_2}{n_1}\frac{m_i}{m_e}, \tag{5.8}$$

where $l = |m_0|$ at $|m_0| = 1$ and 2 and $l = 3$ at $|m_0| \geq 3$.

The maximum values (5.5)—(5.8) of the quantities

(5.4) are attained at $\gamma \sim k_{\parallel}v_{T_2}$. The largest value of these quantities is of the order of

$$\gamma_{max} \sim \min\{(m_e/m_i)^{1/2}, \quad (n_2/n_1)^{1/2}\}\omega_{Bi}. \tag{5.9}$$

6. COHERENT EXCITATION OF OSCILLATIONS BY IONS

We shall assume, just as in the preceding section, that $n_2 \ll n_1T_2/T_1$. Unlike in the preceding section, however, we assume $\gamma \gg k_{\parallel}vT_2$. Then the quantity $\delta\epsilon_2(\omega)$ takes the form (4.1). In this case we can use the dispersion equation (5.1), in which

$$a_0(\omega) = \Omega_2^2 - \Omega_2^2 \sum_{s,m=-\infty}^{\infty} J_m^2(a_E)\frac{\omega + m\omega_0}{\omega + m\omega_0 - s\omega_{B2}} A_s(k^2\rho_2^2). \tag{6.1}$$

If the resonance condition $\omega_\alpha(k) \approx s_0\omega_{B2} - m_0\omega_0$ is satisfied for certain values $s = s_0$ and $m = m_0$, with $\omega_0/\omega_{B2} \neq p/q$, where p/q is an irreducible fraction (p and q are of the order of unity) and $\omega_\alpha - N\omega_0 \neq \omega_\beta$, then the quantity $\epsilon = \omega - \omega_\alpha$ is determined by formula (3.11), in which $\delta = (\omega_\alpha - s_0\omega_{B2} + m_0\omega_0)/2$ and

$$\gamma_m^2 = -\Omega_2^2\frac{s_0\omega_{B2}}{\delta\varepsilon_1'(\omega_\alpha)} J_{m_0}^2(a_E)A_{s_0}(k^2\rho_2^2). \tag{6.2}$$

Obviously, the instability can occur only at $s_0 < 0$. For the maximum value of γ_m defined by (6.2) we can use the order-of-magnitude estimate (5.5)–(5.8).

If $\omega_0/\omega_{B2} = p/q$, then it is necessary to take a large number of resonant terms into account in expression (6.1) for $a_0(\omega)$. In this case γ_m^2 is equal to

$$\gamma_m^2 = -\Omega_2^2\frac{\omega_{B2}}{\delta\varepsilon_1'(\omega_\alpha)} \sum_{r=-\infty}^{\infty} (s_0 + pr) J^2_{m_0+qr}(a_E)A_{s_0+pr}(k^2\rho_2^2). \tag{6.3}$$

The growth increments determined by formulas (6.2) and (6.3) are of equal order of magnitude.

7. EXCITATION OF CYCLOTRON OSCILLATIONS BY IONS UNDER CONDITIONS OF PARAMETRIC RESONANCE

Let us consider, at $n_1 \gg n_2T_1/T_2$ and at arbitrary values of a_E, the coherent excitation of cyclotron oscillations by the ions of the second sort under the parametric-resonance conditions

$$\omega_\alpha - N\omega_0 \approx \omega_\beta, \quad N = \pm1, \pm2, \dots \tag{7.1}$$

Here $\omega_{\alpha,\beta}$ is the solution of the dispersion equation $\lambda(\omega) = 1 + \delta\epsilon_e + \delta\epsilon_1(\omega) = 0$. Assuming that $\gamma \gg k_{\parallel}v_{T_2}$ and using expression (4.1) for $\delta\epsilon_2(\omega)$, we represent the dispersion equation in the form

$$[\lambda(\omega) + a_0(\omega)][\lambda(\omega - N\omega_0) + a_0(\omega - N\omega_0)] - a_N(\omega)a_{-N}(\omega - N\omega_0) = 0, \tag{7.2}$$

where

$$a_n(\omega) = \Omega_2^2\delta_{n,0} - \Omega_2^2\sum_{m=-\infty}^{\infty} J_m(a_E)J_{m+n}(a_E)\frac{\omega + m\omega_0}{\omega + m\omega_0 - s\omega_{B2}} A_s(k^2\rho_2^2). \tag{7.3}$$

From (7.2) we find the following expression for $\epsilon = \omega - \omega_\alpha$

$$\varepsilon = -\delta - \frac{p_{0,\alpha} + p_{0,\beta}}{2} \pm \left[\left(\delta + \frac{p_{0,\beta} - p_{0,\alpha}}{2}\right)^2 + p_{N,\alpha}p_{-N,\beta}\right]^{1/2}, \tag{7.4}$$

where

$$p_{n,\alpha} = \frac{a_n(\omega_\alpha)}{\lambda'(\omega_\alpha)},$$
$$\lambda'(\omega) = \Omega_1^2\sum_{s=1}^{\infty}\frac{4s^2\omega_{B1}^2\omega}{(\omega^2 - s^2\omega_{B1}^2)^2}A_s(k^2\rho_1^2). \tag{7.5}$$

We note that if the parametric resonance (7.1) is realized on one branch of the oscillations ($\omega_\alpha \approx N\omega_\beta/2$), then no instability occurs at odd N, since $a_{-n}(-\omega) = (-1)^n a_n(\omega)$ and $\lambda'(-\omega) = -\lambda'(\omega)$.

The instability does take place is $p_{N,\alpha}p_{-N,\beta} < 0$. The maximum growth increment is

$$\gamma = \left|\frac{a_N(\omega_\alpha)}{[\lambda'(\omega_\alpha)\lambda'(\omega_\beta)]^{1/2}}\right|. \tag{7.6}$$

In order of magnitude, at

$$T_2 \sim T_1 \text{ and } u \geqslant u_0 = v_{Ti}(m_e/m_i + \omega_{Bi}^2/\omega_{pi}^2)^{1/2}$$

we have

$$\gamma_{max} \sim \frac{n_2}{n_1}\frac{u}{v_{Ti}}\omega_{Bi}. \tag{7.7}$$

We have put here

$$\omega_{\alpha,\beta} \sim \omega_0 \sim \omega_{Bi}, \quad a_E \sim 1, \quad |\omega_\alpha + m\omega_0 - s\omega_{B2}| \sim \omega_{Bi}/k\rho_i \ll \omega_{Bi}.$$

We note that if we put $n_2 \sim n_1$ in the estimate (7.7), then it coincides with the estimate (4.8) provided we put in the latter $a_E \sim 1$.

The growth increment given by (7.6) and (7.7) is smaller by a factor of $(n_1/n_2)^{1/2}$ than the maximum growth increment (6.2), i.e., under the conditions of parametric resonance (7.1) the excitation of the oscillations is much weaker than at resonance $\omega_\alpha = s_0\omega_{B2} - m_0\omega_0$.

The expressions obtained in this section are not valid if the parametric resonance condition (7.1) is satisfied simultaneously with the condition (5.3): $\omega_\alpha \approx s_0\omega_{B2} = m_0\omega_0$. We assume that in this case the difference $\omega_\alpha - N'\omega_0$, where $N' \neq N$, is not close to any natural frequency. We then easily obtain from (7.2) for the quantity $\epsilon = \omega - \omega_\alpha$ the equation

$$\varepsilon(\varepsilon + 2\delta)(\varepsilon + \Delta) + \varepsilon p_N(\omega_\alpha) + (\varepsilon + 2\delta)p_0(\omega_\beta) = 0, \tag{7.8}$$

where

$$p_n(\omega) = -\Omega_2^2J^2_{m_0+n}(a_E)A_{s_0}(k^2\rho_2^2)s_0\omega_{B2}/\delta\varepsilon_1'(\omega),$$
$$\delta = {}^1\!/_2(\omega_\alpha - N\omega_0 - \omega_\beta), \quad \Delta = \omega_\alpha + m_0\omega_0 - s_0\omega_{B2}.$$

In the particular case $\delta = 0$ we obtain from (7.8)

$$\varepsilon = -{}^1\!/_2\Delta \pm [{}^1\!/_4\Delta^2 - p_N(\omega_\alpha) - p_0(\omega_\beta)]^{1/2}. \tag{7.9}$$

The growth increments determined by (7.8) agree in order of magnitude with the growth increment (6.2).

Expressions (7.4) were obtained in the case when the excitation of the oscillations by the ions of the second sort is coherent, and $\gamma \gg k_{\parallel}v_{T_2}$. If the temperature T_2 and the value of cos θ are large enough, then the inequality $\gamma \gg k_{\parallel}v_{T_2}$ is not satisfied. The oscillation excitation then takes place with participation of the resonant ions of the second sort. Assuming $\gamma \ll k_{\parallel}v_{T_2}$, we find that in this case ϵ is determined as before by formula (7.4), in which the quantity $a_n(\omega)$ ($n = \pm N$) must be set equal to

$$a_n(\omega) = -\sum_{s,m=-\infty}^{\infty} \Omega_2^2J_m(a_E)J_{m+n}(a_E)i\sqrt{\pi}(z_{02} + m\zeta)w(z_{s2} + m\zeta)A_s(k^2\rho_2^2), \tag{7.10}$$

where $\zeta = \omega_0/\sqrt{2}k_{\parallel}v_{T_2}$.

The growth increments determined by expressions (7.4) and (7.10) are not valid if condition (5.3) is satisfied together with the parametric-resonance condition (7.1). In the case of such a double resonance at $\gamma \ll k_{\parallel}v_{T_2}$, it is necessary to take into account a large number of resonant terms in the expression for $a_n(\omega)$, which is defined by (7.10) and which enters in the dispersion equation (7.4). The quantity ϵ at $k\rho_2 \gg 1$ is determined in this case by the formula

$$\varepsilon = \gamma_0(-i\beta_{m_0, m_0} \pm \beta_{m_0, m_0+N}), \quad (7.11)$$

where

$$\omega_0 / \omega_{B2} = p / q = 2s_0 / (N + m_0),$$

$$\gamma_0 = \sqrt{\frac{\pi}{2}} \Omega_2^2 \frac{\omega_{B2}}{k_\parallel v_{T2}} \frac{1}{\delta\varepsilon_1'(\omega_a) \sqrt{2\pi} k\rho_2},$$

$$\beta_{k,l} = \sum_{r=-\infty}^{\infty} J_{k+qr}(a_E) J_{l+qr}(a_E)(s_0 + pr).$$

The growth increment determined by formulas (7.11) and (5.4) are of the same order of magnitude.

8. EXCITATION OF ION-ION SOUND

In a plasma with a small admixture of "cold" ions there exists a branch of oscillations that can be called ion-ion sound[10]. For these oscillations we have $\omega \gg \omega_{Bi}$, $\omega/k_\parallel v_{T_e} \gg 1$, and $v_{T_2} \ll \omega/k \ll v_{T_1}$, as well as $(m_2/m_e)\cos^2\theta \ll n_2/n_0$, so that we can put $\delta\epsilon_e = \omega_{pe}^2/\omega_{Be}^2$ and $\delta\epsilon_1(\omega) = \Omega_1^2[1 + i\sqrt{\pi} z_1 w(z_1)]$,

$$z_1 = \omega / \sqrt{2} k v_{T1}, \quad \delta\varepsilon_2(\omega) = -\omega_{p2}^2 / \omega^2. \quad (8.1)$$

When $a_E = 0$, the dispersion equation $1 + \delta\epsilon_e + \delta\epsilon_1 + \delta\epsilon_2 = 0$ determines the frequency $\omega(k) = \mathrm{Re}\,\omega$ and the damping decrement $\gamma_0 = -\mathrm{Im}\,\omega$ of the ion-ion sound

$$\omega(k) = \omega_{p2}(1 + \omega_{pe}^2 / \omega_{Be}^2 + \Omega_1^2)^{-1/2}, \quad (8.2)$$

$$\gamma_0(k) = \left(\frac{\pi}{8}\right)^{1/2} \frac{\omega_{p1}^2 \omega^4(k)}{k^3 v_{T1}^3 \omega_{p2}^2}. \quad (8.3)$$

If a_E differs from zero, then at $\omega_0 \gg \omega$ the dispersion equation takes the form

$$1 - \frac{\omega_{p2}^2}{\omega^2} \sum_{m=-\infty}^{\infty} \frac{J_m^2(a_E)}{1 + \delta\varepsilon_e + \Omega_1^2[1 + i\sqrt{\pi}(z_1 + m\zeta) w(z_1 + m\zeta)]} = 0, \quad (8.4)$$

where $\zeta = \omega_0/\sqrt{2} k v_{T_1}$. Assuming that $\omega \ll k v_{T_1}$, we obtain

$$\omega(k) = \pm\omega_{p2} \left\{ \sum_{m=-\infty}^{\infty} \frac{J_m^2(a_E)[1 + \delta\varepsilon_e + \Omega_1^2[1 - \sqrt{\pi} m\zeta v(m\zeta)]]}{[1 + \delta\varepsilon_e + \Omega_1^2(1 - \sqrt{\pi} m\zeta v(m\zeta))]^2 + [\sqrt{\pi} m\zeta e^{-m^2\zeta^2} \Omega_1^2]^2} \right\}^{1/2} \quad (8.5)$$

where

$$v(z) = \mathrm{Im}\, w(z) = \frac{2}{\sqrt{\pi}} e^{-z^2} \int_0^z e^{t^2} dt.$$

Expression (8.5) determines the growth increment of the aperiodic instability. This instability arises if the expression in the curly brackets in (8.5) is smaller than zero. To this end it is necessary to satisfy the inequality

$$1 + \omega_{pe}^2 / \omega_{Be}^2 < \Omega_1^2 \eta, \quad (8.6)$$

where $\eta = \max[\sqrt{\pi} z v(z) - 1] \approx 0.28$. In order of magnitude, we obtain for the growth increment at $\zeta \sim 1$, $a_E \sim 1$ ($u \sim v_{T_2} \gg v_{T_1}$), and $\omega_0 \lesssim \omega_{LH}$

$$\gamma \sim (n_2 / n_1)^{1/2} \omega_0, \quad m_2 \sim m_1. \quad (8.7)$$

Formulas (8.5) and (8.7) are valid if $v_{T_2} \ll \gamma/k \ll v_{T_1}$. Taking (8.7) into account, we represent these inequalities in the form

$$(T_2 / T_1)^{1/2} \ll (n_2 / n_1)^{1/2} \ll 1. \quad (8.8)$$

If the expression in the curly brackets in (8.5) is positive, then at $\zeta \sim 1$ the instability can be due to small terms proportional to z_1. Putting $\omega = \omega(k) + \Delta\omega$, where $\omega(k)$ is determined by formulas (8.5), we obtain

$$\Delta\omega = -i\sqrt{\frac{\pi}{8}} \frac{\omega_{p2}^2 g}{k v_{T1}(1 + \delta\varepsilon_e + \Omega_1^2)} \sum_{m=-\infty}^{\infty} e^{-m^2\zeta^2} J_m^2(a_E) \cdot$$

$$\times \frac{(1 - 2m^2\zeta^2)[1 - \pi m^2\zeta^2 g^2 |w(m\zeta)|^2] + 4m^2\zeta^2[g - \sqrt{\pi} m\zeta v(m\zeta) g^2]}{[(1 - \sqrt{\pi} m\zeta v(m\zeta) g]^2 + (\sqrt{\pi} m\zeta e^{-m^2\zeta^2} g)^2]^2}, \quad (8.9)$$

$$g = \Omega_1^2(1 + \delta\varepsilon_e + \Omega_1^2)^{-1}.$$

The considered ion-ion instability arises only at $\zeta \gtrsim 1$, when $u \gtrsim v_{T_1}$. On the other hand, if $u \ll v_{T_1}$ ($\zeta \ll 1$), then the unstable oscillations with frequency (8.2) can arise under the parametric-resonance conditions $\omega(k) \approx \omega_0$. In this case, however, the dispersion equation (8.4) does not hold, since it was derived under the assumption $\omega \ll \omega_0$, and it is necessary to use the exact dispersion equation (2.13). Substituting the expression for $\delta\epsilon_\alpha(\omega)$ in (2.13) and taking into account the smallness of the quantities $z_1 \ll 1$ and $\zeta \ll 1$, we obtain from (2.13) the approximate dispersion equation

$$\left[\lambda_1(\omega) - \frac{a_1^2}{\lambda_1(2\omega)}\right]\left[\lambda_1(\omega - 2\omega_0) - \frac{a_1^2}{\lambda_1(2\omega_0)}\right] - a_2^2 = 0, \quad (8.10)$$

where

$$\lambda_1(\omega) = 1 + \delta\varepsilon_e + \Omega_1^2\left[1 + i\sqrt{\frac{\pi}{2}} \frac{\omega}{k v_{T1}} - (\zeta a_E)^2\right] - \frac{\omega_{p2}^2}{\omega^2},$$

$$a_1 = i\frac{\sqrt{\pi}}{2} \Omega_1^2 \zeta a_E, \quad a_2 = -\frac{1}{2} \Omega_1^2 (\zeta a_E)^2.$$

From (8.10) we obtain the following expression for the quantities $\epsilon = \omega - \omega(k)$:

$$\varepsilon = -\delta - i\gamma_0 \pm [(\delta + \Delta)^2 - \gamma_m^2]^{1/2}, \quad (8.11)$$

where γ_0 is determined by (8.3), $\delta = \omega(k) - \omega_0$,

$$\Delta = (\zeta a_E)^2 \frac{\omega^3(k) \Omega_1^2}{2\omega_{p2}^2} \left[1 - \frac{\pi}{3}\left(1 + \Omega_1^{-2} + \Omega_1^{-2} \frac{\omega_{pe}^2}{\omega_{Be}^2}\right)^{-1}\right]; \quad (8.12)$$

$$\gamma_m = 1/4 (\zeta a_E)^2 \omega^3(k) \Omega_1^2 / \omega_{p2}^2. \quad (8.13)$$

We have in order of magnitude

$$\omega(k) \sim (n_2/n_1)^{1/2} k v_{T1},$$
$$\gamma_0 \sim (n_2/n_1)^{1/2} \omega, \quad \gamma_m \sim 1/8 (u/v_{T1})^2 \omega. \quad (8.14)$$

Obviously, instability can set in if $\gamma_m > \gamma_0$, and then we have in order of magnitude $\gamma \sim \gamma_m$.

We note that when $u < v_{T_1}$ the parametric excitation of the ion-ion sound is due to the resonant particles.

9. CONCLUSION

The foregoing analysis shows that an alternating electric field of frequency $\omega_0 \sim \omega_{Bi}$ leads to excitation of different ion-cyclotron oscillations in a plasma with a mixture of ions of two sorts. The cause of the instability is the relative motion of the ions in the field of the pump wave. The characteristic value of the growth increment of the ion-cyclotron oscillations is given by

$$\gamma \sim \omega_{Bi} u / v_{Ti}, \quad (9.1)$$

where u is the relative velocity of the ions in the direction perpendicular to the magnetic field and v_{T_i} is their thermal velocity ($u \ll v_{T_i}$). For unstable oscillations, the saturation level is much higher than the saturation level of the electron-ion parametric instabilities, which have at $\omega_0 \sim \omega_{Bi}$ a growth increment of the same order of magnitude. One can therefore expect the considered instabilities to be the cause of strong absorption of waves with frequency $\omega_0 \sim \omega_{Bi}$. It is possible that the effects observed in[7], namely the heating of a plasma containing a mixture of hydrogen and deuterium ions, by electromagnetic waves of frequency $\omega_0 \sim \omega_{B1,2}$ were due to the considered instabilities. We note that the strong smearing of the distribution function of an almost-monoenergetic ion beam injected into the plasma across the magnetic field, a smearing due to the rapid development of ion-ion two-stream instability analogous to the parametric instabilities considered in Sec. 3, with the growth increments (3.27) and (3.31), was observed experimentally[15] and was confirmed also by numerical experiments[16,17].

*$[kE_0] = k \times E_0$.

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