Kinetic phenomena in a Knudsen gas with rotational degrees of freedom

V. D. Borman, L. A. Maksimov, B. I. Nikolaev, and V. I. Troyan
Moscow Engineering Physics Institute

(Submitted August 17, 1972)
Zh. Eksp. Teor. Fiz. **64**, 526-535 (February 1973)

Heat transfer in a very low density gas located in an arbitrarily oriented magnetic field is considered. Boundary conditions for the distribution function of a gas with rotating molecules are discussed. It is shown that if the diffuse reflection coefficient depends on the direction of the angular momentum of the molecules, the heat flow in a Knudsen molecular gas may vary in a magnetic field. From the periodic dependence of the distribution function on distance between two parallel plates it follows that the change of the heat flow in the field should be a periodic function of the product of the molecule precession frequency in the field and the mean free time. The dependence of the effect on field direction is considered. Nonspherical interaction between the molecules and the wall should result in polarization of molecules emitted from a plane gap whose walls have different temperatures.

1. INTRODUCTION

It is known that the transfer coefficients of molecular gases become altered in a magnetic or an electric field (the Senftleben effect)[1-4]. In recent years the tendency has been to investigate the Senftleben effect at ever decreasing pressures. It turns out[5,6] that the effect decreases with decreasing pressure p. The reason is that the main cause of the Senftleben effect, namely collisions of nonspherical molecules with one another, disappears with decreasing pressure. It becomes clear therefore that in the case of a strongly rarefied gas, when the mean free path λ is much larger than the characteristic dimension L of the vessel, transport phenomena either are completely independent of the external field H, or a new effect should appear, namely the dependence of the kinetic phenomena in a Knudsen gas on the external field. This dependence is due to the nonspherical character of the reflection of the molecules from the vessel walls. This new effect was predicted by us earlier[7]. One of the interesting features of this effect is a periodic dependence of the dissipative flux on the products HL. This constitutes the fundamental difference from the classical monotonic dependence of the Senftleben effect on Hλ, i.e., on the ratio H/p. Oscillations of heat flow in a magnetic field were observed in the gases N_2NF_3, and CO[8].

Since the study of a periodic effect in a Knudsen gas can yield essentially new information on the interaction of the molecules with the wall, it is desirable to continue the research. We discuss in the present article the boundary conditions on the wall for the distribution function of molecules with rotational degrees of freedom. The heat-transfer problem[7] is generalized to include the case of an arbitrary field direction. "Thermopolarization" of the molecules emitted from a slit is considered.

We note that back in 1968 Waldmann[9] pointed out the nonspherical character of molecule reflection from a wall as a possible cause of the rotation, in a magnetic field, of a heated cylinder placed in a molecular gas (the Scott effect). Waldmann explained this effect qualitatively, and assumed the collisions of the molecules with the walls to be purely elastic (the "cut tennis ball" model). We show in the present paper (see Sec. 2) that in a Knudsen gas the kinetic phenomena can depend on the external field only if the nonspherical molecules collide inelastically with the wall.

2. BOUNDARY CONDITIONS

The nonsphericity of the collisions of the molecules with the wall, i.e., the dependence of the character of the reflection on the direction of the angular momentum of the molecule, is the main cause of all the phenomena considered in the paper. This nonsphericity should be taken into account in the boundary condition, which plays the role of the collision integral in the kinetic equation for a strongly rarefied gas

$$\frac{\partial f}{\partial t}+(\mathbf{v}\nabla)f+\gamma[\mathbf{MH}]\frac{\partial f}{\partial \mathbf{M}}=0 \qquad (2.1)^*$$

($\mathbf{v}$ and $\mathbf{M}$ are the velocity and angular momentum of the molecules, and γ is the gyromagnetic ratio).

The boundary condition connects the distribution function of the molecules reflected from the wall (f^+) with the distribution function of the molecules incident on the wall (f^-). As usual[10] we assume that the collisions of the molecules with a wall, which lies in the xy plane, are divided into two classes: purely elastic reflection, at which the total energy of the molecules is conserved, and purely diffuse scattering, at which there is no correlation between the initial state (prior to collision with the wall) and final state (after evaporation from the wall) of the molecule. Then

$$f^+(\mathbf{v}, \mathbf{M}) = (1-\alpha)f^-(\mathbf{v}', \mathbf{M}') + \beta f(v^2, M^2), \qquad (2.2)$$

Here $\mathbf{v}$ and $\mathbf{M}$ are the velocity and angular momentum of the molecules moving away from the wall ($\mathbf{v}\cdot\mathbf{k} > 0$, $\mathbf{k}$ is the outward normal to the wall); $\mathbf{v}' = \hat{S}_k\mathbf{v}$ and $\mathbf{M}' = \hat{S}_k\mathbf{M}$ are the velocity and angular momentum prior to elastic collision with the wall; $(1-\alpha)$ is the probability of elastic reflection from the wall; β_f is the distribution of the molecules that experience diffuse reflection.

The nonsphericity of the collisions of the molecules with the wall can become manifest in three different phenomena. First, the law of elastic reflection depends on the orientation of the angular momentum. For example, in the case of the model of reflection of a rough sphere[11] from a rough plane

$$m\mathbf{v}' = m\mathbf{v} + \mathbf{q}, \quad \mathbf{M}' = \mathbf{M} + R[\mathbf{qk}],$$
$$\mathbf{q} = -\frac{2\varkappa}{1+\varkappa}\left\{ m\mathbf{v} + m\mathbf{k}(\mathbf{vk}) + \frac{1}{\varkappa R}[\mathbf{kM}]\right\}. \tag{2.3}$$

Here m is the mass, R is the radius, and $I = \kappa m I^2$ is the moment of inertia of the sphere.

In general, the direction of the angular momentum can influence the probability of adhesion of the molecule to the wall, i.e., the coefficient of diffuse reflection $\alpha = \alpha(\mathrm{vM})$, and, in addition, the distribution of the diffusely reflected molecules which have previously adhered to the wall and were subsequently evaporated. This distribution is Maxwellian, with a temperature equal to the wall temperature, only in the simplest model. More complete models of diffuse reflection[10] take into account the dependence of the probability of evaporation β on the direction of the velocity of the evaporating molecule. For a gas with rotational degrees of freedom, the value of β can also depend on the direction of the angular momentum, $\beta = \beta(\mathbf{v}, \mathbf{M})$. In principle, we can propose many not very convincing models of the interaction and calculate the coefficients α and β for these models. We shall adhere to the phenomenological point of view, according to which the quantities α and β should be expanded in orthogonal polynomials in $\mathbf{v}$ and $\mathbf{M}$, and the coefficient of expansion should be determined directly from experiments on the scattering of the molecules from the wall and from measurements of effects of the type proposed below. Expansions of this kind are complicated because the collision of the molecules with the wall (unlike collisions of molecules with one another) is not invariant with respect to the group of three-dimensional rotations, and can have only axial symmetry. Therefore the invariant expansion of α and β should contain in explicit form the unit vector normal to the wall. For example

$$\beta = \sum_{l_1 l_2 l_3} \beta_{l_1 l_2 l_3} \sum_{m_1 m_2 m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} Y_{l_1 m_1}(\mathbf{v}) Y_{l_2 m_2}(\mathbf{M}) Y_{l_3 m_3}(\mathbf{k}), \tag{2.4}$$

where (...) is the coefficient of vector addition, and the expansion parameters β can depend on v^2 and M^2.

It is clear that at the present status of the experiment the expansion (2.4) with a large number of terms is useless. To reduce the boundary condition to an expression containing the minimum number of parameters, at which one can still speak of effects due to nonsphericity of the molecules, we retain in (2.4) the terms with $l_1 = l_2 = l_3 = 0$ and $l_1 = l_2 = l_3 = 1$, and put

$$\alpha = \beta = \alpha_0\{1 + \mu(\mathbf{k}[\mathbf{vM}])\},$$
$$\mathbf{v}' = \mathbf{v} - 2\mathbf{k}(\mathbf{vk}), \quad \mathbf{M}' = \mathbf{M}, \tag{2.5}$$

μ is a parameter characterizing the evaporation of the molecules from the wall from isotropic diffuse evaporation.

It seems at first glance that to take the nonsphericity of the molecules into account it suffices to include this nonsphericity in the elastic-collision law, as in (2.3), and to assume the coefficients α and β to be constant. In this case, however, the heat transfer between walls turns out to be independent of the external field, since the solution of the critical equation (2.1) under such conditions can be represented in the form of a superposition of Maxwellian distributions with the same temperatures as the walls, and does not change when the field is turned on. It is easily understood that turning on an external field changes the distribution function only in those cases when the distribution in the absence of the field depends on the direction of the angular momentum. This is precisely why we have assumed in (2.5) a specular elastic-reflection law, and took the nonsphericity into account in the coefficients that describe the adhesion probability α and the evaporation β of the molecules.

We shall consider below a gas between plane-parallel identical walls (z = 0 and z = L). The general boundary conditions for this problem are

$$f^+(\mathbf{v}, \mathbf{M}) = [1 - \alpha(\mathbf{k}, \mathbf{v}, \mathbf{M})] f^-(\mathbf{v}', \mathbf{M}') + \beta(\mathbf{k}, \mathbf{v}, \mathbf{M}) f_0(v^2, M^2), \; z = 0,$$
$$f^-(\mathbf{v}, \mathbf{M}) = [1 - \alpha(-\mathbf{k}, \mathbf{v}, \mathbf{M})] f^+(\mathbf{v}'', \mathbf{M}'') + \beta(-\mathbf{k}, \mathbf{v}, \mathbf{M}) f_L(v^2, M^2), \; z = L. \tag{2.6}$$

Here $\mathbf{v}'' = \hat{S}_{-k}\mathbf{v}$, $\mathbf{M}'' = \hat{S}_{-k}\mathbf{M}$, f_0 and f_L are Maxwellian distributions having the same temperatures T_0 and T_L as the walls. These distributions should be normalized in such a way that

$$\int\limits_{\mathbf{kv}>0} d\mathbf{v}\, d\mathbf{M}\, (f^+ + f^-) = n, \; \int\limits_{\mathbf{kv}>0} d\mathbf{v}\, d\mathbf{M}\, \mathbf{kv}\, (f^+ - f^-) = 0,$$

where n is the gas density. The second condition denotes that the molecule flux through the wall is equal to zero.

In the model (2.5), Eqs. (2.6) take the form

$$f^+(\mathbf{v}, \mathbf{M}) = \{1 - \alpha_0[1 + \mu\mathbf{k}[\mathbf{vM}]]\} f^-(\mathbf{v}', \mathbf{M}) + \alpha_0[1 + \mu\mathbf{k}[\mathbf{vM}]] f_0,$$
$$f^-(\mathbf{v}, \mathbf{M}) = \{1 - \alpha_0[1 - \mu\mathbf{k}[\mathbf{vM}]]\} f^+(\mathbf{v}, \mathbf{M}) + \alpha_0[1 - \mu\mathbf{k}[\mathbf{vM}]] f_L. \tag{2.7}$$

3. SOLUTION OF KINETIC EQUATION

We consider the problem of heat transfer in a collisionless molecular gas between parallel walls situated at a distance L from one another, in the presence of a constant homogeneous magnetic field $\mathbf{H}$ having an arbitrary orientation. We choose the coordinate system such that the xy plane coincides with one of the walls, and the second is defined by the equation z = L. The stationary distribution of the gas in the gap is described by the equation

$$v_z \partial f / \partial z + \gamma[\mathbf{MH}] \partial f / \partial \mathbf{M} = 0. \tag{3.1}$$

We seek the solution of (3.1) in the form

$$f = \sum_m F_m \exp\left(im \frac{\omega z}{v_z}\right), \quad F_m = F_m(\mathbf{v}, \mathbf{M}), \tag{3.2}$$

where $\omega = \gamma H$ is the molecule precession frequency in the field. Substituting (3.2) in (3.1), we obtain

$$im\omega F_m + \gamma[\mathbf{MH}] \partial F_m / \partial \mathbf{M} = 0. \tag{3.3}$$

We rotate the coordinate system in space $\mathbf{M}$ in such a way that the z′ axis of the new system coincides with the direction of the field $\mathbf{H}$. Equation (3.3) takes in the rotated coordinate frame the form

$$im\omega F_m - \omega \partial F_m / \partial \tilde{\varphi}_M = 0, \tag{3.4}$$

where $\tilde{\varphi}_M$ is the polar angle of orientation of $\mathbf{M}$ in the new coordinate system. We can write the solution of (3.4) in the form

$$F_m = \sum_l \chi_{lm} Y_{lm}(\tilde{\mathbf{M}}), \tag{3.5}$$

where $Y_{lm}(\tilde{\mathbf{M}})$ is a spherical function of $\tilde{\varphi}_M$ and ϑ_M. The inverse transformation to the old coordinate system corresponds to a linear transformation in the space of the functions Y_{lm}, and therefore

$$F_m = \sum_{lk} \chi_{lm} D_{mk}{}^l Y_{lk}(\mathbf{M}). \tag{3.6}$$

Here $D^l_{mk} = D^l_{mk}(\psi, \vartheta_H, \varphi_H)$ is the finite-rotation matrix[12], ϑ_H and φ_H are the azimuthal and polar angles of the orientation of the magnetic field, and ψ is the angle of rotation about the field direction. Substituting this expression in (3.2), we find that the solution of (3.1) takes the form

$$f=\sum_{lmk}\chi_{lm}D_{mk}^{l}Y_{lk}(\mathbf{M})\exp\left(im\frac{\omega z}{v_z}\right). \tag{3.7}$$

The functions χ_{lm} depend on the vector **v** and on M^2.

The function f is discontinuous in velocity space. The unknown functions $\chi_{lm}^{\pm}=\chi_{lm}^{\pm}(\mathbf{v}, M^2)$ are determined from the boundary conditions (2.7), which take the following form after substituting the expression for $f^{\pm}$, multiplying by $Y_{s\sigma}^{*}(\mathbf{M})$ and integrating over the directions of the vector **M**:

$$\chi_{s\sigma}^{+}=(1-\alpha_0)\chi_{s\sigma}^{-}+\alpha_0f_0\delta_{s0}+{}^4/_3\alpha_0\mu\pi Mvi[Y_{11}(\mathbf{v})D_{\sigma-1}^{1*}-Y_{1-1}(\mathbf{v})D_{\sigma1}^{1*}]f_0\delta_{s1}$$

$$-\frac{4}{3}\pi\alpha_0\mu Mvi\sum_{lmk}(-1)^{m+1}C_{l010}^{s0}C_{s\sigma1k}^{lm}\chi_{lm}^{-}\delta_{l,s\pm1}[Y_{11}(\mathbf{v})D_{k-1}^{1*}-Y_{1-1}(\mathbf{v})D_{k1}^{1*}],$$

$$\chi_{s\sigma}^{-}e^{-i\sigma\eta}=(1-\alpha_0)\chi_{s\sigma}^{+}e^{i\sigma\eta}+\alpha_0f_L\delta_{s0}-{}^4/_3\alpha_0\mu\pi Mvi[Y_{11}(\mathbf{v})D_{\sigma-1}^{1*} \tag{3.8}$$

$$-Y_{1-1}(\mathbf{v})D_{\sigma1}^{1*}]f_L\delta_{s1}+\frac{4}{3}\pi\alpha_0\mu Mvi\sum_{lmk}(-1)^{m+1}C_{l010}^{s0}C_{s\sigma1k}^{lm}\chi_{lm}^{+}e^{-i\sigma\eta}\delta_{l,s\pm1}$$

$$\times[Y_{11}(\mathbf{v})D_{k-1}^{1*}-Y_{1-1}(\mathbf{v})D_{k1}^{1*}],$$

where $\eta=\omega L/v_z$, $D_{km}^{l*}=(-1)^{k-m}D_{-k-m}^{l}$, and C are Clebsch-Gordan coefficients.

Equations (3.8) constitute a system of coupled equations. The last terms on the right in (3.8), which are due to the coupling, are linear in the parameter μ which characterizes, in the chosen model, the deviation from isotropic diffuse evaporation. The system (3.8) can therefore be solved by assuming μ to be a small quantity and by expanding the function $f^{\pm}$ in powers of μ:

$$f^{\pm}=\sum_n f_n^{\pm},\qquad f_n^{\pm}\sim\mu^n. \tag{3.9}$$

To calculate the heat flux and the "thermopolarization" we shall need the first-approximation functions $\chi_{1lm}^{\pm}$. These functions, obtained in the indicated manner, are written in the form

$$\overset{+}{\chi}_{1lm}=\frac{4}{3}\frac{\pi\alpha_0\mu Mvi}{(2-\alpha_0)}[D_{m-1}^{1*}Y_{11}(\mathbf{v})-D_{m1}^{1*}Y_{1-1}(\mathbf{v})][e^{-im\eta}$$

$$+(1-\alpha_0)](f_0-f_L)\Delta_m^{-1}\delta_{l1},$$

$$\overset{-}{\chi}_{1lm}=\frac{4}{3}\frac{\pi\mu\alpha_0Mvi}{(2-\alpha_0)}[D_{m-1}^{1*}Y_{11}(\mathbf{v})-D_{m1}^{1*}Y_{1-1}(\mathbf{v})][(1-\alpha_0)e^{im\eta}+1] \tag{3.10}$$

$$\times(f_0-f_L)\Delta_m^{-1}\delta_{l1},$$

$$\Delta_m=e^{-im\eta}-(1-\alpha_0)^2e^{im\eta}.$$

4. HEAT TRANSFER

The heat flux¹⁾ between two plates in a Knudsen gas is expressed in the form

$$Q=\int_{v_z>0}v_z\varepsilon(f^+-f^-)\,d\mathbf{v}\,d\mathbf{M},$$

$$\varepsilon={}^1/_2mv^2+M^2/2I,\quad d\mathbf{M}=M^2d\cos\vartheta_Md\varphi_MdM. \tag{4.1}$$

The integration in (4.1) is over the velocities of the molecules that move upward from the wall z = 0. After substituting in (4.1) the expressions (3.7) for $f^{\pm}$ and taking the expansion (3.9) into account, we obtain

$$Q=Q_0+Q_1+Q_2+\ldots,$$

$$Q_n\sim\mu^n,\qquad Q_n=\int v_z\varepsilon(\overset{+}{\chi}_{n00}-\overset{-}{\chi}_{n00})\,d\mathbf{v}\,d\mathbf{M}. \tag{4.2}$$

There are no odd terms in the sum of (4.2), since the terms of (3.9) which are odd in μ are odd functions of the vector **M**.

The flux Q_0 does not depend on the field

$$Q_0=\frac{7}{16}\frac{\alpha_0}{(2-\alpha_0)}n_0v_0(T_0-T_L), \tag{4.3}$$

where $v_0=(2T_0/m)^{1/2}$. In the calculation of (4.3) and in what follows it is assumed for simplicity that $|T_0-T_L|\ll T_0$.

Let us find the heat flux Q_2, which is the first field-dependent term in the series (4.2). The second-approximation functions $\chi_{200}^{\pm}$ are defined, in accordance with (3.8) and (3.9), by the equations

$$\overset{+}{\chi}_{200}=\frac{\mu}{(2-\alpha_0)}\langle Y_{00}(\mathbf{M}),\mathbf{k}[\mathbf{vM}][(1-\alpha_0)f_1^+-f_1^-]\rangle,$$

$$\overset{-}{\chi}_{200}=\frac{\mu}{(2-\alpha_0)}\langle Y_{00}(\mathbf{M}),\mathbf{k}[\mathbf{vM}][f_1^+-(1-\alpha_0)f_1^-]\rangle, \tag{4.4}$$

$$\langle F_1,F_2\rangle=\int F_1^*F_2\,d\cos\vartheta_M\,d\varphi_M.$$

The calculation of the integrals in the right-hand side of (4.4) with allowance for (3.10) yields, for example,

$$\langle Y_{00}(\mathbf{M}),\mathbf{k}[\mathbf{vM}]f_1^+\rangle=\frac{4}{3}\frac{\pi\alpha_0\mu}{(2-\alpha_0)}M^2v^2$$

$$\times\sum_{m=-2}^{2}\left\{\frac{1}{2}[(2-m^2)+(3m^2-2)\cos^2\vartheta_H]\left[\frac{1}{2}\pi^{-1/2}+5^{-1/2}Y_{20}(\mathbf{v})\right]\right.$$

$$\left.-\frac{1}{8\sqrt{\pi}}(3m^2-2)\sin^2\vartheta_H\sin^2\vartheta_v\cos2(\varphi_v+\varphi_H)\right\}[1+(1-\alpha_0)e^{im\eta}]\Delta_m^{-1}(f_0-f_L). \tag{4.5}$$

In the derivation of (4.5) we used the following relation for the function D[12]:

$$D_{m_1m_1'}^{l_1}D_{m_2m_2'}^{l_2}=\sum_l C_{l_1m_1l_2m_2}^{lm}C_{l_1m_1'l_2m_2'}^{lm'}D_{mm'}^{l}.$$

Using (4.2)—(4.4), we obtain an expression for the heat flux Q_2:

$$Q_2=-\frac{8\pi\alpha_0^2\mu^2}{9(2-\alpha_0)}\sum_{m=-2}^{2}\left[\left(1-\frac{m^2}{2}\right)+\frac{3}{2}\cos^2\vartheta_H\left(m^2-\frac{2}{3}\right)\right]$$

$$\times\int v_z\varepsilon M^2v^2[{}^1/_2\pi^{-1/2}+5^{-1/2}Y_{20}(\mathbf{v})](f_0-f_L)\varphi_m\,d\mathbf{v}\,d\mathbf{M}, \tag{4.6}$$

$$\varphi_m=\frac{(1-\alpha_0)e^{im\eta}+1}{e^{-im\eta}-(1-\alpha_0)^2e^{im\eta}}\qquad\eta=\frac{\omega L}{v_z}=\frac{\gamma HL}{v_z}$$

Expression (4.6) together with (4.3) describes the heat transfer between flat plates in an arbitrarily directed magnetic field.

We separate in the flux Q_2 the part that depends on the magnetic field. In the absence of a field (η = 0) we have

$$Q_2(0)=-\frac{33}{2}\frac{\alpha_0\pi^{1/2}}{(2-\alpha_0)^2}\Psi^2n_0v_0(T_0-T_L), \tag{4.7}$$

where $\Psi=\mu T(I/m)^{1/2}$. The flux $Q_2(0)$ is a correction to Q_0 and takes into account the deviation of the character of the diffuse reflection from isotropic.

Equations (4.6) and (4.7) lead to expressions for the relative change in the heat flux for cases when the field is parallel ($\Delta Q_{\parallel}/Q_0$) and perpendicular ($\Delta Q_{\perp}/Q_0$) to the plates:

$$\frac{\Delta Q_{\parallel}}{Q_0}=\frac{2\alpha_0}{(2-\alpha_0)}\Psi^2\left[I(\xi,\alpha_0)+\frac{132}{7\alpha_0}\right]$$

$$\frac{\Delta Q_{\perp}}{Q_0}=\frac{\alpha_0}{(2-\alpha_0)}\Psi^2\left[I(\xi,\alpha_0)+\frac{132}{7\alpha_0}\right]; \tag{4.8}$$

$$I(\xi,\alpha_0)=\frac{48}{7}\int_0^{\infty}(x^3+4.5x)$$

$$\times\frac{(1-\alpha_0)^2-\alpha_0(2-\alpha_0)\cos(\xi/x)-(1-\alpha_0)\cos(2\xi/x)}{1-2(1-\alpha_0)^2\cos(2\xi/x)+(1-\alpha_0)^4}\exp(-x^2)\,dx,$$
$$x=v_z(m/2T)^{1/2},\quad \xi=2\gamma HL/v_0\pi^{1/2}. \tag{4.9}$$

It follows from (4.8) and (4.9) that the effect should depend on the dimensionless parameter ξ, which is a product of the molecule precession frequency in the field $\omega = \gamma H$ and the free path time of the molecules from wall to wall $\tau = L/v_0$. A plot of the relative change of the heat flux against the parameter ξ is shown in Fig. 1. It is seen from the figure that in a magnetic field there should be observed heat-flux oscillations whose period increases as amplitude decreases with increasing product HL ($\xi \sim HL$). These oscillations correspond to single, double, etc. precession of the molecule as it moves from wall to wall (see the z-dependence in (3.7)).

The aperiodicity of the oscillations is due to the averaging of the heat flux over the molecule velocities. It follows from (4.4), (4.6), and (4.9) that the damping of the oscillations will be weaker in comparison with the molecule-wall interaction model (2.5) considered above if $\mu \sim v^k$ where $k \gg 1$ ($k = 0$ in the present model).

The effect saturates with increasing product $\omega\tau$. For example, in the case of a field parallel to the walls we have

$$\left(\frac{\Delta Q_\parallel}{Q_0}\right)_{\omega\tau\to\infty}=\frac{264}{7}\Psi^2\frac{1}{(2-\alpha_0)}. \tag{4.10}$$

Oscillations of the heat flux in a magnetic field were observed experimentally in the gases N_2, NF_3, and CO[8], and was observed later in an electric field in NF_3 by L. L. Gorelik and A. I. Pisanko (private communication). A quantitative comparison of the presented theory with the experimental results[8], however, is difficult because the geometry used in these experiments is not the same as that considered by us. In addition, our experiments[8] were performed at gas pressures at which $\lambda \gtrsim L$, and not at $\lambda \gg L$. Under these conditions, the change in the heat flux occurs not only because of the mechanism proposed here, but also because the average cross section for collisions of the molecules with one another changes in the field (the Senftleben effect).

It follows from (4.3), (4.7), and (4.8) that the heat flux should increase in a magnetic field. The heat flux Q_2, which depends on the molecule orientation, vanishes as $\omega\tau \to \infty$ if the field is perpendicular to the plates, and decreases by a factor of 2 when the field is parallel to the plates.

At any fixed value of the field we have

$$\Delta Q_\parallel/\Delta Q_\perp=2. \tag{4.11}$$

This result should be connected with the chosen model of the interaction of the molecules with the wall (2.5), when the most probable is evaporation of the molecules with an angular momentum perpendicular to the direction of emission of the molecules from the surface. It is obvious that another dependence of the coefficient of evaporation and adhesion of the molecules on the directions of the velocity and of the angular momentum of the molecules should lead to a change in the ratio $\Delta Q_\parallel/\Delta Q_\perp$ in comparison with (4.11). An analogous situation obtains at high pressures ($\lambda \ll L$), namely, the ratio of the maximum changes of the heat-conduction coefficient in fields parallel and perpendicular to the temperature gradient depends directly on the anisotropy of the distribution function in the (**v**, **M**) space, which in turn is determined by the dependence of the scattering probability on the directions of the angular-momentum vectors of the colliding molecules and their relative velocity. It follows therefore that measurement of the ratio $\Delta Q_\parallel/\Delta Q_\perp$ can yield information on the angular dependence of the diffuse-scattering coefficient and the anisotropy of the distribution function of a Knudsen gas in (**v**, **M**) space.

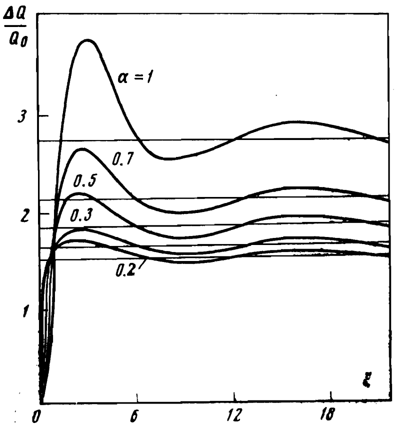

FIG. 1. Plots of $C\Delta Q/Q_0 = f(\xi)$ at different values of α_0; $C = 7/96\sqrt{\pi}(\mu\Psi)^2$

As seen from Fig. 1, the effect should increase with increasing diffuse-scattering coefficient α_0. The amplitude of the oscillations and the value of the parameter ξ at which $\Delta Q/Q_0$ reaches the first maximum are then also increased. Plots of $(\Delta Q/Q_0)_{max}$ and ξ_{max} against α_0 are shown in Fig. 2. This behavior of $(\Delta Q/Q_0)_{max}$ with increasing α_0 is due obviously to the increase of the fraction of the molecules that are anisotropically evaporated from the wall.

5. THERMAL POLARIZATION

In this section we consider the polarization of molecules emitted from a flat slit whose walls have different temperatures ("thermopolarization"). Such a polarization can arise in the absence of a field, owing to anisotropic evaporation of the molecules from the wall. We calculate the average value of the angular momentum of the molecules emitted from the slit in the positive direction of the y axis:

$$\overline{\mathbf{M}}=\frac{1}{n}\int_{v_{y,z}>0}\mathbf{M}(f^++f^-)\,d\mathbf{v}\,d\mathbf{M}. \tag{5.1}$$

It is necessary to substitute in (5.1) the expressions obtained for $f^\pm$ in the preceding section, with allowance for anisotropic scattering by the wall. A nonzero average angular momentum can occur in first approximation in the parameter μ. In the absence of a field, in accordance with (3.10), the functions $f_1^\pm$ are given by

$$f_1^\pm=\frac{2\mu}{(2-\alpha_0)}(v_xM_y-v_yM_x)(f_0-f_L) \tag{5.2}$$

(y is the axis perpendicular to the edge of the slit). Taking (5.2) into account, the integration in (5.1) yields

$$\overline{\mathbf{M}}=\mathbf{i}\overline{M}_x;\qquad \overline{M}_x=-\frac{\mu}{(2-\alpha_0)}M_0\left(\frac{I}{m}\right)^{1/2}(T_0-T_L), \tag{5.3}$$

$M_0 = (3T_0I)^{1/2}$. In the calculations of (5.3) it was assumed that $|T_0 - T_L| \ll T_L$.

It follows from (5.3) that the average angular momentum of the molecules emitted from the slit lies in the plane of the slit and is directed along the x axis. The

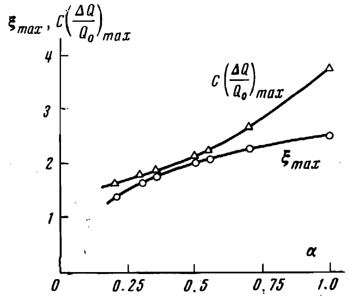

FIG. 2. Plots of $C(\Delta Q/Q)_{0max}$ and of ξ_{max} against α_0: $C = 7/96\sqrt{\pi}(\mu\Psi)^2$

quantity M_x is proportional to the anisotropy parameter μ and to the temperature difference between the slit walls. The very occurrence of vector polarization of the angular momenta of the molecules, as well as the magnitude of the ratio of the changes in the heat flux in a field parallel and perpendicular to the plane of the slit (see (4.11)), are obviously connected with the chosen model (2.5) of anisotropic evaporation of the molecules. If the nonzero terms in the expansions of β and α (2.4) are those with $l_2 \geq 2$, then we get in first approximation in μ not vector but tensor thermopolarization.

The angular dependence of the evaporation and adhesion coefficients on the directions of the velocity and angular momentum of the molecules (see (2.4)) can be deduced from the dependence of the average angular momenta of the molecules on the direction of their emission from the slit at sufficiently large distances from the edge of the slit. If, for example, the interaction of the molecules with the wall is described by the model (2.5), then

$$\begin{aligned} \overline{M}_x &= -\frac{\mu}{2\pi(2-\alpha_0)} M_0 \left(\frac{I}{m}\right)^{1/2} (T_0 - T_L) \sin\vartheta_v \sin\varphi_v, \\ \overline{M}_y &= -\frac{\mu}{2\pi(2-\alpha_0)} M_0 \left(\frac{I}{m}\right)^{1/2} (T_0 - T_L) \sin\vartheta_v \cos\varphi_v, \qquad \overline{M}_z = 0. \end{aligned} \tag{5.4}$$

Indirect proof of the existence of "thermopolarization" of the angular momenta of the molecules is the observed[8] periodic variation of the heat flux in the magnetic field. Both phenomena are due to the same cause, the anisotropic evaporation of the molecules from the wall. Our estimates[8] of μ yield $\mu T(I/m)^{1/2} \sim 0.1$ for the N_2 and NF_3 molecules at room temperature. We can therefore expect $\overline{M}_x$ to amount to 10% of M_0.

---

*[MH] ≡ M × H.

[1)] I.e., the energy flux not connected with the macroscopic motion of the gas.

[1] H. Senftleben, Phys. Zs., **31**, 961, 1930; Ann. der Phys. **7**, 273, 1965.
[2] Yu. Kagan and L. A. Maksimov, Zh. Eksp. Teor. Fiz. **41**, 842 (1961) [Sov. Phys.-JETP **14**, 604 (1962)]; **51**, 1893 (1966) [Sov. Phys.-JETP **24**, 1272 (1967)].
[3] L. L. Gorelik and V. V. Sinitsyn, Zh. Eksp. Teor. Fiz. **46**, 401 (1964) [Sov. Phys.-JETP **19**, 272 (1964)]. ZhETF Pis. Red. **3**, 145 (1966) [JETP Lett. 3, 91 (1966)].
[4] J. J. M. Beenakker and F. R. McCourt, Ann. Rev. Phys. Chem. **21**, 47, 1970.
[5] L. J. F. Hermans, J. M. Koks, A. F. Hengeveld and H. F. P. Knaap, Physica **50**, 410, 1970.
[6] V. D. Borman, L. A. Maksimov, B. I. Nikolaev, and V. I. Troyan, Izv. Akad. Nauk SSSR, MzHG No. 1 (1973).
[7] V. D. Borman, L. A. Maksimov, B. I. Nikolaev, and V. I. Troyan, Dokl. Akad. Nauk SSSR **207**, 1082 (1972) [Sov. Phys.-Doklady **17**, 1170 (1973)].
[8] V. D. Borman, V. S. Laz'ko, and B. I. Nikolaev, Zh. Eksp. Teor. Fiz. **63**, 886 (1972) [Sov. Phys.-JETP **36**, 466 (1973)].
[9] L. Waldmann, Zs. Naturforsch., **22a**, 1678, 1968.
[10] M. N. Kogan, Dinamika razrezhennogo gaza (Dynamics of Rarefied Gas), Nauka, 1967.
[11] S. Chapman and T. G. Cowling, Mathematical Theory of Nonuniform Gases, Cambridge, 1952.
[12] V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, Relyativistskaya kvanotovaya teoriya (Relativistic Quantum Theory), Vol. 1, 1968.

Translated by J. G. Adashko
57