Development of instabilities in a Z-pinch

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The development of the m = 0 and m = 1 magnetohydrodynamic instabilities is experimentally investigated for the linear Z-pinch. It is shown that the mode and the characteristic wavelength of the instability are determined mainly by the radius of the discharge tube and the rate of current buildup. The results obtained indicate the development under the conditions considered of Rayleigh-Taylor type of instabilities. Analysis of these results under the assumption that such instabilities do develop leads to relations between the wavelength of the instability and the discharge parameters. The relations obtained are in good agreement with the experimental results.

Investigations of the method of dynamic stabilization of Z-pinches by a high-frequency (HF) transverse magnetic field with a steep gradient[1,2] have yielded quite unexpected results. It turned out that when the stabilization conditions for long-wave perturbations were fulfilled, the plasma filament remained stable with respect to kink deformations for practically all waves, including short waves for which the stabilization conditions are clearly not fulfilled. These results were at variance with the generally accepted ideas of the existing linear theory of the stability of the stationary Z-pinch, according to which theory the increment of the perturbations should increase with decreasing wavelength.

Let us write down the stability condition for a Z-pinch with a quasi-direct discharge current stabilized by such a method. The HF field is produced by the currents of a multipole located at the periphery of the discharge tube. This stability condition amounts to the inequality

$$\left(\frac{\partial \tilde{H}}{\partial r}\right)_a > \frac{H_\varphi}{a}\frac{K_0(\varkappa a)}{K_1(\varkappa a)+K_1(\varkappa a)/\varkappa a}. \quad (1)$$

Here $(\partial\tilde{H}/\partial r)_a$ is the gradient of the HF field at the surface of a pinch of radius a; $H_\varphi = 2I/ca$ is the field of the pinch current also at the boundary; $\kappa = 2\pi/\lambda$ is the wave number of an m = 1 instability of wavelength λ; $K_0(\kappa a)$ and $K_1(\kappa a)$ are the Macdonald functions of orders zero and one. For long waves, when $\kappa a \ll 1$, the condition (1) amounts to the well-known relation (see [3]),

$$\left(\frac{\partial \tilde{H}}{\partial r}\right)_a > \frac{2I}{c}\left(\frac{2\pi}{\lambda}\right)^2 \ln\frac{\lambda}{\pi a}. \quad (1a)$$

It follows from these expressions that such a method of stabilization can, it would appear, be used only for the suppression of long-wave perturbations, and that the stabilization of short-wave perturbations and of the m = 0 mode must be realized by some other methods.

In order that perturbations of all wavelengths, even those for which $\kappa a \gg 1$, will become stabilized, it is necessary to meet the condition

$$(\partial\tilde{H}/\partial r)_a > H_\varphi/a, \quad (1b)$$

to which the condition (1) reduces at large κa. This requires HF fields of so large gradients that there are hardly any real prospects for the stabilization of fairly strong currents. However, as has been noted above, the experiments indicate that short-wave perturbations do not appear.

The basic question in the whole problem of dynamic stabilization—perhaps in the whole problem of the dynamic systems of plasma physics—remains the question as to what determines the mode and the minimum wavelength of the observed instabilities. It was for the purpose of elucidating this question that we set up this series of experiments in which we investigated the development of the m = 0 and m = 1 instabilities in a fairly wide range of values of the parameters amenable to the control of experimentalists. Here the dimensions of the discharge tubes (the length l and the radius a_0), the initial current growth rate $(dI/dt)_0$, and the initial pressure p_0 of the gas were varied.

The experiments were performed in cylindrical glass tubes with copper electrodes. The discharge power was supplied by a 50-μF capacitor bank at a voltage of 40 kV. The investigations were carried out in the following range of variation of the current growth rate:

$$10^9 \leqslant (dI/dt)_0 \leqslant 10^{11}\ \text{A/sec}$$

which was attained by including an auxiliary inductance in the circuit. The discharge tubes had diameters ranging from 10 to 40 cm and lengths from 60 to 180 cm. As the working gas, we used hydrogen. The initial pressure varied within a comparatively narrow range around 0.1 Torr.

The development of the instability of the Z-pinch was recorded by the method of high-speed photography and by measuring the longitudinal fields H_Z that arise during the kinking and curling of the plasma filament. For this purpose we used two types of magnetic probes: external and internal probes. The external probe, which was in the form of several loops around the discharge chamber, recorded the flux Φ_Z inside the tube if the probe's signal was integrated, or $d\Phi_Z/dt$ if this signal from the coil was fed directly to the cathode-ray oscillograph. The internal probe, which was in the form of a coil of diameter 1 cm, was placed inside the discharge chamber on the axis and recorded with sufficient accuracy the moment of the appearance of the Z-component of the magnetic field near the axis. In order to avoid distortion by the probe, the conducting wires from the probe to the oscillograph were led out not through the side walls of the discharge chamber, but along the axis through the electrodes. In any case, no distortions by the probe were observed at the initial stage of the development of the discharge. The sensitivity of both probes was $\sim 5 \times 10^{-3}$ V/Mx.

The aim of the experiments performed was, as indicated above, to determine the wavelength λ—the dimension characterizing the mean curling period of the plasma filament. Figure 1 shows separate high-speed frame photographs phased in with the oscillograms of

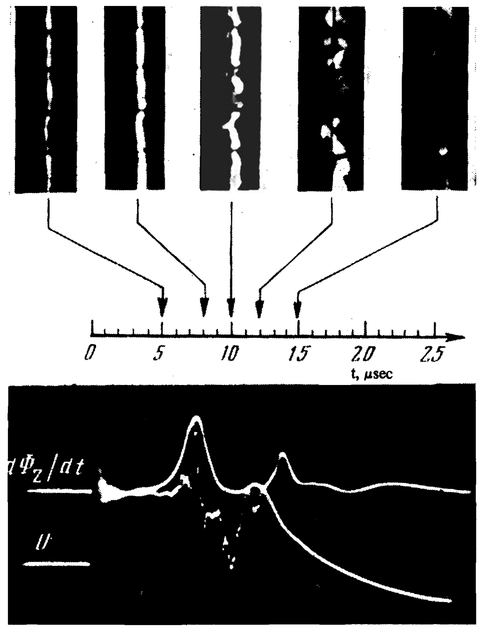

FIG. 1. Oscillograms of the voltage on the discharge chamber and of the derivative of the longitudinal magnetic field. The upper part of the figure shows high-speed frame photographs pertaining to a definite moment of time during the discharge.

the voltage U on the discharge tube and with the $d\Phi_Z/dt$ signals of the external probe. It can be seen that to the maximum amplitude of the plasma-filament deformation correspond the peaks in the voltage curve and the maxima of H_Z.

It should be noted that not only is the direction of the curling not preserved from discharge to discharge, but in an individual discharge it varies along the length of the filament. To make this more precise, we measured the field H_Z with four identical probes located at equal intervals along the axis of the tube. Examination of the oscillograms obtained in 26 identical discharges showed that in 4 cases all the probes recorded a field of one sign, in 6 cases a field of the opposite sign, and in 16 cases the direction of the field in each discharge changed from probe to probe.

For the purpose of elucidating the dependence of the wavelength λ on the length of the discharge tube, we carried out the corresponding measurements for three tubes of different lengths. Figure 2 shows photographs of the filament for an instant close to the maximum-compression phase. It can be seen that there is no significant difference in wavelength for these three photographs. Of course all the remaining parameters (a_0, dI/dt, p_0) for these three cases remained uncharged.

A completely different picture was observed in the experiments in which the quantity $(dI/dt)_0$ and the radius a_0 of the tube were varied. It can be seen from the photographs in Figs. 3 and 4 that the wave length of the instability increases with the tube radius and decreases with increasing current growth rate. We recall that the experiments were performed in the range of values of $(dI/dt)_0$ from 10^9 to 10^{11} A/sec. For values of $(dI/dt)_0 \geq 10^{11}$ A/sec the m = 0 instability (constrictions) developed. At relatively low rates, $(dI/dt)_0 \geq 10^9$ A/sec, a picture completely different from the usual "fast" process is observed. The characteristic deformation for such a process is a large-scale kinking or curling of the plasma filament (the m = 1 mode) with a relatively long instability wavelength.

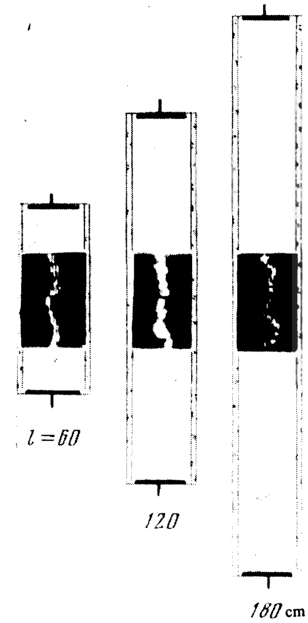

FIG. 2. Photographs of the filament for three different lengths of the discharge chamber; p_0 = 0.1 Torr, a_0 = 10 cm, $(dI/dt)_0 = 10^{10}$ A/sec.

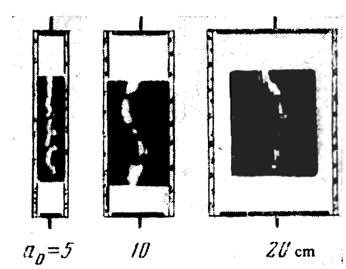

FIG. 3. Photographs of the filament in the case of three different values of the diameter of the discharge chamber; p_0 = 0.1 Torr, l = 60 cm, $(dI/dt)_0 = 5 \times 10^9$ A/sec.

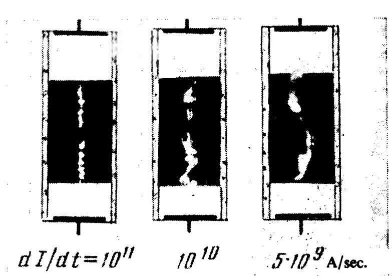

FIG. 4. Photographs of the filament in the case of three different values of the current growth rate; p_0 = 0.1 Torr, a_0 = 10 cm, l = 60 cm.

Using the set of data thus obtained, we can uncover a definite dependence between the wavelength λ of the instability and the parameters characterizing the discharge. It turns out that when the experimental conditions were such that the parameters a_0 and $(dI/dt)_0$ varied in the fairly wide ranges:

$$4.5 \leqslant a_0 \leqslant 20\,\text{cm}, \quad 5\cdot 10^9 \leqslant (dI/dt)_0 \leqslant 10^{11}\ \text{A/sec}$$

the quantity $\lambda a_0^{-1}(dI/dt)_0^{1/2} \equiv \theta$ remained more or less constant. The table illustrates this assertion. The values in the last column are normalized to those in the second column. Assuming that θ = const, we find, normalizing to the second column, that

$$\lambda \approx 2\cdot 10^5 a_0 (dI/dt)_0^{-1/2}\ [\text{cm}].$$

As for the moment when the deformation of the plasma

$(dI/dt)_0$, A/sec	a_0, cm	λ, cm	θ, rel. units
10^{11}	10	5	1.27
10^{10}	10	20	1.0
$5\cdot10^9$	10	35	0.82
$5\cdot10^9$	20	70	0.82
$5\cdot10^9$	4.5	10	1.29

plasma filament appears, comparison of the readings of the external and internal probes shows that the development of the instabilities precedes the maximum compression phase.

Figure 5 shows the photosweep of the radiation of the plasma filament. The photosweep is synchronized with oscillograms of the external (Φ_Z) and internal (Φ_{Z_0}) probes. It should be pointed out that the readings of the external magnetic probe prove to be more sensitive at the stage of the development of the instability when the "curling" plasma filament expands and approaches the walls of the chamber. The most complete information about the initial phase of the development of the instability is given by the internal probe, which fixes precisely the instant when a longitudinal magnetic field appears near the axis of the discharge. The plasma filament then appears as though it has been carrying the instability long before the total constriction. Therefore, we can apparently assume that an instability of definite wavelength λ is established in the filament before its detachment from the walls of the container.

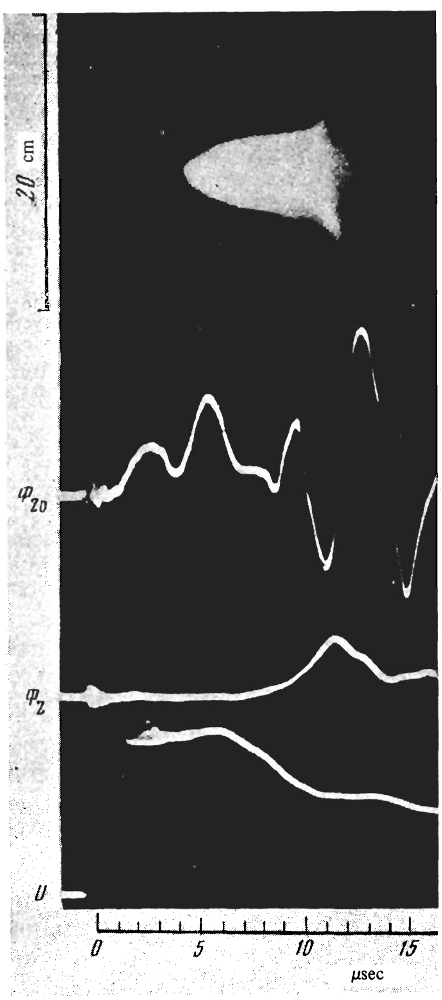

FIG. 5. Photosweep of the radiation of a diameter of the plasma filament, taken synchronously with the oscillograms of the magnetic field.

The experimental results cited indicate that both the m = 0 and m = 1 instabilities appear at the initial stage of the development of the Z-pinch: at that stage when it has only just broken away from the walls. The experiments[4] performed by Phillips on the "Colombo" facility already indicate this.

The above-discussed results of the experimental investigations cast doubt on the applicability here of the existing theory of the magnetohydrodynamic instabilities that has been developed for the stationary Z-pinch. In fact the fact that the instability develops at the early stage of the formation of the pinch and the wavelength turns out to be dependent upon the initial radius and growth rate of the current does not fit into the framework of the existing theory. According to this theory, in the absence of the longitudinal field, the Z-pinch is unstable with respect to all wavelengths for both the m = 1 and m = 0 modes. Further, the shorter the wavelength the larger the increment of the perturbation. The magnitude of the increment is determined by the current and radius of the pinch in its constricted state, etc. All this is at variance with the observed facts, and it is clear that in our case, of course, there develops an instability connected with the compression process, i.e., with the dynamics of the process and not with the development of an instability of the pinched filament.

An investigation of a magnetohydrodynamic type of instability usually amounts to the analysis of the buildup of small stochastic perturbations for a system in some quasi-stationary equilibrium state. In our case it is natural to assume that the Rayleigh-Taylor type of instabilities being observed do develop as a result of the acceleration of the plasma during its compression from the periphery towards the center. Such instabilities can develop prior to the detachment of the entire plasma from the walls of the discharge chamber because of the motion of the compression wave. It was postulated quite a long time ago that in "fast" Z-pinches the m = 0 instability had a Rayleigh-Taylor origin, and experiments were performed which quite convincingly confirmed that hypothesis. The bibliography is given in Harris's paper,[5] where it is shown that such an instability arises with the m = 1 mode. However, in all these investigations it is found that perturbations of all wavelengths are unstable. This occurs because, as was noted by Harris, what was solved was in fact the surface problem: the plasma was assumed to be a perfect conductor, the magnetic pressure being considered as applied to the plasma surface. Here, as indeed in most theoretical investigations of the magnetohydrodynamic instabilities, the processes inside the plasma are not considered. The solution of the problem which includes the investigation of the processes inside the plasma is attended by certain as yet unsurmounted difficulties. Therefore, let us restrict ourselves to some qualitative analysis based rather on dimensional analysis.

We shall proceed from the well-known Kruskal-Schwarzschild relation which indicates the relation between the increment τ of the instability and the wave number κ characterizing the wavelength of the instability for a semi-infinite plasma maintained by a magnetic field in the gravitational fields:

$$\tau^{-1} = {}^1\!/_2 (g\kappa)^{1/2}, \qquad (5)$$

where g is the acceleration of the plasma boundary. Since the results of the experiments carried out clearly indicate that an instability develops in the process of "detachment" of the plasma from the walls of the container, the sole quantity determining the time scale is the "detachment" time t_{det}. Let us assume that $\tau = t_{det}$, where t_{det} is the time during which the discharge current rises to the value at which the whole mass of gas is detached from the walls and dragged in the course of the compression towards the axis.

We determine the acceleration g of the plasma boundary on the basis of the equation of motion of the contracting gaseous column under the action of the magnetic pressure of the field of the current flowing through the gas. Let us write this equation in the form (see, for example[6])

$$r\frac{d}{dt}M_0\frac{dr}{dt}=-\frac{I^2}{c^2}. \qquad (6)$$

Here M_0 is the mass of the gas involved in the acceleration process; $M_0 = \pi(a_0^2 - r^2)\rho_0$, where ρ_0 is the initial density: $\rho_0 = MN/\pi a_0^2$, N is the number of particles per unit length of the chamber, M is the ion mass, and r is the instantaneous radius.

Let us consider the beginning of the motion. We shall regard this motion, as a uniformly accelerated motion, so that $r = a_0 - gt^2/2$, with $gt^2/2 \ll a_0$. We shall also assume that $I = (dI/dt)_0 t$, where $(dI/dt)_0 = \text{const}$. Then, under these assumptions, we find from (6) that

$$g\approx\left(\frac{dI}{dt}\right)_0\frac{1}{ca_0(3\rho_0\pi)^{1/2}}. \qquad (7)$$

Since the detachment current $I_{det} = (dI/dt)_0 t_{det}$, we obtain from (5) and (7)

$$\lambda=\frac{\pi}{2}\frac{I_{det}^2}{c(dI/dt)_0(3MN)^{1/2}}. \qquad (8)$$

We take the expression for the detachment current in the form[7]

$$I_{det}^2=c^2\left[\frac{80\pi a_0{}^2\varepsilon}{\sigma_{in}c}\left(\frac{dI}{dt}\right)_0(MN)^{1/2}\right]^{1/2}. \qquad (9)$$

We recall that in[7] a weakly ionized plasma was considered in which the neutral particles were drawn into the motion from the periphery to the axis largely by the charge-transfer mechanism. The interaction cross section σ_{in} for this process in the region of low energies for hydrogen considerably exceeds the cross section for other elementary events and is $\sim 3\times10^{-15}$ cm^2.

In the course of the ionization of a plasma by slow electrons with energies of 5–10 eV, the energy losses (the cost of one electron ϵ) in one ionization event are $\sim$ 100 eV. In this case the energy is largely expended in the excitation of the neutral atoms. Substituting these values ($\sigma_{in} = 3\times10^{-15}$ cm^2 and ϵ = 100 eV) and going over to the practical system of units, we obtain from (8) and (9)

$$\lambda=\frac{\pi}{2}\frac{a_0}{(MN)^{1/4}}\left[\frac{80\pi c\varepsilon}{\sigma_{in}(dI/dt)_0}\right]^{1/2}\approx\frac{5\cdot10^9a_0}{(dI/dt)_0{}^{1/2}N^{1/4}}. \qquad (10)$$

Here all the quantities pertain to hydrogen, which was used as the working gas in all the cited experiments. In the above-described experiments the number of particles in a cross section was not too different from $N = 10^{18}$ particles/cm. Then

$$\lambda=1.8\cdot10^5a_0(dI/dt)_0^{-1/2}\ [\text{cm}],$$

which is in good agreement with the above-cited results. It is clear that since N enters into the expression with a power of 1/4, the dependence on the initial pressure is not noticeable in the experiments, especially as the experiments were performed in a comparatively narrow range of initial pressures.

From the relation (10) we can conclude that the m = 0 perturbation appears when $\lambda/a_0 < 1$, and this means that instabilities of the "constriction" type develop when

$$(dI/dt)_0>2.5\cdot10^{19}/N^{1/2}. \qquad (11)$$

For example, for $N = 10^{18}$, this type of instability appears when

$$(dI/dt)_0>2.5\cdot10^{10}\,\text{A/sec}$$

which is in good agreement with experiment.

Of course the relations cited here have not been sufficiently well validated. The main argument in their favor is, in truth, the surprisingly good agreement with the experimental results—not only with those cited in the present paper, but also with the results of measurements carried out by other experimentalists on similar facilities.[8] This gives ground to suppose that a future theory will lead to relations that will be sufficiently close to those cited here, and that as long as such a theory does not exist we can with sufficient confidence use the results of the present paper.

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